Al-Ayen University
College of Petroleum Engineering

Reservoir Engineering II

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Lecture 5: Steady-State Flow of Reservoir Fluids (Part 1)

Ref.: ***Reservoir Engineering Handbook*** by **Tarek Ahmed**

Outlines

- **Darcy's Law**
- **Steady-State Flow**
  - Horizontal Linear Flow of Incompressible Fluids
  - Inclined Flow of Incompressible Fluids
  - Linear Flow of Compressible Fluids (Gases)
  - Radial Flow of Incompressible Fluids

Darcy's Law

- For a horizontal linear system $$\nu = \frac{q}{A} = -\frac{k}{\mu}\frac{dp}{dx}$$

- For a horizontal-radial system, the pressure gradient is positive:

$$\nu = \frac{q_r}{A_r} = \frac{k}{\mu}\left(\frac{\partial p}{\partial r}\right)_r$$

$$A_r = 2\pi rh$$

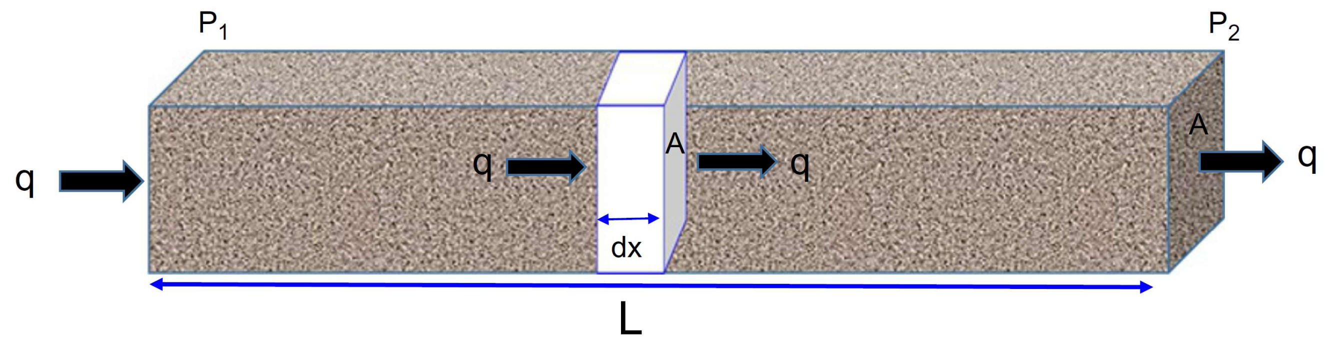
Darcy's Law applies only when the following conditions exist:

- Laminar (viscous) flow
- Steady-state flow
- Homogeneous formation
- No fluid-rock reaction

Steady-State Flow

Horizontal Linear Flow of Incompressible Fluids

$$\nu = \frac{q}{A} = -\frac{k}{\mu}\frac{dp}{dx}$$



$$\frac{q}{A}\int_0^L dx = -\frac{k}{\mu}\int_{p_1}^{p_2} dp$$

$$q = \frac{kA(p_1 - p_2)}{\mu L}$$

$$q = \frac{0.001127\, kA(p_1 - p_2)}{\mu L}$$

q = flow rate, bbl/day
k = absolute permeability, md
p = pressure, psia
μ = viscosity, cp
L = distance, ft
A = cross-sectional area, ft^2

Example 1

An incompressible fluid flows in a linear porous media with the following properties:

$$L = 2000\,\text{ft} \qquad h = 20' \qquad \text{width} = 300'$$
$$k = 100\,\text{md} \qquad \phi = 15\% \qquad \mu = 2\,\text{cp}$$
$$p_1 = 2000\,\text{psi} \qquad p_2 = 1990\,\text{psi}$$

Calculate:

a. Flow rate in bbl/day
b. Apparent fluid velocity in ft/day
c. Actual fluid velocity in ft/day

Solution

Calculate the cross-sectional area A:

$$A = (h)(\text{width}) = (20)(300) = 6000\ \text{ft}^2$$

a. Calculate the flow rate

$$q = \frac{0.001127\, kA(p_1 - p_2)}{\mu L}$$

$$q = \frac{(0.001127)(100)(6000)(2000 - 1990)}{(2)(2000)} = 1.6905\ \text{bbl/day}$$

b. Calculate the apparent velocity:

$$v = \frac{q}{A} = \frac{(1.6905)(5.615)}{6000} = 0.0016\ \text{ft/day}$$

c. Calculate the actual fluid velocity:

$$v = \frac{q}{\phi A} = \frac{(1.6905)(5.615)}{(0.15)(6000)} = 0.0105\ \text{ft/day}$$

Inclined Flow of Incompressible Fluids

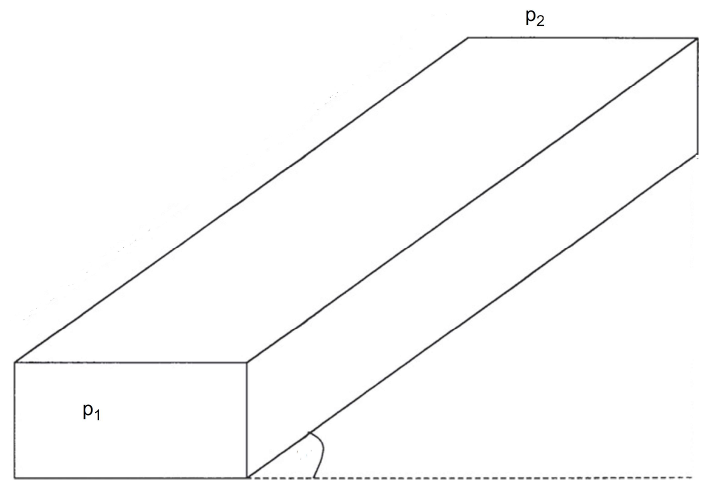
$$q = \frac{0.001127\, kA(\Phi_1 - \Phi_2)}{\mu L}$$

$$\Phi_i = p_i - \left(\frac{\rho}{144}\right)\Delta z_i$$

Expressing the fluid density in gm/cc in the equation gives:

$$\Phi_i = p_i - 0.433\ \gamma\ \Delta z_i$$

Note: 1 gm/cm3 = 62.4 lb/ft3

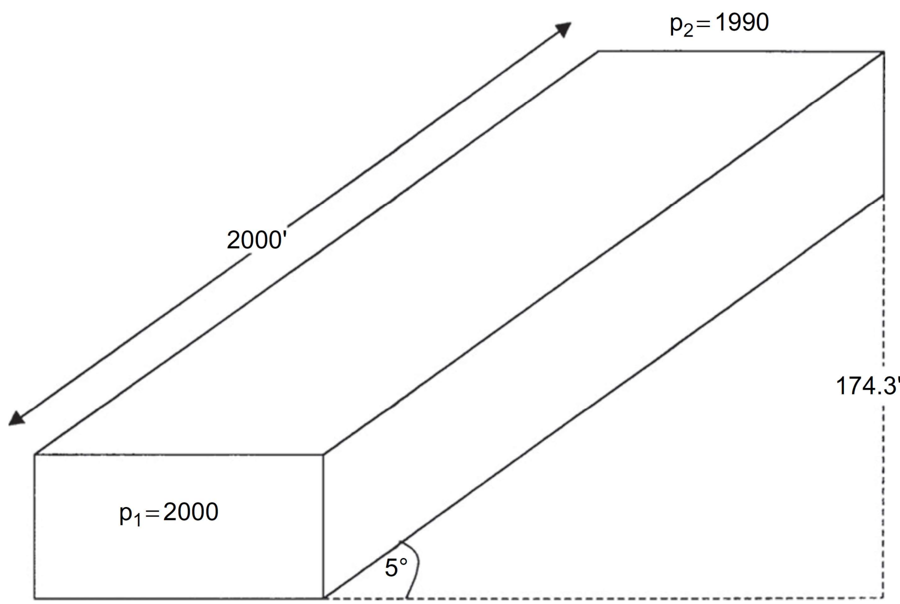


Φ_i = fluid potential at point i, psi
p_i = pressure at point i, psi
Δz_i = vertical distance from point i to the selected datum level
ρ = fluid density, lb/ft^3
γ = fluid density, gm/cm^3

- The datum is usually selected at the gas-oil contact, oil-water contact, or at the highest point in formation.
- The vertical distance Δz_i is assigned as a positive value when the point i is below the datum level and as a negative when it is above the datum level.

Example 2

Assume that the porous media with the properties as given in the previous example is tilted with a dip angle of 5° as shown in Figure . The incompressible fluid has a density of 42 lb/ft^3. Resolve Example 1 using this additional information.



Solution

Let the datum level at half the vertical distance between the two points, i.e., at 87.15 feet.

$$\Phi_1 = p_1 - \left(\frac{\rho}{144}\right)\Delta z_1 = 2000 - \left(\frac{42}{144}\right)(87.15) = 1974.58 \text{ psi}$$

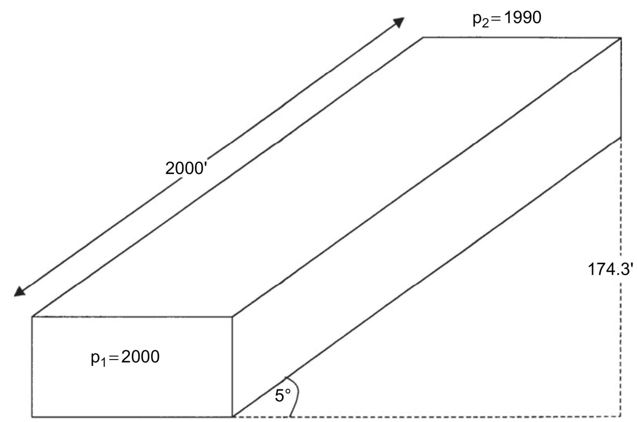
$$\Phi_2 = p_2 + \left(\frac{\rho}{144}\right)\Delta z_2 = 1990 + \left(\frac{42}{144}\right)(87.15) = 2015.42 \text{ psi}$$

Since $\Phi_2 > \Phi_1$, the fluid flows downward from Point 2 to Point 1.

$$q = \frac{0.001127\, kA(\Phi_2 - \Phi_1)}{\mu L} = 6.9 \text{ bbl/day}$$

$$\text{Apparent velocity} = \frac{(6.9)(5.615)}{6000} = 0.0065 \text{ ft/day}$$

$$\text{Actual velocity} = \frac{(6.9)(5.615)}{(0.15)(6000)} = 0.043 \text{ ft/day}$$



Linear Flow of Compressible Fluids (Gases)

- The number of gas moles n at pressure p, temperature T, and volume V: $n = \frac{pV}{zRT}$

- At standard conditions, the volume occupied by the above n moles is given by: $V_{sc} = \frac{n\, z_{sc}\, R\, T_{sc}}{p_{sc}}$

- Combining the above two expressions and assuming **z**$_{sc}$ = 1 gives: $\frac{pV}{zT} = \frac{p_{sc}\, V_{sc}}{T_{sc}}$

- Equivalently, the above relation can be expressed in terms of the flow rate as: $\frac{5.615\, pq}{zT} = \frac{p_{sc}\, Q_{sc}}{T_{sc}}$

Rearranging: $$\left(\frac{p_{sc}}{T_{sc}}\right)\left(\frac{zT}{p}\right)\left(\frac{Q_{sc}}{5.615}\right) = q$$

q = gas flow rate at pressure p and temperature T, bbl/day
Q_{sc} = gas flow rate at standard conditions, scf/day
z = gas compressibility factor
T_{sc}, p_{sc} = standard temperature and pressure in °R and psia, respectively

$$\left(\frac{p_{sc}}{T_{sc}}\right)\left(\frac{zT}{p}\right)\left(\frac{Q_{sc}}{5.615}\right) = q$$

- Replacing the gas flow rate q with that of Darcy's Law, i.e., the equation gives:

$$\frac{q}{A} = \left(\frac{p_{sc}}{T_{sc}}\right)\left(\frac{zT}{p}\right)\left(\frac{Q_{sc}}{5.615}\right)\left(\frac{1}{A}\right) = -0.001127\,\frac{k}{\mu}\,\frac{dp}{dx}$$

- The constant 0.001127 is to convert from Darcy's units to field units. Separating variables and arranging yields:

$$\left[\frac{q_{sc}\,p_{sc}\,T}{0.006328\,k\,T_{sc}\,A}\right]\int_0^L dx = -\int_{p1}^{p2} \frac{p}{z\mu_g}dp$$

- Assuming constant z and μ_g over the specified pressures, i.e., p_1 and p_2, and integrating gives:

$$Q_{sc} = \frac{0.003164\;T_{sc}\,A\,k\,\left(p_1^2 - p_2^2\right)}{p_{sc}\;T\,L\,z\,\mu_g}$$

$$Q_{sc} = \frac{0.111924\,A\,k\,\left(p_1^2 - p_2^2\right)}{T\,L\,z\,\mu_g}$$

Q_{sc} = gas flow rate at standard conditions, scf/day
k = permeability, md
T = temperature, °R
μ_g = gas viscosity, cp
A = cross-sectional area, ft^2
L = total length of the linear system, ft

- It is essential to notice that those gas properties z and μg are a very strong function of pressure, but they have been removed from the integral to simplify the final form of the gas flow equation.
- These equations are valid for applications when the pressure < 2000 psi. The gas properties must be evaluated at the average pressure p as defined

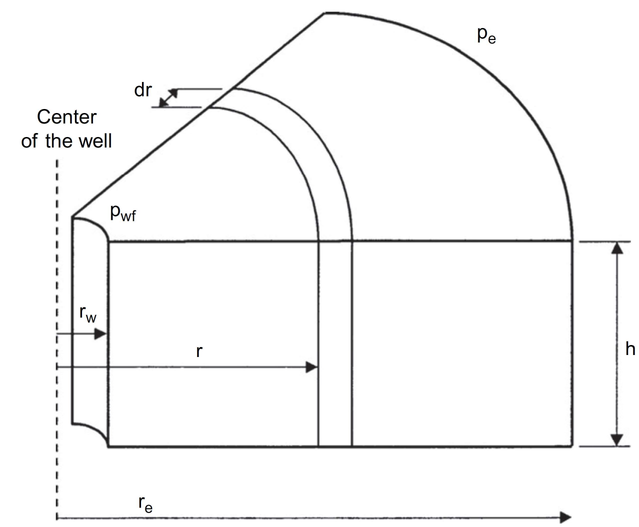
$$\overline{p} = \sqrt{\frac{p_1^2 + p_2^2}{2}}$$

Read Example 6-4,page 348, Reservoir Engineering Handbook

Radial Flow of Incompressible Fluids

$$v = \frac{q}{A_r} = \frac{q}{2\pi rh} = 0.001127\,\frac{k}{\mu}\,\frac{dp}{dr}$$

v = apparent fluid velocity, bbl/day-ft^2
q = flow rate at radius r, bbl/day
k = permeability, md
μ = viscosity, cp
0.001127 = conversion factor to express the equation in field units
A_r = cross-sectional area at radius r



$$\frac{Q_o\,B_o}{2\pi rh} = 0.001127\,\frac{k}{\mu_o}\,\frac{dp}{dr}$$

For incompressible fluid Qo is constant, thus: $$\frac{Q_o}{2\pi h}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{0.001127k}{\mu_o\,B_o}\int_{P_1}^{P_2} dp$$

Performing the integration, gives:

$$Q_o = \frac{0.00708\ k\ h\ (p_2 - p_1)}{\mu_o\ B_o\ \ln\ (r_2/r_1)}$$

or

$$Q_o = \frac{0.00708\ k\,h\,(p_e - p_w)}{\mu_o\ B_o\ \ln(r_e/r_w)}$$

Q_o = oil, flow rate, STB/day
p_e = external pressure, psi
p_{wf} = bottom-hole flowing pressure, psi
k = permeability, md
μ_o = oil viscosity, cp
B_o = oil formation volume factor, bbl/STB
h = thickness, ft
r_e = external or drainage radius, ft
r_w = wellbore radius, ft

Example

An oil well in the Nameless Field is producing at a stabilized rate of 600 STB/day at a stabilized bottom-hole flowing pressure of 1800 psi. The reservoir permeability is 120 md and the reservoir thickness is 25 ft. The well drains an area of approximately 40 acres. The following additional data is available: rw =0.25 ft, Bo = 1.25 bbl/STB, μo =2.5 cp. Calculate the pressure at the outer boundary of the well drainage area.

Solution

$$r_e = \sqrt{\frac{43,560\,A}{\pi}} = 744.73 \text{ ft}$$

$$Q_o = \frac{0.00708\,k\,h\,(p_e - p_w)}{\mu_o\,B_o\,\ln(r_e/r_w)}$$

$$p = p_{wf} + \left[\frac{Q_o\,B_o\,\mu_o}{0.00708\,kh}\right] \ln\left(\frac{r_e}{r_w}\right) = 2506.15 \text{ psi}$$

THANK YOU