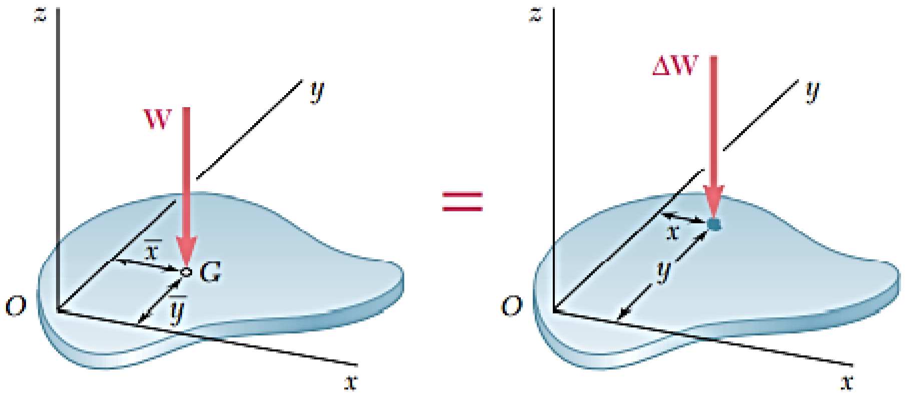
Al-Ayen University
Petroleum Engineering College

Mechanics

Dr. Mohaimen Al-Thamir

Title: centers of gravity; not curved shapes **Lec. Number: 4**

Center of gravity of at two dimensions: Areas



$\Sigma M_y: \quad \bar{x}W = \Sigma x\, \Delta W$

$\Sigma M_x: \quad \bar{y}W = \Sigma y\, \Delta W$

Let us first consider a flat horizontal plate in the figure shown left. We can divide the plate into n small elements. The coordinates of the first element are denoted by x1 and y1, those of the second element by x2 and y2, etc. The forces exerted by the earth on the elements of plate will be denoted, respectively, by ΔW1, ΔW2, . . . , ΔWn. These forces or weights are directed toward the center of the earth; however, for all practical purposes they can be assumed to be parallel. Their resultant is therefore a single force in the same direction. The magnitude W of this force is obtained by adding the magnitudes of the elemental weights

Force equation

$$\Sigma F_z: \quad W = \Delta W_1 + \Delta W_2 + \cdots + \Delta W_n$$

To obtain the coordinates x and y of the point G where the resultant W should be applied, we write that the moments of W about the y and x axes are equal to the sum of the corresponding moments of the elemental weights,

Momentum equation

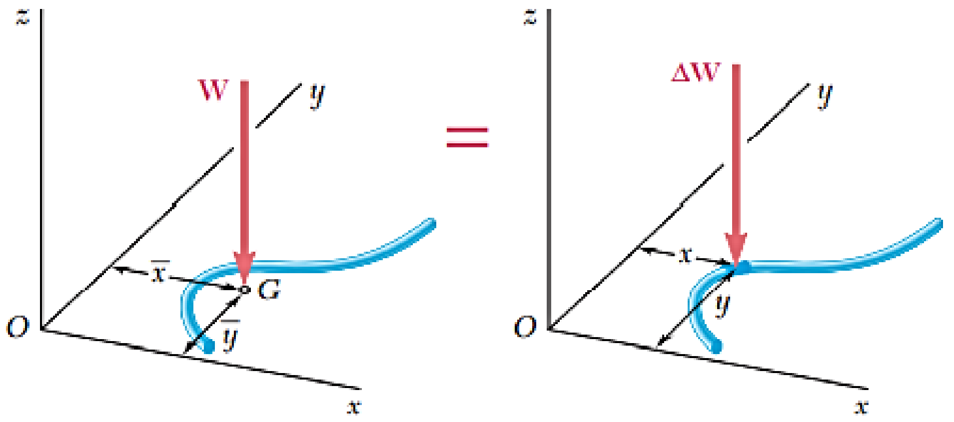
$$\Sigma M_y: \quad \bar{x}W = x_1\,\Delta W_1 + x_2\,\Delta W_2 + \cdots + x_n\,\Delta W_n$$
$$\Sigma M_x: \quad \bar{y}W = y_1\,\Delta W_1 + y_2\,\Delta W_2 + \cdots + y_n\,\Delta W_n$$

Or

$$W = \int dW \qquad \bar{x}W = \int x\,dW \qquad \bar{y}W = \int y\,dW$$

Center of gravity of at two dimensions: Lines

The same equations in the previous slide (centroid of areas) can be derived for a wire lying in the xy plane.
We note that the center of gravity G of a wire is usually not located on the wire.



$$\Sigma M_y: \quad \bar{x}\,W = \Sigma x\,\Delta W$$

$$\Sigma M_x: \quad \bar{y}\,W = \Sigma y\,\Delta W$$

In the case of a flat homogeneous plate of uniform thickness, the magnitude ΔW of the weight of an element of the plate can be expressed as

$$\Delta W = \gamma t \, \Delta A$$

where γ = specific weight (weight per unit volume) of the material
t = thickness of the plate
ΔA = area of the element

Similarly, we can express the magnitude W of the weight of the entire plate as

$$W = \gamma t A$$

where A is the total area of the plate.

Substituting for DW and W in the moment equations and dividing throughout by γt, we obtain

$$\Sigma M_y: \quad \bar{x}A = x_1 \, \Delta A_1 + x_2 \, \Delta A_2 + \cdots + x_n \, \Delta A_n$$
$$\Sigma M_x: \quad \bar{y}A = y_1 \, \Delta A_1 + y_2 \, \Delta A_2 + \cdots + y_n \, \Delta A_n$$

→

$$\Sigma M_y: \quad \bar{x}\, A = \Sigma x \, \Delta A$$
$$\Sigma M_x: \quad \bar{y}\, A = \Sigma y \, \Delta A$$

In the case of a homogeneous wire of uniform cross section, the magnitude ΔW of the weight of an element of wire can be expressed as

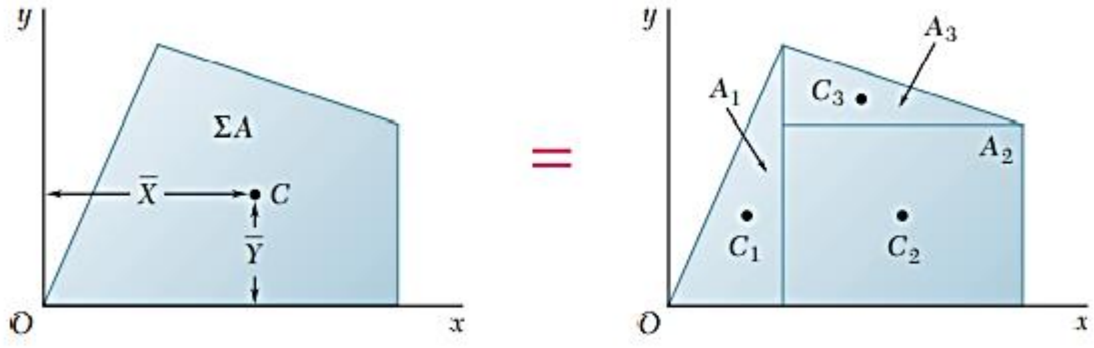
$$\Delta W = \gamma a \, \Delta L$$

where γ = specific weight of the material
a = cross-sectional area of the wire
ΔL = length of the element

→

$$\Sigma M_y: \quad \bar{x} L = \Sigma x \, \Delta L$$
$$\Sigma M_x: \quad \bar{y} L = \Sigma y \, \Delta L$$

Centroid of a composite area



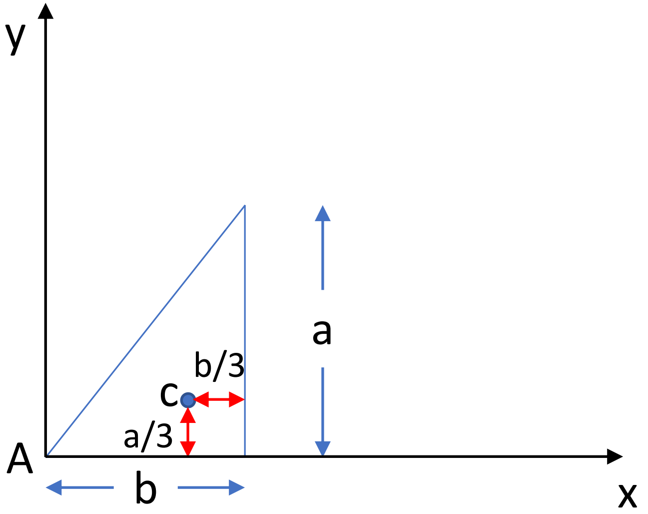
The abscissa $\overline{X}$ of the centroid of the area can be determined by noting that the first moment Q_y of the composite area with respect to the y axis can be expressed both as the product of X and the total area and as the sum of the first moments of the elementary areas with respect to the y axis. The ordinate Y of the centroid is found in a similar way by considering the first moment Q_x of the composite area. We have

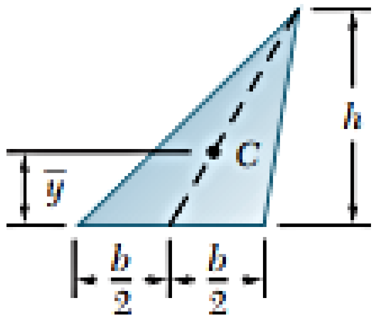
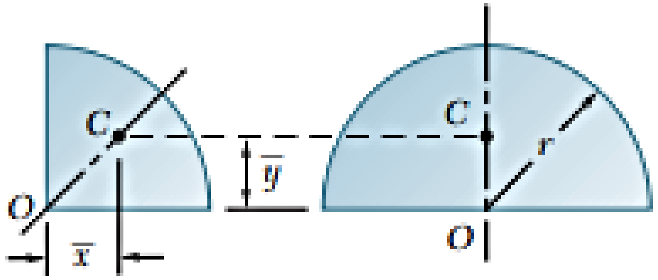
$$Q_y = \overline{X}\Sigma A = \Sigma \overline{x} A \qquad Q_x = \overline{Y}\Sigma A = \Sigma \overline{y} A$$

If the plate is homogeneous and of uniform thickness, the center of gravity coincides with the centroid C of its area. This also true for lines.

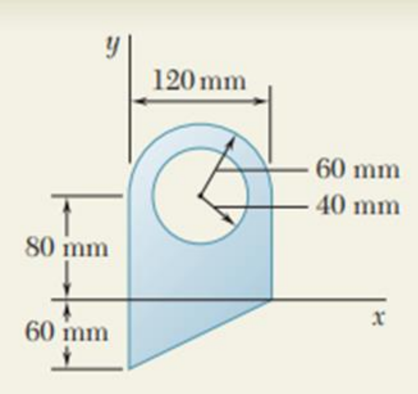
Note: The case of composite lines will be explained later in a solved example.

Centroids of some shapes of areas



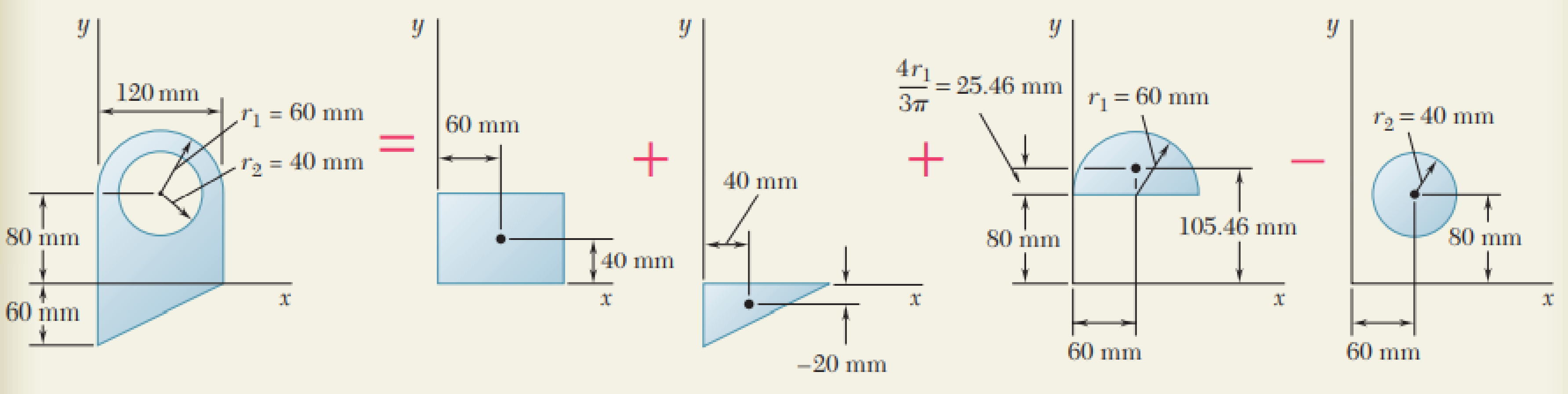
Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$

Example 1

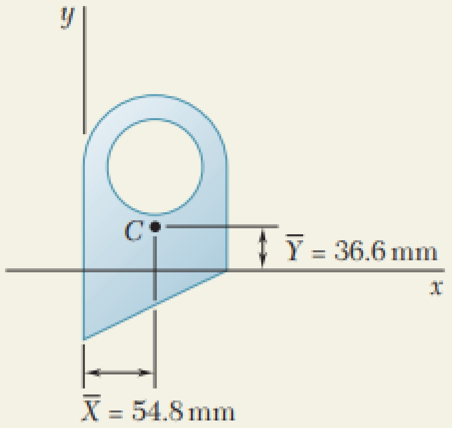


For the plane area shown, determine (a) the first moments with respect to the x and y axes, (b) the location of the centroid.

Solution



Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$



a. First Moments of the Area.

$$Q_x = \Sigma\bar{y}A = 506.2 \times 10^3 \text{ mm}^3 \qquad Q_x = 506 \times 10^3 \text{ mm}^3$$

$$Q_y = \Sigma\bar{x}A = 757.7 \times 10^3 \text{ mm}^3 \qquad Q_y = 758 \times 10^3 \text{ mm}^3$$

b. Location of Centroid. Substituting the values given in the table into the equations defining the centroid of a composite area, we obtain

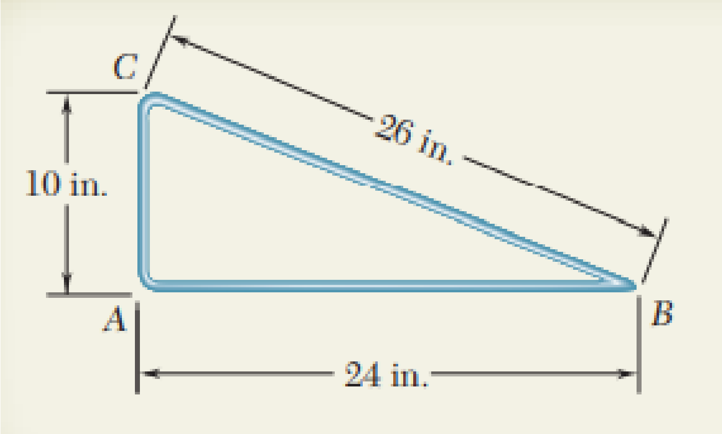
$$\bar{X}\Sigma A = \Sigma\bar{x}A: \qquad \bar{X}(13.828 \times 10^3 \text{ mm}^2) = 757.7 \times 10^3 \text{ mm}^3$$

$$\bar{X} = 54.8 \text{ mm}$$

$$\bar{Y}\Sigma A = \Sigma\bar{y}A: \qquad \bar{Y}(13.828 \times 10^3 \text{ mm}^2) = 506.2 \times 10^3 \text{ mm}^3$$

$$\bar{Y} = 36.6 \text{ mm}$$

Example 2



The figure shown is made from a piece of thin, homogeneous wire. Determine the location of its center of gravity.

Solution

Since the figure is formed of homogeneous wire, its center of gravity coincides with the centroid of the corresponding line. Therefore, that centroid will be determined. Choosing the coordinate axes shown, with origin at *A*, we determine the coordinates of the centroid of each line segment and compute the first moments with respect to the coordinate axes.

Segment	L, in.	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}L$, in^2	$\bar{y}L$, in^2
AB	24	12	0	288	0
BC	26	12	5	312	130
CA	10	0	5	0	50
	$\Sigma L = 60$			$\Sigma \bar{x}L = 600$	$\Sigma \bar{y}L = 180$

Substituting the values obtained from the table into the equations defining the centroid of a composite line, we obtain

$\bar{X}\Sigma L = \Sigma \bar{x}L$: $\bar{X}(60 \text{ in.}) = 600 \text{ in}^2$ $\quad \bar{X} = 10 \text{ in.}$ ◀

$\bar{Y}\Sigma L = \Sigma \bar{y}L$: $\bar{Y}(60 \text{ in.}) = 180 \text{ in}^2$ $\quad \bar{Y} = 3 \text{ in.}$ ◀