$$S_g = \frac{\displaystyle\sum_{i=1}^{n} \phi_i\, h_i\, S_{gi}}{\displaystyle\sum_{i=1}^{n} \phi_i\, h_i} \tag{4-18}$$

where the subscript i refers to any individual measurement and h_i represents the depth interval to which ϕ_i, S_{oi}, S_{gi}, and S_{wi} apply.

Example 4-3

Calculate average oil and connate water saturation from the following measurements:

Sample	h_i, ft	ϕ, %	S_o, %	S_{wc}, %
1	1.0	10	75	25
2	1.5	12	77	23
3	1.0	11	79	21
4	2.0	13	74	26
5	2.1	14	78	22
6	1.1	10	75	25

Solution

Construct the following table and calculate the average saturation for the oil and water phase:

Sample	h_i, ft	ϕ	ϕh	S_o	$S_o\phi h$	S_{wc}	$S_{wc}\phi h$
1	1.0	.10	.100	.75	.0750	.25	.0250
2	1.5	.12	.180	.77	.1386	.23	.0414
3	1.0	.11	.110	.79	.0869	.21	.0231
4	2.0	.13	.260	.74	.1924	.26	.0676
5	2.1	.14	.294	.78	.2293	.22	.0647
6	1.1	.10	.110	.75	.0825	.25	.0275
			1.054		0.8047		0.2493

Calculate average oil saturation by applying Equation 4-16:

$$S_o = \frac{.8047}{1.054} = 0.7635$$

Calculate average water saturation by applying Equation 4-17:

$$S_w = \frac{0.2493}{1.054} = 0.2365$$

WETTABILITY

Wettability is defined as the tendency of one fluid to spread on or adhere to a solid surface in the presence of other immiscible fluids. The concept of wettability is illustrated in Figure 4-1. Small *drops* of three liquids—mercury, oil, and
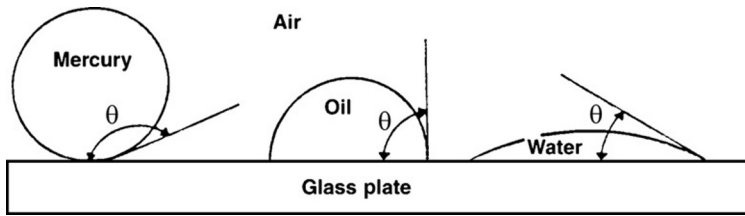
FIGURE 4-1 Illustration of wettability.

water—are placed on a clean glass plate. The three droplets are then observed from one side as illustrated in Figure 4-1. It is noted that the mercury retains a spherical shape, the oil droplet develops an approximately hemispherical shape, but the water tends to spread over the glass surface.

The tendency of a liquid to spread over the surface of a solid is an indication of the *wetting* characteristics of the liquid for the solid. This spreading tendency can be expressed more conveniently by measuring the angle of contact at the *liquid-solid* surface. This angle, which is always measured through the liquid to the solid, is called the contact angle θ.

The contact angle θ has achieved significance as a measure of wettability. As shown in Figure 4-1, as the contact angle decreases, the wet characteristics of the liquid increase. Complete wettability would be evidenced by a zero contact angle, and complete nonwetting would be evidenced by a contact angle of 180°. There have been various definitions of *intermediate* wettability but, in much of the published literature, contact angles of 60° to 90° will tend to repel the liquid.

The wettability of reservoir rocks to the fluids is important in that the distribution of the fluids in the porous media is a function of wettability. Because of the attractive forces, the wetting phase tends to occupy the smaller pores of the rock and the nonwetting phase occupies the more open channels.

SURFACE AND INTERFACIAL TENSION

In dealing with multiphase systems, it is necessary to consider the effect of the forces at the interface when two immiscible fluids are in contact. When these two fluids are liquid and gas, the term *surface tension* is used to describe the forces acting on the interface. When the interface is between two liquids, the acting forces are called *interfacial tension*.

Surfaces of liquids are usually blanketed with what acts as a thin film. Although this apparent film possesses little strength, it nevertheless acts like a thin membrane and resists being broken. This is believed to be caused by attraction between molecules within a given system. All molecules are attracted one to the other in proportion to the product of their masses and inversely as the squares of the distance between them.

Consider the two immiscible fluids, air (or gas) and water (or oil) as shown schematically in Figure 4-2. A liquid molecule, which is remote from the
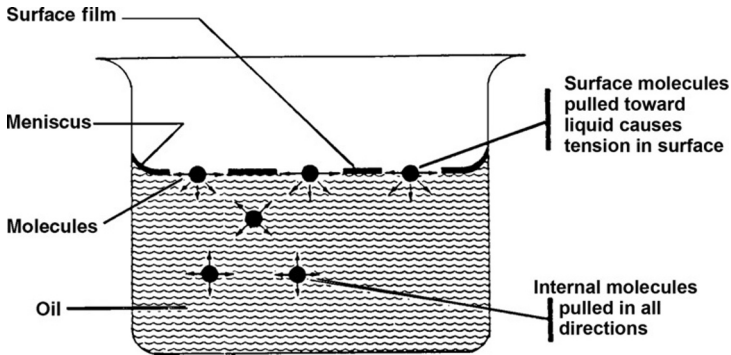
FIGURE 4-2 Illustration of surface tension. *(After Clark, N. J., Elements of Petroleum Reservoirs, SPE, 1969).*

interface, is surrounded by other liquid molecules, thus having a resulting net attractive force on the molecule of zero. A molecule at the interface, however, has a force acting on it from the air (gas) molecules lying immediately above the interface and from liquid molecules lying below the interface.

Resulting forces are unbalanced and give rise to surface tension. The unbalanced attraction force between the molecules creates a membrane-like surface with a measurable tension, i.e., surface tension. As a matter of fact, if carefully placed, a needle will float on the surface of the liquid, supported by the thin membrane even though it is considerably more dense than the liquid.

The surface or interfacial tension has the units of force per unit of length, e.g., dynes/cm, and is usually denoted by the symbol σ.

If a glass capillary tube is placed in a large open vessel containing water, the combination of surface tension and wettability of tube to water will cause water to rise in the tube above the water level in the container outside the tube as shown in Figure 4-3.

The water will rise in the tube until the total force acting to pull the liquid upward is balanced by the weight of the column of liquid being supported in the tube. Assuming the radius of the capillary tube is r, the total upward force F_{up}, which holds the liquid up, is equal to the force per unit length of surface times the total length of surface, or

$$F_{up} = (2\pi r)(\sigma_{gw})(\cos \theta) \qquad (4\text{-}19)$$

where

σ_{gw} = surface tension between air (gas) and water (oil), dynes/cm
θ = contact angle
r = radius, cm

The upward force is counteracted by the weight of the water, which is equivalent to a downward force of mass times acceleration, or
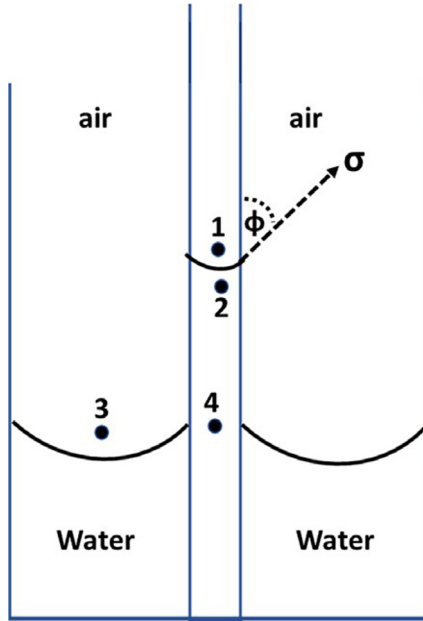
FIGURE 4-3   Pressure relations in capillary tubes.

$$F_{down} = \pi r^2 h(\rho_w - \rho_{air})g \qquad (4\text{-}20)$$

where

h = height to which the liquid is held, cm
g = acceleration due to gravity, cm/sec^2
ρ_w = density of water, gm/cm^3
ρ_{air} = density of gas, gm/cm^3

Because the density of air is negligible in comparison with the density of water, Equation 4-20 is reduced to:

$$F_{down} = \pi r^2 \rho_w g \qquad (4\text{-}21)$$

Equating Equation 4-19 with 4-21 and solving for the surface tension gives:

$$\sigma_{gw} = \frac{r h \rho_w g}{2 \cos \theta} \qquad (4\text{-}22)$$

The generality of Equations 4-19 through 4-22 will not be lost by applying them to the behavior of two liquids, i.e., water and oil. Because the density of oil is not negligible, Equation 4-22 becomes

$$\sigma_{ow} = \frac{r h g (\rho_w - \rho_o)}{2 \cos \theta} \qquad (4\text{-}23)$$

where

ρ_o = density of oil, gm/cm^3
σ_{ow} = interfacial tension between the oil and the water, dynes/cm

The interfacial tension varies between 72 dynes/cm for water-gas system and 20–30 dynes/cm for water-oil system. Ramey (1973) proposed a graphical correlation for estimating water-hydrocarbon interfacial tension that was curve-fit by Whitson-Brule (2000) by the following expressions:

$$\sigma_{ow} = 20 + 0.57692 \, (\rho_w - \rho_o)$$

where

σ_{ow} = interfacial tension, dynes/cm
ρ_w = density of the water phase, lb/ft^3
ρ_o = density of the oil, lb/ft^3

The surface tension between the gas and crude oil depends on pressure, temperature, composition of the oil and gas, gas-solubility, among others. The surface tension can be roughly approximated by the following expression:

$$\sigma_{go} = 60 - 0.036 \, T - 0.0104 \, P + 1.5 \left(10^{-6}\right) p^2$$

where

σ_{go} = surface tension between gas and oil, dynes/cm
T = Temperature, °R
P = Pressure, psia

CAPILLARY PRESSURE

The capillary forces in a petroleum reservoir are the result of the combined effect of the surface and interfacial tensions of the rock and fluids, the pore size and geometry, and the wetting characteristics of the system. Any curved surface between two immiscible fluids has the tendency to contract into the smallest possible area per unit volume. This is true whether the fluids are oil and water, water and gas (even air), or oil and gas. When two immiscible fluids are in contact, a discontinuity in pressure exists between the two fluids, which depends upon the curvature of the interface separating the fluids. We call this pressure difference the *capillary pressure* and it is referred to by p_c.

The displacement of one fluid by another in the pores of a porous medium is either aided or opposed by the surface forces of capillary pressure. As a consequence, in order to maintain a porous medium partially saturated with nonwetting fluid and while the medium is also exposed to wetting fluid, it is necessary to maintain the pressure of the nonwetting fluid at a value greater than that in the wetting fluid.

Denoting the pressure in the wetting fluid by p_w and that in the nonwetting fluid by p_{nw}, the capillary pressure can be expressed as:

$$\text{Capillary pressure} = (\text{pressure of the nonwetting phase})$$
$$- (\text{pressure of the wetting phase})$$

$$p_c = p_{nw} - p_w \tag{4-24}$$

That is, the pressure excess in the nonwetting fluid is the capillary pressure, and this quantity is a function of saturation. This is the defining equation for capillary pressure in a porous medium.

There are three types of capillary pressure:

○ Water-oil capillary pressure (denoted as P_{cwo})
○ Gas-oil capillary pressure (denoted as P_{cgo})
○ Gas-water capillary pressure (denoted as P_{cgw})

Applying the mathematical definition of the capillary pressure as expressed by Equation 4-24, the three types of the capillary pressure can be written as:

$$P_{cwo} = P_o - P_w$$

$$P_{cgo} = P_g - P_o$$

$$P_{cgw} = P_g - P_w$$

where p_g, p_o, and p_w represent the pressure of gas, oil, and water, respectively. If all the three phases are continuous, then:

$$P_{cgw} = P_{cgo} + P_{cwo}$$

Referring to Figure 4-3, the pressure difference across the interface between Points 1 and 2 is essentially the capillary pressure, i.e.:

$$p_c = p_1 - p_2 \tag{4-25}$$

The pressure of the water phase at Point 2 is equal to the pressure at point 4 minus the head of the water, or:

$$p_2 = p_4 - gh\rho_w \tag{4-26}$$

The pressure just above the interface at Point 1 represents the pressure of the air and is given by:

$$p_1 = p_3 - gh\rho_{air} \tag{4-27}$$

It should be noted that the pressure at Point 4 within the capillary tube is the same as that at Point 3 outside the tube. Subtracting Equation 4-26 from 4-27 gives:

$$p_c = gh(\rho_w - \rho_{air}) = gh\Delta\rho \tag{4-28}$$

where $\Delta\rho$ is the density difference between the wetting and nonwetting phase. The density of the air (gas) is negligible in comparison with the water density.

In practical units, Equation 4-28 can be expressed as:

$$p_c = \left(\frac{h}{144}\right)\Delta\rho$$

where

p_c = capillary pressure, psi
h = capillary rise, ft
$\Delta\rho$ = density difference, lb/ft^3

In the case of an oil-water system, Equation 4-28 can be written as:

$$p_c = gh(\rho_w - \rho_o) = gh\Delta\rho \qquad (4\text{-}29)$$

and in practical units

$$p_c = \left(\frac{h}{144}\right)(\rho_w - \rho_o)$$

The capillary pressure equation can be expressed in terms of the surface and interfacial tension by combining Equations 4-28 and 4-29 with Equations 4-22 and 4-23 to give:

- **Gas-liquid system**

$$p_c = \frac{2\sigma_{gw}(\cos\theta)}{r} \qquad (4\text{-}30)$$

and

$$h = \frac{2\sigma_{gw}(\cos\theta)}{rg\left(\rho_w - \rho_{gas}\right)} \qquad (4\text{-}31)$$

where

ρ_w = water density, gm/cm^3
σ_{gw} = gas-water surface tension, dynes/cm
r = capillary radius, cm
θ = contact angle
h = capillary rise, cm
g = acceleration due to gravity, cm/sec^2
p_c = capillary pressure, dynes/cm^2

- **Oil-water system**

$$p_c = \frac{2\sigma_{ow}(\cos\theta)}{r} \qquad (4\text{-}32)$$

and

$$h = \frac{2\sigma_{wo}(\cos\theta)}{rg(\rho_w - \rho_o)} \qquad (4\text{-}33)$$

where σ_{wo} is the water-oil interfacial tension.