Al-Ayen University
College of Petroleum Engineering

Reservoir Engineering II

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Lecture 12: Pseudo-Steady State Flow of Reservoir Fluids (Part 2)

Ref.: Reservoir Engineering Handbook by Tarek Ahmed

Outline

- Pseudo-Steady State of Flow Radial Flow for Slightly Compressible Fluids
  - **Shapes of the Well Drainage Area**
  - **Example**
- Pseudo-Steady State of Radial Flow for Compressible Fluids (Gases)
  - **Pressure-Squared Approximation Method**
  - **Pressure- Approximation Method**
- Summary
- Discussion

Semi (Pseudo)-Steady State of Radial Flow for Slightly Compressible Fluids

Shapes of the Well Drainage Area

- It should be pointed out that the pseudosteady-state flow occurs regardless of the geometry of the reservoir.
- Ramey and Cobb (1971) introduced a correction factor that is called the ***shape factor***, *C_A*, *which is designed to account for the deviation of the drainage area from the ideal circular form*.
- In terms of the volumetric average pressure, $\bar{P}_r$:

$$p_{wf} = \bar{p}_r - \frac{162.6 Q B \mu}{kh} \log\left[\frac{4A}{1.781 C_A r_w^2}\right]$$

- In terms of the initial reservoir pressure, P_i :

$$\bar{p}_r = p_i - \frac{0.23396 q t}{c_t A h \phi}$$

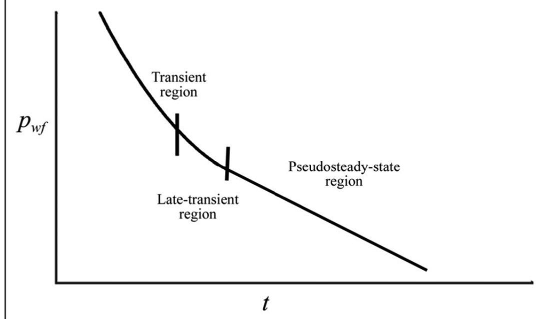
$$p_{wf} = \left[p_i - \frac{0.23396 Q B t}{A h \phi c_t}\right] - \frac{162.6 Q B \mu}{kh} \log\left[\frac{4A}{1.781 C_A r_w^2}\right]$$

where k = permeability, mD
A = drainage area, ft^2
C_A = shape factor
Q = flow rate, STB/day
t = time, hr
c_t = total compressibility coefficient, psi^{-1}

Shape Factors for Various Single-Well Drainage Areas

In Bounded Reservoirs	C_A	$\ln C_A$	$\frac{1}{2}\ln\left(\frac{2.2458}{C_A}\right)$	Exact for t_{DA} >	Less than 1% Error For t_{DA} >	Use Infinite System Solution with Less Than 1% Error for t_{DA} <
	31.62	3.4538	−1.3224	0.1	0.06	0.10
	31.6	3.4532	−1.3220	0.1	0.06	0.10
	27.6	3.3178	−1.2544	0.2	0.07	0.09
60°	27.1	3.2995	−1.2452	0.2	0.07	0.09
1/3	21.9	3.0865	−1.1387	0.4	0.12	0.08
3 4	0.098	−2.3227	+1.5659	0.9	0.60	0.015
	30.8828	3.4302	−1.3106	0.1	0.05	0.09
	12.9851	2.5638	−0.8774	0.7	0.25	0.03
	4.5132	1.5070	−0.3490	0.6	0.30	0.025
	3.3351	1.2045	−0.1977	0.7	0.25	0.01
1, 2	21.8369	3.0836	−1.1373	0.3	0.15	0.025
1, 2	10.8374	2.3830	−0.7870	0.4	0.15	0.025
1, 2	4.5141	1.5072	−0.3491	1.5	0.50	0.06
1, 2	2.0769	0.7309	−0.0391	1.7	0.50	0.02
1, 2	3.1573	1.1497	−0.1703	0.4	0.15	0.005

In Bounded Reservoirs	C_A	$\ln C_A$	$\frac{1}{2}\ln\left(\frac{2.2458}{C_A}\right)$	Exact for t_{DA} >	Less than 1% Error For t_{DA} >	Use Infinite System Solution with Less than 1% Error for t_{DA} <
1, 2	0.5813	−0.5425	+0.6758	2.0	0.60	0.02
1, 2	0.1109	−2.1991	+1.5041	3.0	0.60	0.005
1, 4	5.3790	1.6825	−0.4367	0.8	0.30	0.01
1, 4	2.6896	0.9894	−0.0902	0.8	0.30	0.01
1, 4	0.2318	−1.4619	+1.1355	4.0	2.00	0.03
1, 4	0.1155	−2.1585	+1.4838	4.0	2.00	0.01
1, 5	2.3606	0.8589	−0.0249	1.0	0.40	0.025
IN VERTICALLY FRACTURED RESERVOIRS	Use $(x_e/x_f)^2$ in place of A/r_w^2 for fractured systems					
0.1, x_f/x_e	2.6541	0.9761	−0.0835	0.175	0.08	cannot use
0.2	2.0348	0.7104	+0.0493	0.175	0.09	cannot use
0.3	1.9986	0.6924	+0.0583	0.175	0.09	cannot use
0.5	1.6620	0.5080	+0.1505	0.175	0.09	cannot use
0.7	1.3127	0.2721	+0.2685	0.175	0.09	cannot use
1.0	0.7887	−0.2374	+0.5232	0.175	0.09	cannot use
IN WATER-DRIVE RESERVOIRS	19.1	2.95	−1.07	—	—	—
IN RESERVOIRS OF UNKNOWN PRODUCTION CHARACTER	25.0	3.22	−1.20	—	—	—



Flow Regimes

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Example

An oil well is developed on the center of a 40-acre square-drilling pattern. The well is producing at a constant flow rate of 800 STB/day under a semisteady-state condition. The reservoir has the following properties:

$\phi = 15\%$	$h = 30$ ft	$k = 200$ md
$\mu = 1.5$ cp	$B_o = 1.2$ bbl/STB	$c_t = 25 \times 10^{-6}$ psi-1
$p_i = 4500$ psi	$r_w = 0.25$ ft	$A = 40$ acres

a. Calculate and plot the bottom-hole flowing pressure as a function of time.

b. Based on the plot, calculate the pressure decline rate. What is the decline in the average reservoir pressure from t = 10 to t = 200 hr?

Solution

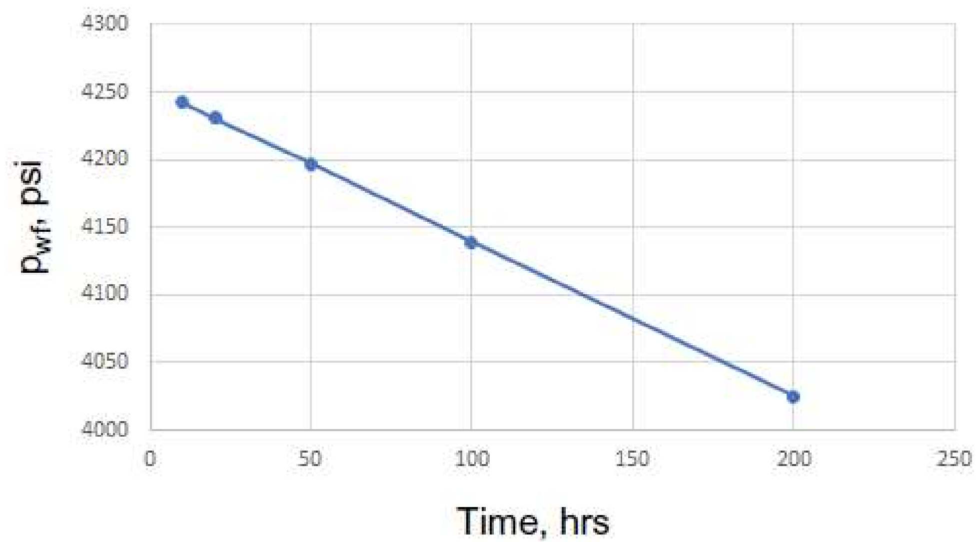
(a) From Table: $C_A = 30.8828$

$A = (40)(43{,}560) = 1{,}742{,}400\ ft^2$

$$p_{wf} = \left[p_i - \left(\frac{0.23396\,QB}{Ah\phi c_t}\right)t\right] - \frac{162.6\ Q\ B\mu}{kh}\log\left[\frac{4A}{1.781C_A r_w^2}\right]$$

$$p_{wf} = 4253.878 - 1.14581\,t$$

t, hours	Pwf, psia
10	4242.42
20	4230.961
50	4196.587
100	4139.297
200	4024.716



From the Pwf Eq.: $\frac{dp}{dt} = -1.14581$ psi/hr

$$\Delta \overline{p}_r = (1.14581)(200 - 10) = 217.704 \text{ psi}$$

Pseudo-Steady State of Radial Flow for Compressible Fluids (Gases)

$$\frac{\partial^2 m(p)}{\partial r^2} + \frac{1}{r}\frac{\partial m(p)}{\partial r} = \frac{\phi \mu c_t}{0.000264\,k}\frac{\partial m(p)}{\partial t}$$

Radial diffusivity equation of compressible fluid under unsteady-state conditions

For the semisteady-state flow: $\frac{\partial m(p)}{\partial t} = \text{constant}$

Using the same technique used for the slightly compressible fluid, gives the following exact solution to the diffusivity equation:

$$Q_g = \frac{kh[m(\bar{p}_r) - m(p_{wf})]}{1422\,T\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]}$$

where Qg = gas flow rate, Mscf/day
T = temperature, °R
k = permeability, md

Two approximations to the above solution are widely used. These approximations are:

- Pressure-squared approximation
- Pressure-approximation

Pressure-Squared Approximation Method

This method provides us with compatible results to that of the exact solution approach when $p < 2000$. The solution has the following familiar form:

$$Q_g = \frac{kh(\bar{p}_r^2 - p_{wf}^2)}{1422\,T\bar{\mu}\,\bar{z}\left(\ln\frac{r_e}{r_w} - 0.75\right)}$$

The average gas properties z and μ are evaluated at: $\bar{p} = \sqrt{\frac{(\bar{p}_r)^2 + p_{wf}^2}{2}}$

Pressure-Approximation Method

This approximation method is applicable at $p > 3000$ psi and has the following mathematical form:

$$Q_g = \frac{kh(\bar{p}_r - p_{wf})}{1422\,\bar{\mu}\,\bar{B}_g\left(\ln\frac{r_e}{r_w} - 0.75\right)}$$

with the gas properties evaluated at:

$$\bar{p} = \frac{\bar{p}_r + p_{wf}}{2}$$

$$\bar{B}_g = 0.00504\,\frac{\bar{z}\,T}{\bar{p}}$$

where Qg = gas flow rate, Mscf/day
k = permeability, md
Bg = gas formation volume factor at average pressure, bbl/scf

Summary

- The pseudosteady-state flow occurs regardless of the geometry of the reservoir.
- The shape factor, C_A, is designed to account for the deviation of the drainage area from the ideal circular form
- For radial flow of gases under the pseudo-steady state, there are three forms of the mathematical solution to the diffusivity equation:
  - ➢ The m(p)-Solution Method (Exact Solution)
  - ➢ The Pressure-Squared Method (p^2 - Approximation Method)
  - ➢ The Pressure Method (p - Approximation Method)

Discussion

From the table of shape factor (C_A), investigate how the value of C_A changes with the boundary shape and well location. Discuss influence of these changes in C_A values on the pressure drop in a reservoir with a single well produces at a constant flow rate.

THANK YOU