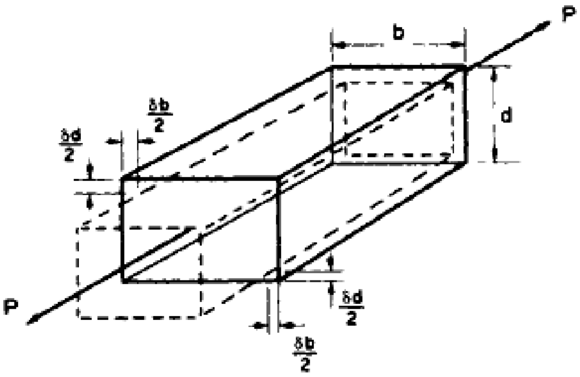
<u>Poisson's ratio (v) :</u>

Another type of elastic deformation is the change in transverse dimensions accompanying axial tension or compression .

If the bar is extended by axial tension , there is a reduction in the transverse dimensions .The ratio of lateral strain to the longitudinal strain is called ″ *Poisson's ratio* ″ . This ratio is constant for each material like [for steel (v)=0.25-0.3 , for concrete (v)=0.2 , and for rubber (v) $\leq$ 0.49] .

when the load (P) in X-direction : (P_x)

$$\upsilon = \frac{-\varepsilon_y}{\varepsilon_x} = \frac{-\varepsilon_z}{\varepsilon_x}$$



when the load (P) in Y-direction : (P_y)

$$\upsilon = \frac{-\varepsilon_x}{\varepsilon_y} = \frac{-\varepsilon_z}{\varepsilon_y}$$

when the load (P) in Z-direction : (P_z)

$$\upsilon = \frac{-\varepsilon_x}{\varepsilon_z} = \frac{-\varepsilon_y}{\varepsilon_z}$$

From above figure $\Rightarrow \varepsilon_{Lateral} = \frac{\delta d}{d}$ **(in Y-direction)**

$$\varepsilon_{Lateral} = \frac{\delta b}{b} \quad \textbf{(in Z-direction)}$$

$$\varepsilon_{Longitudinal} = \frac{\delta L}{L} \quad \textbf{(in X-direction)}$$

$$\therefore \upsilon = \frac{\varepsilon_{Lateral}}{\varepsilon_{Longitudinal}}$$

$$\upsilon \varepsilon_x = -\varepsilon_y = -\varepsilon_z$$

$$\text{n}\sigma_x = \varepsilon_x * E \Rightarrow \varepsilon_x = \frac{\sigma_x}{E}$$

$$n -\upsilon * \frac{\sigma_x}{E} = \varepsilon_y = \varepsilon_z$$ **For one axial loading**

$$\varepsilon_{X_{(1)}} = \frac{\sigma_x}{E}$$ **(due to stress along X-direction)**

$$\varepsilon_{X_{(2)}} = -\upsilon \frac{\sigma_y}{E}$$ **(due to stress along Y-direction)**

$\therefore$ **Total strain in X-direction** $(\varepsilon_{X_t}) = \frac{\sigma_x}{E} - \upsilon \frac{\sigma_y}{E}$ **----------(1)**

$\therefore$ **Total strain in Y-direction** $(\varepsilon_{Y_t}) = \frac{\sigma_y}{E} - \upsilon \frac{\sigma_x}{E}$ **-----------(2)**

From (1) and (2) :

$$\therefore \quad \sigma_x = \frac{(\varepsilon_x + \upsilon * \varepsilon_y)E}{1-\upsilon^2}, \quad \sigma_y = \frac{(\varepsilon_y + \upsilon * \varepsilon_x)E}{1-\upsilon^2}$$ → **For Bi axial loading**

and for Tri axial loading :

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \upsilon(\sigma_y + \sigma_z)]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \upsilon(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \upsilon(\sigma_x + \sigma_y)]$$

Ex: -8- A square steel plate has a tensile strains of (ε_x=2.37*10^{-7} , ε_y=3.95*10^{-4}) along X and Y directions . Find (σ_x and σ_y) when (E= 210 GPa) and (υ=0.28) .

<u>Sol:</u>

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon \frac{\sigma_y}{E}$$

$$2.37*10^{-7} = \frac{1}{210*10^9}[\sigma_x - 0.28\sigma_y]$$

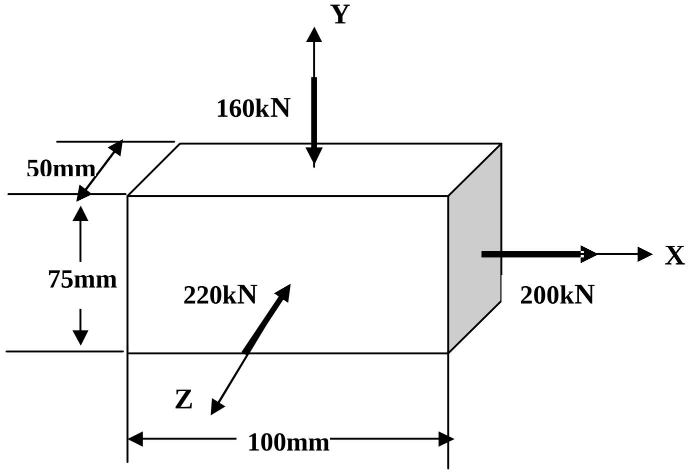
$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon \frac{\sigma_x}{E}$$

$$3.95*10^{-4} = \frac{1}{210*10^9}[\sigma_y - 0.28\sigma_x]$$

$$\therefore \sigma_x = 0.344\, MPa$$

$$\sigma_y = 1.05\, MPa$$

Ex :-9- A rectangular Aluminum block is (100mm) long in X-direction , (75mm) wide in Y-direction and (50mm) thick in Z-direction . It is subjected to tri axial loading consisting of uniformly distributed tensile force of (200kN) in the X-direction and uniformly distributed compressive forces of 160kN in Y-direction and (220kN) in Z-direction. If the Poisson's ratio (υ=0.333) and (E=70GPa) . Determine a single distributed load that must applied in X-direction that would produce the same deformation in Z-direction as original loading .



$$\varepsilon_z = \frac{\sigma_z}{E} - \upsilon\frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E}$$

$$\sigma_x = \frac{P_x}{A} = \frac{200*10^3}{0.05*0.075} = 53.3 MPa\ (tension)$$

$$\sigma_y = \frac{160*10^3}{0.05*0.1} = 32 MPa\ (compression)$$

$$\sigma_z = \frac{220*10^3}{0.075*0.1} = 24.34 MPa\ (compression)$$

$$\therefore \varepsilon_z = \frac{1}{70*10^9}[-24.34 - 0.333(-32+53.3)]*10^6$$

$$\varepsilon_z = -0.52*10^{-3}$$

$$\varepsilon_z = -\upsilon\frac{\sigma_x}{E} = -\upsilon\frac{P_x}{A*E} \Rightarrow -0.52*10^{-3} = -0.333\frac{P_x}{0.075*0.05*70*10^9}$$

$$\therefore P_x = 410\ kN\ (Tension)$$