The above expression indicates that during the displacement of oil by waterflood, an **increase in f_w** at any point in the reservoir will cause a proportional decrease in:

- Oil-cut "f_o" and
- Oil mobility**.**

Therefore, the objective is to select the proper injection scheme that could possibly reduce the water fractional flow. This can be achieved by investigating the effect of the injected water viscosity, formation dip angle, and water-injection rate on the water cut. The overall effect of these parameters on the water fractional flow curve are discussed next that includes the impact of:

- Oil and water viscosities
- Water injection rate as related to the formation dip angle

Effect of Water and Oil Viscosities

Figure 14-13 shows the general effect of oil viscosity on the fractional flow curve for both water-wet and oil-wet rock systems. This illustration reveals that regardless of the system wettability, a higher oil viscosity results in an upward shift (an increase) in the fractional flow curve. The apparent effect of the water viscosity on the water fractional flow is clearly indicated by examining

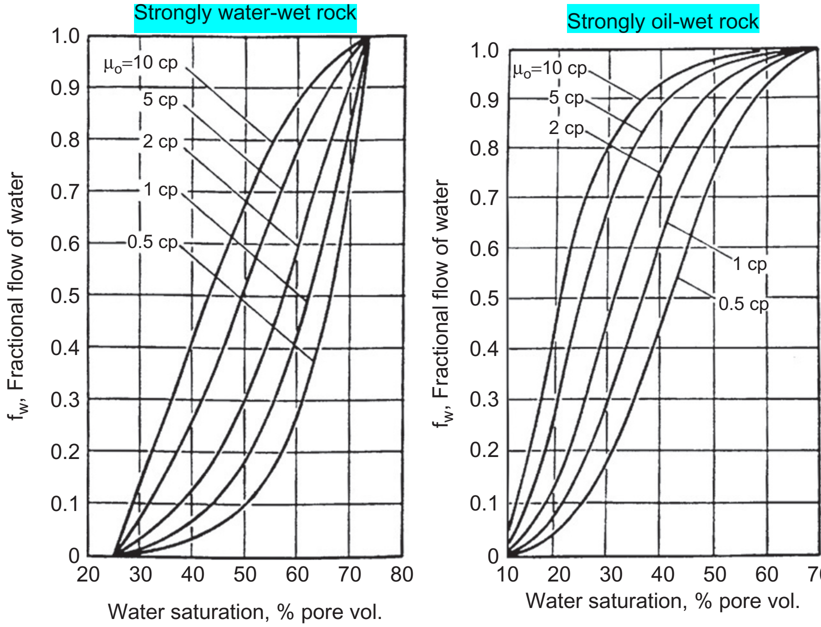

FIGURE 14-13 Effect of oil viscosity on f_w.

Equation 14-22. Higher injected water viscosities will result in an increase in the value of the denominator of Equation 14-22 with an overall reduction in f_w (i.e., a downward shift).

Effect of Dip Angle and Injection Rate

To study the effect of the formation dip angle α and the injection rate on the displacement efficiency, consider the water fractional flow equation as represented by Equation 14-22. Assuming a constant injection rate and realizing that $(\rho_w - \rho_o)$ is always positive and in order to isolate the effect of the dip angle and injection rate on f_w, Equation 14-22 is expressed in the following simplified form:

$$f_w = \frac{1 - \left[X \frac{\sin(\alpha)}{i_w}\right]}{1 + Y} \tag{14-23}$$

where the variables X and Y are a collection of different terms that are all considered positives and given by:

$$X = \frac{(0.001127)(0.433)(kk_{ro})A(\rho_w - \rho_o)}{\mu_o}$$

$$Y = \frac{k_{ro}}{k_{rw}} \frac{\mu_w}{\mu_o}$$

Consider the following two possible cases for the placement of the injection well in a tilted reservoir:

I Case 1: Injection Well is Located Downdip. In this case scenario, the **sin(α)** in Equation (14-23) is treated as positive number and, therefore, the negative sign in the numerator will always remain negative. Figure 14-14 shows that when the injection well located downdip and injected water displaces oil *updip*, a more efficient performance is obtained. This improvement is due to the fact that the term "$\mathbf{X \sin(\alpha)/i_w}$" will always remain positive, which leads to a decrease (downward shift) in the f_w curve. Equation 14-23 also reveals that a *lower water-injection rate* i_w is desirable since the nominator $\mathbf{1 - [X \sin(\alpha)/i_w]}$ of Equation 14-23 will decrease with a lower injection rate i_w, resulting in an overall downward shift in the f_w curve. Case 2: Injection Well is Located Updip. In this case scenario, the **sin(α)** in Equation (14-23) is treated as negative and, therefore, the negative sign in the numerator of Equation (14-23) will change to positive. When the oil is displaced *downdip* (i.e., injection well is located updip), the term $X \sin(\alpha)/i_w$ will always remain negative and, therefore, the numerator of Equation 14-23 will be $1 + [X \sin(\alpha)/i_w]$, i.e.:

$$f_w = \frac{1 + \left[X \frac{\sin(\alpha)}{i_w}\right]}{1 + Y}$$

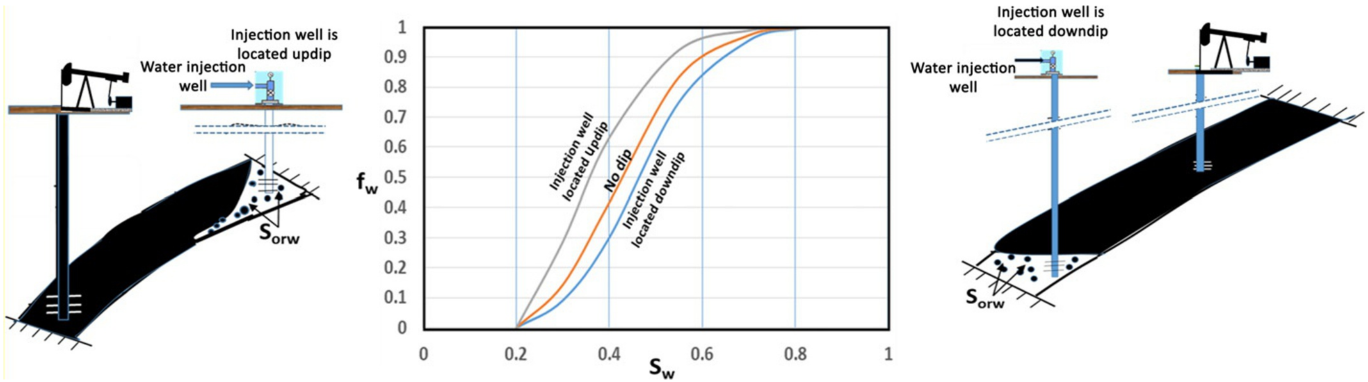

FIGURE 14-14 Impact of injection well location on f_w.

which causes an increase (upward shift) in the f_w curve. It is beneficial, therefore, when injection wells are located at the top of the structure to inject the water at a higher injection rate to improve the displacement efficiency.

It is interesting to reexamine the fractional flow equation when the injection well is located updip and the injection water displacing the oil *downdip*. Combining the product X sin(α) as C, Equation 14-23 can be written:

$$f_w = \frac{1 + \left(\frac{C}{i_w}\right)}{1 + Y}$$

The above expression shows that the possibility exists that the water cut f_w could reach a value greater than unity ($f_w > 1$) if:

$$\frac{C}{i_w} > Y$$

This could only occur when displacing the oil downdip at a low water-injection rate i_w. The resulting effect of this possibility is called a **counterflow**, where the oil phase is moving in a direction opposite to that of the water (i.e., oil is moving upward and the water downward). When the water injection wells are located at the top of a tilted formation, the injection rate must be high to avoid oil migration to the top of the formation.

Notice that for a horizontal reservoir, i.e., sin(α) = 0, the injection rate has no effect on the fractional flow curve. When the dip angle α is zero, Equation 14-22 is reduced to the following simplified form:

$$f_w = \frac{1}{1 + \left(\frac{k_{ro}}{k_{rw}} \frac{\mu_w}{\mu_o}\right)} \tag{14-24}$$

In waterflooding calculations, the reservoir water cut $\mathbf{f_w}$ and the water–oil ratio **WOR** are both traditionally expressed in two different units: bb/bbl and STB/STB, i.e.:

- Q_o = oil flow rate, STB/day
- q_o = oil flow rate, bbl/day
- Q_w = water flow rate, STB/day
- q_w = water flow rate, bbl/day
- WOR_s = surface water–oil ratio, STB/STB
- WOR_r = reservoir water–oil ratio, bbl/bbl
- f_{ws} = surface water cut, STB/STB
- f_w = reservoir water cut, bbl/bbl

The interrelationships that exist between the water-oil ratio "WOR" and water-cut "f_w" are conveniently presented below:

Reservoir f_w and Reservoir WOR_r Relationship:

$$f_w = \frac{q_w}{q_w + q_o} = \frac{\left(\frac{q_w}{q_o}\right)}{\left(\frac{q_w}{q_o}\right) + 1}$$

Substituting for WOR gives:

$$f_w = \frac{WOR_r}{WOR_r + 1} \tag{14-25}$$

Solving for WOR_r gives:

$$WOR_r = \frac{1}{\frac{1}{f_w} - 1} = \frac{f_w}{1 - f_w} \tag{14-26}$$

Reservoir f_w and Surface WOR_s Relationship

By definition:

$$f_w = \frac{q_w}{q_w + q_o} = \frac{Q_w B_w}{Q_w B_w + Q_o B_o} = \frac{\left(\frac{Q_w}{Q_o}\right) B_w}{\left(\frac{Q_w}{Q_o}\right) B_w + B_o}$$

Introducing the surface WOR_s into the above expression gives:

$$f_w = \frac{B_w WOR_s}{B_w WOR_s + B_o} \tag{14-27}$$

Solving for WOR_s yields:

$$WOR_s = \frac{B_o}{B_w\left(\frac{1}{f_w} - 1\right)} = \frac{B_o f_w}{B_w(1 - f_w)} \tag{14-28}$$

Reservoir WOR_r and Surface WOR_s Relationship

From the definition of WOR:

$$WOR_r = \frac{q_w}{q_o} = \frac{Q_w B_w}{Q_o B_o} = \frac{\left(\frac{Q_w}{Q_o}\right) B_w}{B_o}$$

Introducing the surface WOR_s into the above expression gives:

$$WOR_r = (WOR)_s \left(\frac{B_w}{B_o}\right) \tag{14-29}$$

or

$$WOR_s = (WOR)_r \left(\frac{B_o}{B_w}\right)$$

Surface f_{ws} – Surface WOR_s Relationship

$$f_{ws} = \frac{Q_w}{Q_w + Q_o} = \frac{\left(\frac{Q_w}{Q_o}\right)}{\left(\frac{Q_w}{Q_o}\right) + 1}$$

or

$$f_{ws} = \frac{WOR_s}{WOR_s + 1} \tag{14-30}$$

Surface f_{ws} and Reservoir f_w Relationship

$$f_{ws} = \frac{B_o}{B_w\left(\frac{1}{f_w} - 1\right) + B_o} \tag{14-31}$$

Example 14-5

Use the relative permeability as shown in Figure 14-15 to plot the fractional flow curve for a linear reservoir system with the following properties:

Dip angle = 0		Absolute permeability	= 50 md
B_o	= 1.20 bbl/STB	B_w	= 1.05 bbl/STB
ρ_o	= 45 lb/ft^3	ρ_w	= 64.0 lb/ft^3
μ_w	= 0.5 cp	Cross – sectional area A	= 25,000 ft^2

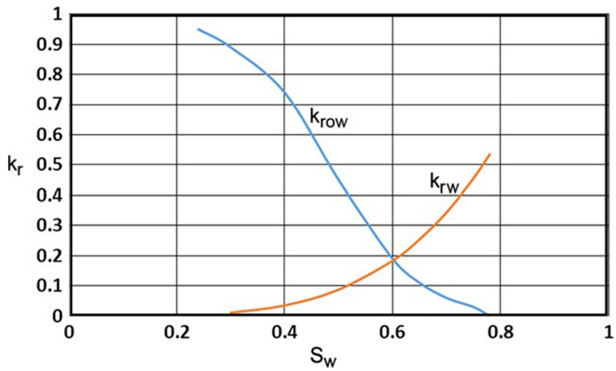

FIGURE 14-15 Relative permeability data for Example 14-5.

Perform the calculations for the following values of oil viscosity: $\mu_0 = 0.5$, 1.0, 5, and 10 cp.

Solution

For a horizontal system, Equation 14-24 can be used to calculate f_w as a function of saturation.

				$f_w = \dfrac{1}{1 + \dfrac{k_{ro}}{k_{rw}}\dfrac{\mu_w}{\mu_o}}$			
S_w	k_{ro}	k_{rw}	k_{ro}/k_{rw}	$\mu_o = 0.5$	$\mu_o = 1.0$	$\mu_o = 5$	$\mu_o = 10$
0.24	0.95	0.00	00	0	0	0	0
0.30	0.89	0.01	89.0	0.011	0.022	0.101	0.183
0.40	0.74	0.04	18.5	0.051	0.098	0.351	0.519
0.50	0.45	0.09	5.0	0.17	0.286	0.667	0.800
0.60	0.19	0.17	1.12	0.47	0.641	0.899	0.947
0.65	0.12	0.28	0.43	0.70	0.823	0.459	0.979
0.70	0.06	0.22	0.27	0.79	0.881	0.974	0.987
0.75	0.03	0.36	0.08	0.93	0.962	0.992	0.996
0.78	0.00	0.41	0	1.00	1.000	1.000	1.000

Results of the above example are documented graphically in Figure 14-16, which shows the apparent effect of oil viscosity on the fractional flow curve.

Example 14-6

The linear system in Example 14-5 is under consideration for a waterflooding project with a water injection rate of 1000 bbl/day. The oil viscosity is considered constant at 1.0 cp. Calculate the fractional flow curve for the reservoir dip angles of 10, 20, and 30°, assuming (a) updip displacement and (b) downdip displacement.

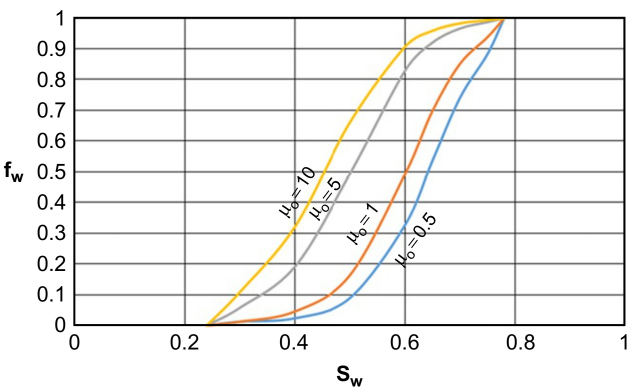

FIGURE 14-16 Impact of oil viscosity on shifting the fractional flow curve.

Solution

Step 1. Calculate the density difference $(\rho_w - \rho_o)$ in g/cm^3:

$$(\rho_w - \rho_o) = (64 - 45)/62.4 = 0.304 g/cm^3$$

Step 2. Simplify Equation 14-22 by using the given fixed data:

$$f_w = \frac{1 - \left(\frac{(0.001127(kk_{ro})A)}{\mu_o i_w}\right)[0.433(\rho_w - \rho_o)\sin(\alpha)]}{1 + \frac{k_{ro}}{k_{rw}}\frac{\mu_w}{\mu_o}}$$

$$f_w = \frac{1 - \frac{0.001127(50k_{ro})(25{,}000)}{(1)(1000)}[0.433(0.304)\sin(\alpha)]}{1 + \left(\frac{0.5}{1}\right)\left(\frac{k_{ro}}{k_{rw}}\right)}$$

$$f_w = \frac{1 - 0.185k_{ro}[\sin(\alpha)]}{1 + 0.5\left(\frac{k_{ro}}{k_{rw}}\right)}$$

Injection well located downdip (updip displacement), **sin(α) is positive,** therefore:

$$f_w = \frac{1 - 0.185k_{ro}\sin(\alpha)}{1 + 0.5\left(\frac{k_{ro}}{k_{rw}}\right)}$$

Injection well located updip (downdip displacement), **sin(α) is negative**, therefore:

$$f_w = \frac{1 + 0.185k_{ro}\sin(\alpha)}{1 + 0.5\left(\frac{k_{ro}}{k_{rw}}\right)}$$

Step 3. Perform the fractional flow calculations in the following tabulated form:

			f_w, Updip Displacement			f_w, Downdip Displacement		
S_w	k_{ro}	k_{ro}/k_{rw}	10°	20°	30°	10°	20°	30°
0.24	0.95	00	0	0	0	0	0	0
0.30	0.89	89	0.021	0.021	0.020	0.023	0.023	0.024
0.40	0.74	18.5	0.095	0.093	0.091	0.100	0.102	0.104
0.50	0.45	5.0	0.282	0.278	0.274	0.290	0.294	0.298
0.60	0.19	1.12	0.637	0.633	0.630	0.645	0.649	0.652
0.65	0.12	0.43	0.820	0.817	0.814	0.826	0.830	0.832
0.70	0.06	0.27	0.879	0.878	0.876	0.883	0.884	0.886
0.75	0.03	0.08	0.961	0.960	0.959	0.962	0.963	0.964
0.78	0.00	0	1.000	1.000	1.000	1.000	1.000	1.000