Al-Ayen University
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# Reservoir Engineering II

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Lecture 8: Unsteady-State Flow of Reservoir Fluids (Part 2)
Ref.: Reservoir Engineering Handbook by Tarek Ahmed

# Outlines

## ❖ *Unsteady-State Flow*

- **Solution of the Diffusivity Equation**
  - **Radial Flow of the Slightly Compressible Fluids**
    - **The constant-terminal-rate solution**
      - **The Ei-function solution**
      - **The dimensionless pressure PD solution**
        - **Infinite-acting reservoir**
        - **Finite-radial system**

# *Unsteady-State Flow*

## Solution of the Diffusivity Equation:

## Radial Flow of the Slightly Compressible Fluids

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu c_t}{0.000264\,k}\frac{\partial p}{\partial t}$$

1. **Constant-terminal-rate solution:**
   It solves for the pressure change throughout the radial system providing that the flow rate is held constant at one terminal end of the radial system.

2. **Constant-terminal-pressure solution:**
   It is designed to provide the cumulative flow at any particular time for a reservoir in which the pressure at one boundary of the reservoir is held constant. *The constant-pressure solution is widely used in water influx calculations. A detailed description of the solution is discussed in the water influx chapter.*

## The constant-terminal-rate solution

**There are two commonly used forms of the constant-terminal-rate solution:**

- The Ei-function solution
- The dimensionless pressure $P_D$ solution

### The Ei-Function Solution

Matthews and Russell (1967) proposed a solution that is based on the following assumptions:

1. Infinite acting reservoir, i.e., the reservoir is infinite in size.
2. The well is producing at a constant flow rate.
3. The reservoir is at a uniform pressure, pi, when production begins.
4. The well, with a wellbore radius of rw, is centered in a cylindrical reservoir of radius re.
5. No flow across the outer boundary, i.e., at re.

$$\boxed{p(r,t) = p_i + \left[\frac{70.6\, Q_o\, \mu_o B_o}{kh}\right] E_i\left[\frac{-948\phi\mu_o c_t r^2}{kt}\right]}$$

It is commonly referred to as the ***line-source solution.***

where P (r,t) = pressure at radius r from the well after t hours

t = time, hrs

k = permeability, md

Qo = flow rate, STB/day

The mathematical function, Ei, is called the ***exponential integral*** and is defined by:

$$E_i(-x) = -\int_x^\infty \frac{e^{-u}\,du}{u} = \left[\ln x - \frac{x}{1!} + \frac{x^2}{2(2!)} - \frac{x^3}{3(3!)} + \text{etc.}\right]$$

Craft, Hawkins, and Terry (1991) presented the values of the Ei-function in tabulated and graphical forms.

- The exponential integral Ei can be approximated (with less than 0.25% Error) by the following equation when its argument x is less than 0.01:

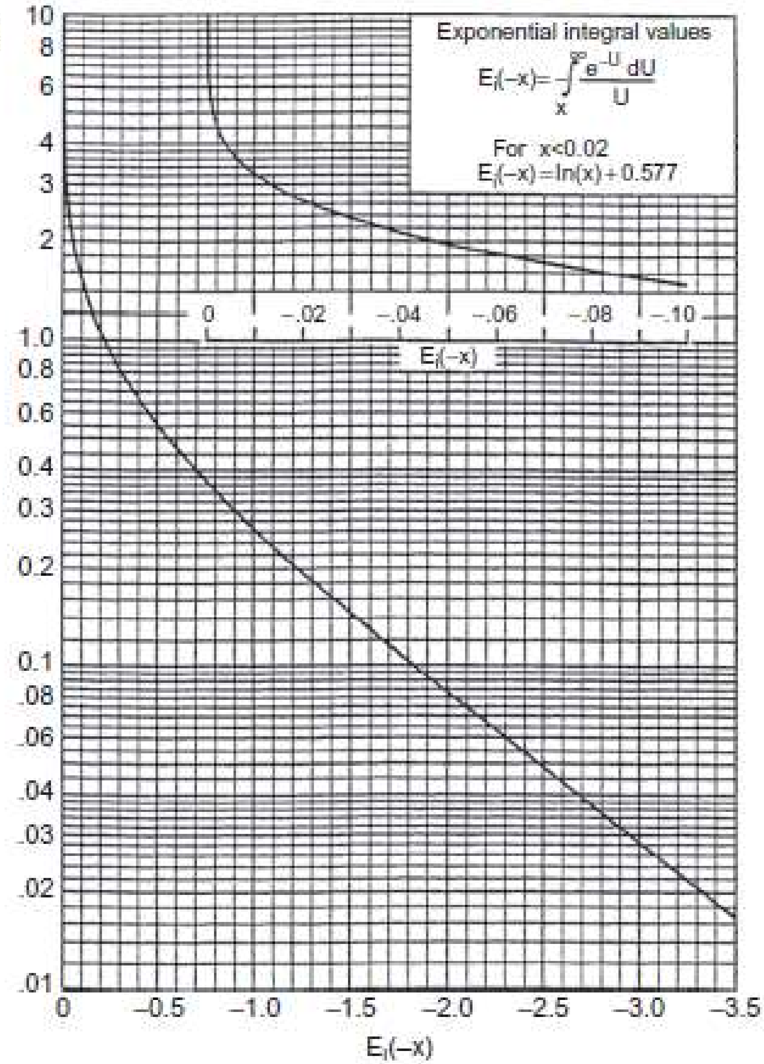
$$E_i(-x) = \ln(1.781x)$$

where the term x in this case is given by: $x = \dfrac{948\phi\mu\, c_t\, r^2}{k\,t}$

$$\ln(x) = 2.303\, log(x)$$

Thus Eq.: $$p(r,t) = p_i + \left[\frac{70.6\, Q_o\, \mu_o Bo}{kh}\right] E_i\left[\frac{-948\phi\, \mu_o\, c_t r^2}{kt}\right]$$

becomes: $$p(r,t) = p_i - \frac{162.6\, Q_o\, B_o\, \mu_o}{kh}\left[\log\left(\frac{k\,t}{\phi\mu_o\, c_t\, r^2}\right) - 3.23\right]$$



- For the behavior of the bottom-hole flowing pressure at the wellbore, i.e., $r = r_w$:

$$\boxed{p_{wf} = p_i - \frac{162.6\, Q_o\, B_o\, \mu_o}{k\, h}\left[\log\left(\frac{k\, t}{\phi \mu_o\, c_t\, r_w^2}\right) - 3.23\right]}$$

where:

$k$ = permeability, md

$t$ = time, hr

$c_t$ = total compressibility, psi$^{-1}$

- It should be noted that these approximated equations cannot be used until *the flow time t exceeds the limit imposed by the following constraint*:

$$\boxed{t > 9.48 \times 10^4 \frac{\phi \mu_o\, c_t\, r^2}{k}}$$

- It should be pointed out that ***for x > 10.9, the Ei (−x) can be considered zero*** for all practical reservoir engineering calculations.

# Example 1

An oil well is producing at a constant flow rate of 300 STB/day under unsteady state flow conditions. The reservoir has the following rock and fluid properties:

$$B_o = 1.25\,\text{bbl/STB} \qquad \mu_o = 1.5\,\text{cp} \qquad c_t = 12 \times 10^{-6}\text{psi}^{-1}$$
$$k_o = 60\,\text{md} \qquad h = 15\,\text{ft} \qquad p_i = 4000\,\text{psi}$$
$$\phi = 15\% \qquad r_w = 0.25\,\text{ft}$$

Calculate pressure at radii of 0.25, 5, 10, 50, 100, 500, 1000, 1500, 2000, and 2500 feet, for 1 hour. Plot the results as:

a. Pressure versus logarithm of radius
b. Pressure versus radius

Exercise:
Repeat the example for t = 12 hours and 24 hours.

# Solution

$$p(r,t) = p_i + \left[\frac{70.6\,Q_o\,\mu_o\,Bo}{kh}\right] E_i\left[\frac{-948\phi\,\mu_o\,c_t r^2}{kt}\right]$$

$$p(r,t) = 4000 + \left[\frac{70.6(300)(1.5)(1.25)}{(60)(15)}\right] \times E_i\left[\frac{-948(0.15)(1.5)(12\times10^{-6})\,r^2}{(60)(t)}\right]$$

$$p(r,t) = 4000 + 44.125\,E_i\left[-42.6(10^{-6})\frac{r^2}{t}\right]$$

| Elapsed Time t = 1 hr | | | |
|---|---|---|---|
| r, ft | $x = -42.6(10^{-6})\frac{r^2}{1}$ | $E_i\,(-x)$ | $p(r, 1) = 4000 + 44.125\,E_i\,(-x)$ |
| 0.25 | $-2.6625(10^{-6})$ | −12.26* | 3459 |
| 5 | −0.001065 | −6.27* | 3723 |
| 10 | −0.00426 | −4.88* | 3785 |
| 50 | −0.1065 | −1.76† | 3922 |
| 100 | −0.4260 | −0.65† | 3971 |
| 500 | −10.65 | 0 | 4000 |
| 1000 | −42.60 | 0 | 4000 |
| 1500 | −95.85 | 0 | 4000 |
| 2000 | −175.40 | 0 | 4000 |
| 2500 | −266.25 | 0 | 4000 |

*As calculated from Equation $E_i\,(-x) = \ln(1.781x)$
†From Figure 6-19

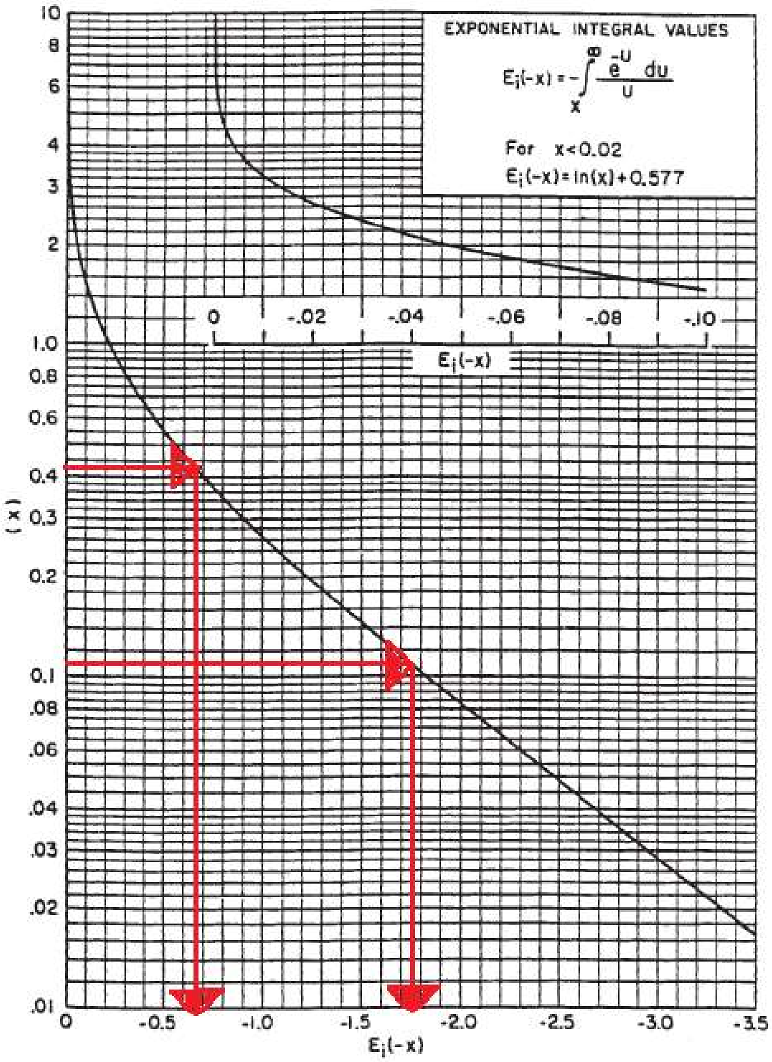
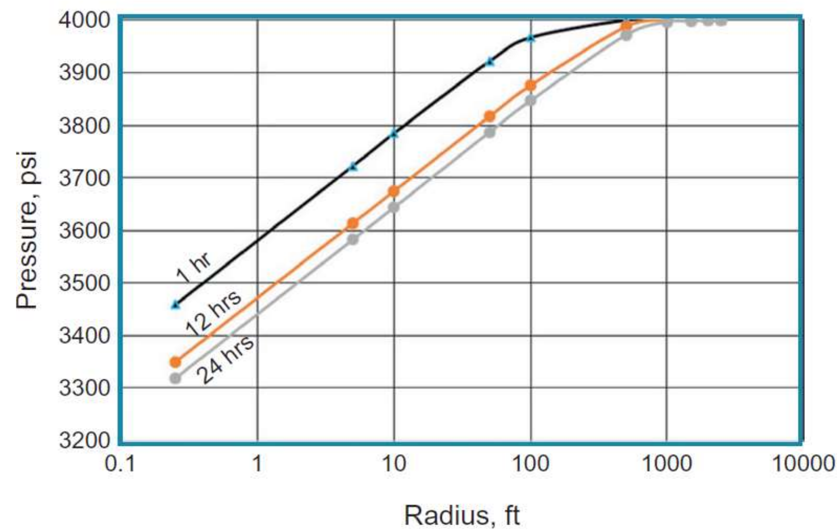
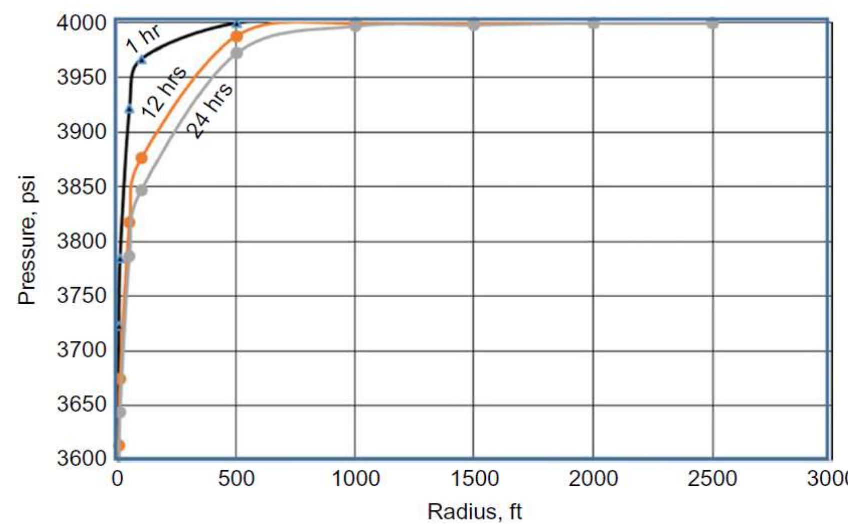



**Figure 6-19.** The $E_i$-function. (*After Craft, Hawkins, and Terry, 1991.*)



# Example 2

Using the data in Example 1, estimate the bottom-hole flowing pressure after *10 hours* of production.

## Solution

To use the approximated equation, the following condition should be satisfied:

$$\boxed{t > 9.48 \times 10^4 \frac{\phi \mu_o c_t r^2}{k}}$$

$$t = 9.48\left(10^4\right)\frac{(0.15)(1.5)\left(12 \times 10^{-6}\right)(0.25)^2}{60} = 0.000267 \text{ hr}$$

Since t= 10 hrs > 0.000267 hr, then Pwf can be calculated from the following equation:

$$\boxed{p_{wf} = p_i - \frac{162.6 Q_o B_o \mu_o}{kh}\left[\log\left(\frac{kt}{\phi \mu_o c_t r_w^2}\right) - 3.23\right]}$$

$$p_{wf} = 4000 - \frac{162.6(300)(1.25)(1.5)}{(60)(15)} \times \left[\log\left(\frac{(60)(10)}{(0.15)(1.5)\left(12 \times 10^{-6}\right)(0.25)^2}\right) - 3.23\right] = 3358 \text{ psi}$$

## The dimensionless pressure PD solution

To introduce the concept of the dimensionless pressure drop solution, consider for example Darcy's equation in a radial form:

$$Q_o = 0.00708 \frac{kh(p_e - p_{wf})}{\mu_o B_o \ln(r_e / r_w)}$$

Rearrange the above equation to give:

$$\frac{p_e - p_{wf}}{\left(\dfrac{Q_o B_o \mu_o}{0.00708\, kh}\right)} = \ln\left(\frac{r_e}{r_w}\right)$$

It is obvious that the right hand side of the above equation has no units (i.e., dimensionless) and, accordingly, the left-hand side must be dimensionless

Therefore, above equation can be written in a dimensionless form as:

$$p_D = \ln(r_{eD})$$

where $p_D = \dfrac{p_e - p_{wf}}{\left(\dfrac{Q_o B_o \mu_o}{0.00708\, k\, h}\right)}$ and $r_{eD} = \dfrac{r_e}{r_w}$

This concept can be extended to consider unsteady state equations as follows:

In transient flow analysis, the dimensionless pressure $p_D$ is always a function of dimensionless time that is defined by the following expression:

$$p_D = \frac{p_i - p(r,t)}{\left(\dfrac{Q_o\, B_o\, \mu_o}{0.00708\, k\, h}\right)}$$

Another definition in common usage is $t_{DA}$, the dimensionless time based on total drainage area:

$$t_{DA} = \frac{0.000264\, kt}{\phi \mu c_t\, A} = t_D \left(\frac{r_w^2}{A}\right) \quad \text{and} \quad t_D = \frac{0.000264\, kt}{\phi \mu c_t\, r_w^2}$$

And the dimensionless radial distances $r_D$ and $r_{eD}$ are defined by:

$$r_D = \frac{r}{r_w} \quad \text{and} \quad r_{eD} = \frac{r_e}{r_w}$$

where

$A$ = total drainage area = $\pi r_e^2$

$r_e$ = drainage radius, ft

$r_w$ = wellbore radius, ft

$p_D$ = dimensionless pressure drop

$r_{eD}$ = dimensionless external radius

$t_D$ = dimensionless time

$r_D$ = dimensionless radius

$t$ = time, hr

$p(r,t)$ = pressure at radius r and time t

$k$ = premeability, md

$\mu$ = viscosity, cp

- The dimensionless groups (i.e., $p_D$, $t_D$, and $r_D$) can be introduced into the diffusivity equation to be in the following dimensionless form:

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D}$$

- Van Everdingen and Hurst (1949) proposed an analytical solution to the above equation by assuming:

  1. Perfectly radial reservoir system
  2. The producing well is in the center and producing at a constant production rate of Q
  3. Uniform pressure $P_i$ throughout the reservoir before production
  4. No flow across the external radius $r_e$.

- Chatas (1953) and Lee (1982) conveniently tabulated these solutions for the following two cases:

  - Infinite-acting reservoir
  - Finite-radial system

## ➢ Infinite-acting reservoir

For an infinite-acting reservoir, i.e., $r_{eD} = \infty$, the dimensionless pressure drop function $P_D$ is strictly a function of the dimensionless time $t_D$, or:

$$p_D = f(t_D)$$

Chatas and Lee tabulated the $p_D$ values for the infinite-acting reservoir as shown in Table 6-2 ( Tarek Ahmed ). The following mathematical expressions can be used to approximate these tabulated values of $P_D$:

- For $t_D < 0.01$: $p_D = 2\sqrt{\frac{t_D}{\pi}}$
- For $t_D > 100$: $p_D = 0.5[\ln(t_D) + 0.80907]$
- For $0.02 < t_D < 1000$:

$$p_D = a_1 + a_2 \ln(t_D) + a_3 [\ln(t_D)]^2 + a_4 [\ln(t_D)]^3 + a_5 t_D + a_6 (t_D)^2 + a_7 (t_D)^3 + a_8/t_D$$

where

$a_1 = 0.8085064$
$a_2 = 0.29302022$
$a_3 = 3.5264177(10^{-2})$
$a_4 = -1.4036304(10^{-3})$
$a_5 = -4.7722225(10^{-4})$
$a_6 = 5.1240532(10^{-7})$
$a_7 = -2.3033017(10^{-10})$
$a_8 = -2.6723117(10^{-3})$

**TABLE 6-2** $p_D$ vs. $t_D$–Infinite-Radial System, Constant-Rate at the Inner Boundary (*After Lee, J.,* Well Testing, *SPE Textbook Series.*) *(Permission to publish by the SPE, copyright SPE, 1982)*

| $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ |
|---|---|---|---|---|---|
| 0 | 0 | 0.15 | 0.3750 | 60.0 | 2.4758 |
| 0.0005 | 0.0250 | 0.2 | 0.4241 | 70.0 | 2.5501 |
| 0.001 | 0.0352 | 0.3 | 0.5024 | 80.0 | 2.6147 |
| 0.002 | 0.0495 | 0.4 | 0.5645 | 90.0 | 2.6718 |
| 0.003 | 0.0603 | 0.5 | 0.6167 | 100.0 | 2.7233 |
| 0.004 | 0.0694 | 0.6 | 0.6622 | 150.0 | 2.9212 |
| 0.005 | 0.0774 | 0.7 | 0.7024 | 200.0 | 3.0636 |
| 0.006 | 0.0845 | 0.8 | 0.7387 | 250.0 | 3.1726 |
| 0.007 | 0.0911 | 0.9 | 0.7716 | 300.0 | 3.2630 |
| 0.008 | 0.0971 | 1.0 | 0.8019 | 350.0 | 3.3394 |
| 0.009 | 0.1028 | 1.2 | 0.8672 | 400.0 | 3.4057 |
| 0.01 | 0.1081 | 1.4 | 0.9160 | 450.0 | 3.4641 |
| 0.015 | 0.1312 | 2.0 | 1.0195 | 500.0 | 3.5164 |
| 0.02 | 0.1503 | 3.0 | 1.1665 | 550.0 | 3.5643 |
| 0.025 | 0.1669 | 4.0 | 1.2750 | 600.0 | 3.6076 |
| 0.03 | 0.1818 | 5.0 | 1.3625 | 650.0 | 3.6476 |
| 0.04 | 0.2077 | 6.0 | 1.4362 | 700.0 | 3.6842 |
| 0.05 | 0.2301 | 7.0 | 1.4997 | 750.0 | 3.7184 |
| 0.06 | 0.2500 | 8.0 | 1.5557 | 800.0 | 3.7505 |
| 0.07 | 0.2680 | 9.0 | 1.6057 | 850.0 | 3.7805 |
| 0.08 | 0.2845 | 10.0 | 1.6509 | 900.0 | 3.8088 |
| 0.09 | 0.2999 | 15.0 | 1.8294 | 950.0 | 3.8355 |
| 0.1 | 0.3144 | 20.0 | 1.9601 | 1,000.0 | 3.8584 |
| | | 30.0 | 2.1470 | | |
| | | 40.0 | 2.2824 | | |
| | | 50.0 | 2.3884 | | |

*Notes:* For $t_D < 0.01$, $p_D \cong 2\sqrt{t_D/\pi}$.
For $100 < t_D < 0.25 r_{eD}^2$, $p_D \cong 0.5(\ln t_D + 0.80907)$.

## ➢ Finite-Radial System

For a finite radial system, the $P_D$-function is a function of both the dimensionless time and radius, or:

$$p_D = f\,(t_D,\, r_{eD})$$

where

$$r_{eD} = \frac{\text{external radius}}{\text{wellbore radius}} = \frac{r_e}{r_w}$$

- Van Everdingen and Hurst principally applied *the $P_D$ function solution to model the performance of water influx into oil reservoirs*.
- Table 6-3 ( Tarek Ahmed ) presents $P_D$ as a function of $t_D$ for $1.5 < r_{eD} < 10$.
- ' wellbore radius $r_w$ is in this case the external radius of the reservoir and the $r_e$ is the external boundary radius of the aquifer.
- Chatas (1953) proposed the following mathematical expression for calculating $P_D$:

For $\mathbf{25 < t_D}$ and $\mathbf{0.25\, r_{eD}^2 < t_D}$

$$p_D = \frac{0.5 + 2t_D}{r_{eD}^2 - 1} - \frac{r_{eD}^4\,[3 - 4\ln(r_{eD})] - 2r_{eD}^2 - 1}{4\,(r_{eD}^2 - 1)^2}$$

A special case arises when $r_{eD}^2 \gg 1$, then:

$$p_D = \frac{2\,t_D}{r_{eD}^2} + \ln(r_{eD}) - 0.75$$

**TABLE 6-3** $p_D$ vs. $t_D$–Finite-Radial System, Constant-Rate at the Inner Boundary (*After Lee, J.,* Well Testing, *SPE Textbook Series.*) (*Permission to publish by the SPE, copyright SPE, 1982*)

| $r_{eD}=1.5$ | | $r_{eD}=2.0$ | | $r_{eD}=2.5$ | | $r_{eD}=3.0$ | | $r_{eD}=3.5$ | | $r_{eD}=4.0$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ |
| 0.06 | 0.251 | 0.22 | 0.443 | 0.40 | 0.565 | 0.52 | 0.627 | 1.0 | 0.802 | 1.5 | 0.927 |
| 0.08 | 0.288 | 0.24 | 0.459 | 0.42 | 0.576 | 0.54 | 0.636 | 1.1 | 0.830 | 1.6 | 0.948 |
| 0.10 | 0.322 | 0.26 | 0.476 | 0.44 | 0.587 | 0.56 | 0.645 | 1.2 | 0.857 | 1.7 | 0.968 |
| 0.12 | 0.355 | 0.28 | 0.492 | 0.46 | 0.598 | 0.60 | 0.662 | 1.3 | 0.882 | 1.8 | 0.988 |
| 0.14 | 0.387 | 0.30 | 0.507 | 0.48 | 0.608 | 0.65 | 0.683 | 1.4 | 0.906 | 1.9 | 1.007 |
| 0.16 | 0.420 | 0.32 | 0.522 | 0.50 | 0.618 | 0.70 | 0.703 | 1.5 | 0.929 | 2.0 | 1.025 |
| 0.18 | 0.452 | 0.34 | 0.536 | 0.52 | 0.628 | 0.75 | 0.721 | 1.6 | 0.951 | 2.2 | 1.059 |
| 0.20 | 0.484 | 0.36 | 0.551 | 0.54 | 0.638 | 0.80 | 0.740 | 1.7 | 0.973 | 2.4 | 1.092 |
| 0.22 | 0.516 | 0.38 | 0.565 | 0.56 | 0.647 | 0.85 | 0.758 | 1.8 | 0.994 | 2.6 | 1.123 |
| 0.24 | 0.548 | 0.40 | 0.579 | 0.58 | 0.657 | 0.90 | 0.776 | 1.9 | 1.014 | 2.8 | 1.154 |
| 0.26 | 0.580 | 0.42 | 0.593 | 0.60 | 0.666 | 0.95 | 0.791 | 2.0 | 1.034 | 3.0 | 1.184 |
| 0.28 | 0.612 | 0.44 | 0.607 | 0.65 | 0.688 | 1.0 | 0.806 | 2.25 | 1.083 | 3.5 | 1.255 |
| 0.30 | 0.644 | 0.46 | 0.621 | 0.70 | 0.710 | 1.2 | 0.865 | 2.50 | 1.130 | 4.0 | 1.324 |
| 0.35 | 0.724 | 0.48 | 0.634 | 0.75 | 0.731 | 1.4 | 0.920 | 2.75 | 1.176 | 4.5 | 1.392 |
| 0.40 | 0.804 | 0.50 | 0.648 | 0.80 | 0.752 | 1.6 | 0.973 | 3.0 | 1.221 | 5.0 | 1.460 |
| 0.45 | 0.884 | 0.60 | 0.715 | 0.85 | 0.772 | 2.0 | 1.076 | 4.0 | 1.401 | 5.5 | 1.527 |
| 0.50 | 0.964 | 0.70 | 0.782 | 0.90 | 0.792 | 3.0 | 1.328 | 5.0 | 1.579 | 6.0 | 1.594 |
| 0.55 | 1.044 | 0.80 | 0.849 | 0.95 | 0.812 | 4.0 | 1.578 | 6.0 | 1.757 | 6.5 | 1.660 |
| 0.60 | 1.124 | 0.90 | 0.915 | 1.0 | 0.832 | 5.0 | 1.828 | | | 7.0 | 1.727 |
| 0.65 | 1.204 | 1.0 | 0.982 | 2.0 | 1.215 | | | | | 8.0 | 1.861 |
| 0.70 | 1.284 | 2.0 | 1.649 | 3.0 | 1.506 | | | | | 9.0 | 1.994 |
| 0.75 | 1.364 | 3.0 | 2.316 | 4.0 | 1.977 | | | | | 10.0 | 2.127 |
| 0.80 | 1.444 | 5.0 | 3.649 | 5.0 | 2.398 | | | | | | |

| $r_{eD}=4.5$ | | $r_{eD}=5.0$ | | $r_{eD}=6.0$ | | $r_{eD}=7.0$ | | $r_{eD}=8.0$ | | $r_{eD}=9.0$ | | $r_{eD}=10.0$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ |
| 2.0 | 1.023 | 3.0 | 1.167 | 4.0 | 1.275 | 6.0 | 1.436 | 8.0 | 1.556 | 10.0 | 1.651 | 12.0 | 1.732 |
| 2.1 | 1.040 | 3.1 | 1.180 | 4.5 | 1.322 | 6.5 | 1.470 | 8.5 | 1.582 | 10.5 | 1.673 | 12.5 | 1.750 |
| 2.2 | 1.056 | 3.2 | 1.192 | 5.0 | 1.364 | 7.0 | 1.501 | 9.0 | 1.607 | 11.0 | 1.693 | 13.0 | 1.768 |
| 2.3 | 1.702 | 3.3 | 1.204 | 5.5 | 1.404 | 7.5 | 1.531 | 9.5 | 1.631 | 11.5 | 1.713 | 13.5 | 1.784 |
| 2.4 | 1.087 | 3.4 | 1.215 | 6.0 | 1.441 | 8.0 | 1.559 | 10.0 | 1.653 | 12.0 | 1.732 | 14.0 | 1.801 |
| 2.5 | 1.102 | 3.5 | 1.227 | 6.5 | 1.477 | 8.5 | 1.586 | 10.5 | 1.675 | 12.5 | 1.750 | 14.5 | 1.817 |
| 2.6 | 1.116 | 3.6 | 1.238 | 7.0 | 1.511 | 9.0 | 1.613 | 11.0 | 1.697 | 13.0 | 1.768 | 15.0 | 1.832 |

**TABLE 6-3** $p_D$ vs. $t_D$–Finite-Radial System, Constant-Rate at the Inner Boundary (*After Lee, J.,* Well Testing, *SPE Textbook Series.*) (*Permission to publish by the SPE, copyright SPE, 1982*)—cont'd

| $r_{eD}=4.5$ | | $r_{eD}=5.0$ | | $r_{eD}=6.0$ | | $r_{eD}=7.0$ | | $r_{eD}=8.0$ | | $r_{eD}=9.0$ | | $r_{eD}=10.0$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ | $t_D$ | $p_D$ |
| 2.7 | 1.130 | 3.7 | 1.249 | 7.5 | 1.544 | 9.5 | 1.638 | 11.5 | 1.717 | 13.5 | 1.786 | 15.5 | 1.847 |
| 2.8 | 1.144 | 3.8 | 1.259 | 8.0 | 1.576 | 10.0 | 1.663 | 12.0 | 1.737 | 14.0 | 1.803 | 16.0 | 1.862 |
| 2.9 | 1.158 | 3.9 | 1.270 | 8.5 | 1.607 | 11.0 | 1.711 | 12.5 | 1.757 | 14.5 | 1.819 | 17.0 | 1.890 |
| 3.0 | 1.171 | 4.0 | 1.281 | 9.0 | 1.638 | 12.0 | 1.757 | 13.0 | 1.776 | 15.0 | 1.835 | 18.0 | 1.917 |
| 3.2 | 1.197 | 4.2 | 1.301 | 9.5 | 1.668 | 13.0 | 1.810 | 13.5 | 1.795 | 15.5 | 1.851 | 19.0 | 1.943 |
| 3.4 | 1.222 | 4.4 | 1.321 | 10.0 | 1.698 | 14.0 | 1.845 | 14.0 | 1.813 | 16.0 | 1.867 | 20.0 | 1.968 |
| 3.6 | 1.246 | 4.6 | 1.340 | 11.0 | 1.757 | 15.0 | 1.888 | 14.5 | 1.831 | 17.0 | 1.897 | 22.0 | 2.017 |
| 3.8 | 1.269 | 4.8 | 1.360 | 12.0 | 1.815 | 16.0 | 1.931 | 15.0 | 1.849 | 18.0 | 1.926 | 24.0 | 2.063 |
| 4.0 | 1.292 | 5.0 | 1.378 | 13.0 | 1.873 | 17.0 | 1.974 | 17.0 | 1.919 | 19.0 | 1.955 | 26.0 | 2.108 |
| 4.5 | 1.349 | 5.5 | 1.424 | 14.0 | 1.931 | 18.0 | 2.016 | 19.0 | 1.986 | 20.0 | 1.983 | 28.0 | 2.151 |
| 5.0 | 1.403 | 6.0 | 1.469 | 15.0 | 1.988 | 19.0 | 2.058 | 21.0 | 2.051 | 22.0 | 2.037 | 30.0 | 2.194 |
| 5.5 | 1.457 | 6.5 | 1.513 | 16.0 | 2.045 | 20.0 | 2.100 | 23.0 | 2.116 | 24.0 | 2.906 | 32.0 | 2.236 |
| 6.0 | 1.510 | 7.0 | 1.556 | 17.0 | 2.103 | 22.0 | 2.184 | 25.0 | 2.180 | 26.0 | 2.142 | 34.0 | 2.278 |
| 7.0 | 1.615 | 7.5 | 1.598 | 18.0 | 2.160 | 24.0 | 2.267 | 30.0 | 2.340 | 28.0 | 2.193 | 36.0 | 2.319 |
| 8.0 | 1.719 | 8.0 | 1.641 | 19.0 | 2.217 | 26.0 | 2.351 | 35.0 | 2.499 | 30.0 | 2.244 | 38.0 | 2.360 |
| 9.0 | 1.823 | 9.0 | 1.725 | 20.0 | 2.274 | 28.0 | 2.434 | 40.0 | 2.658 | 34.0 | 2.345 | 40.0 | 2.401 |
| 10.0 | 1.927 | 10.0 | 1.808 | 25.0 | 2.560 | 30.0 | 2.517 | 45.0 | 2.817 | 38.0 | 2.446 | 50.0 | 2.604 |
| 11.0 | 2.031 | 11.0 | 1.892 | 30.0 | 2.846 | | | | | 40.0 | 2.496 | 60.0 | 2.806 |
| 12.0 | 2.135 | 12.0 | 1.975 | | | | | | | 45.0 | 2.621 | 70.0 | 3.008 |
| 13.0 | 2.239 | 13.0 | 2.059 | | | | | | | 50.0 | 2.746 | 80.0 | 3.210 |
| 14.0 | 2.343 | 14.0 | 2.142 | | | | | | | 60.0 | 2.996 | 90.0 | 3.412 |
| 15.0 | 2.447 | 15.0 | 2.225 | | | | | | | 70.0 | 3.246 | 100.0 | 3.614 |

*Notes: For $t_D$ smaller than values listed in this table for a given $r_{eD}$, reservoir is infinite acting. Find $p_D$ in Table 6-2.*
*For $25 < t_D$ and $t_D$ larger than values in table.*

$$p_D \cong \frac{\left(^1/_2 + 2t_D\right)}{\left(r_{eD}^2 - 1\right)} - \frac{3r_{eD}^4 - 4r_{eD}^4 \ln r_{eD} - 2r_{eD}^2 - 1}{4\left(r_{eD}^2 - 1\right)^2}$$

*For wells in rebounded reservoirs with $r_{eD}^2 \gg 1$*

$$p_D \cong \frac{2t_D}{r_{eD}^2} + \ln r_{eD} - {}^3/_4.$$

## Example 3

A well is producing at a constant flow rate of 300 STB/day under unsteady-state flow condition. The reservoir has the following rock and fluid properties:

$$B_o = 1.25\,\text{bbl/STB} \qquad \mu_o = 1.5\,\text{cp} \qquad c_t = 12 \times 10^{-6}\,\text{psi}^{-1}$$
$$k = 60\,\text{md} \qquad h = 15\,\text{ft} \qquad p_i = 4000\,\text{psi}$$
$$\phi = 15\% \qquad r_w = 0.25'$$

Assuming an infinite acting reservoir, i.e., $r_{eD} = \infty$, calculate the bottomhole flowing pressure after one hour of production using the dimensionless pressure approach.

### Solution

$$t_D = \frac{0.000264\ kt}{\phi \mu\, c_t\, r_w^2}$$

$$t_D = \frac{0.000264(60)(1)}{(0.15)(1.5)(12 \times 10^{-6})(0.25)^2} = 93,866.67$$

Since $t_D > 100$, we can use Equation:

$$p_D = 0.5[\ \ln(t_D) + 0.80907]$$

$$p_D = 0.5[\ \ln(93,866.67) + 0.80907] = 6.1294$$

From the definition of PD:

$$p_D = \frac{p_i - p(r,t)}{\left(\dfrac{Q_o\ B_o\ \mu_o}{0.00708\,k\ h}\right)}$$

$$p(r_w, t) = p_i - \left(\frac{Q_o B_o \mu_o}{0.00708\ kh}\right) p_D$$

$$p_D p(0.25, 1) = 4000 - \left[\frac{(300)(1.25)(1.5)}{0.00708(60)(15)}\right](6.1294) = 3459\ \text{psi}$$

***THANK YOU***