Lecture Two

2.1 Oil Well Performance

This lecture presents the practical reservoir engineering equations that are designed to predict the performance of vertical oil wells. The lecture also describes some of the factors that are governing the flow of fluids from the formation to the wellbore and how these factors may affect the production performance of the well. The analysis of the production performance is essentially based on the following fluid and well characteristics:

- Fluid PVT properties
- Relative permeability data
- Inflow-performance-relationship (IPR)

2.2 Vertical Oil Well Performance

Productivity Index and IPR

A commonly used measure of the ability of the well to produce is the Productivity Index. Defined by the symbol **J**, the productivity index is the ratio of the total liquid flow rate to the pressure drawdown. For a water-free oil production, the productivity index is given by:

$$J = \frac{Q_o}{\overline{p}_r - p_{wf}} = \frac{Q_o}{\Delta p} \quad \text{.................. (2-1)}$$

Where:

Q_o = oil flow rate, STB/day

J = productivity index, STB/day/psi

P_r = volumetric average drainage area pressure (static pressure)

p_{wf} = bottom-hole flowing pressure

Δp = drawdown, psi

The productivity index is generally measured during a production test on the well. The well is shut-in until the static reservoir pressure is reached. The well is then allowed to produce at a constant flow rate of Q and a stabilized bottomhole flow pressure of p_{wf}. Since a stabilized pressure at surface does not necessarily indicate a stabilized p_{wf}, the bottom-hole flowing pressure should be recorded continuously from the time the well is to flow. The productivity index is then calculated from Equation 2-1.

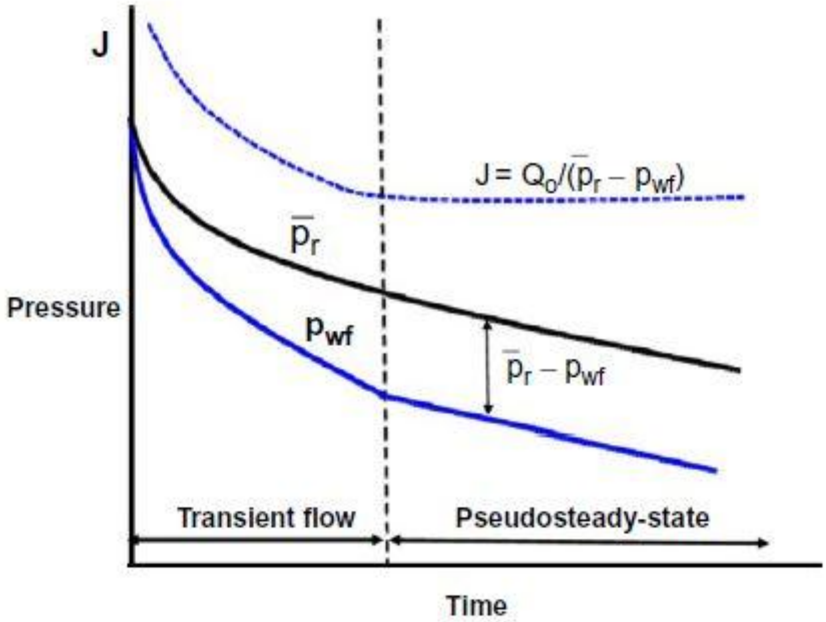

FIGURE 2-1: Productivity index during semi-steady and unsteady fluid flow.

It is important to note that the productivity index is a valid measure of the well productivity potential only if the well is flowing at pseudo-steady-state conditions. Therefore, in order to accurately measure the productivity index of a well, it is essential that the well is allowed to flow at a constant flow rate for a sufficient amount of time to reach the pseudo-steady-state as illustrated in Figure 2-1. The figure indicates that during the transient flow period, the calculated values of the productivity index will vary depending upon the time at which the measurements of p_{wf} are made.

The productivity index can be numerically calculated by recognizing that J must be defined in terms of semi-steady-state flow conditions.

$$Q_o = \frac{0.00708\, k_o\, h(\bar{p}_r - p_{wf})}{\mu_o B_o \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \qquad \ldots\ldots\ldots\ldots\ldots\ (2\text{-}2)$$

The above equation is combined with Equation 2-1 to give:

$$J = \frac{0.00708\, k_o\, h}{\mu_o B_o \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \qquad \ldots\ldots\ldots\ldots\ldots\ (2\text{-}3)$$

Where:

J = productivity index, STB/day/psi

k_o = effective permeability of the oil, md

s = skin factor

h = thickness, ft

The oil relative permeability concept can be conveniently introduced into Equation 2-3 to give:

$$J = \frac{0.00708\,h\,k}{\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]}\left(\frac{k_{ro}}{\mu_o B_o}\right) \quad \text{…………..} \ (2\text{-}4)$$

Since most of the well life is spent in a flow regime that is approximating the Pseudo-steady-state, the productivity index is a valuable methodology for predicting the future performance of wells. Further, by monitoring the productivity index during the life of a well, it is possible to determine if the well has become damaged due to completion, workover, production, injection operations, or mechanical problems. If a measured J has an unexpected decline, one of the indicated problems should be investigated.

A comparison of productivity indices of different wells in the same reservoir should also indicate some of the wells might have experienced unusual difficulties or damage during completion. Since the productivity indices may vary from well to well because of the variation in thickness of the reservoir, it is helpful to normalize the indices by dividing each by the thickness of the well. This is defined as the specific productivity index J_s, or:

$$J_s = \frac{J}{h} = \frac{Q_o}{h(\overline{p}_r - p_{wf})} \quad \text{………….} \ (2\text{-}5)$$

Assuming that the well's productivity index is constant, Equation 2-1 can be rewritten as:

$$Q_o = J(\overline{p}_r - p_{wf}) = J\,\Delta p \quad \text{……………} \ (2\text{-}6)$$

Where:

Δp = drawdown, psi

J = productivity index

Equation 2-6 indicates that the relationship between Q_o and Δp is a straight line passing through the origin with a slope of J as shown in Figure 2-2.

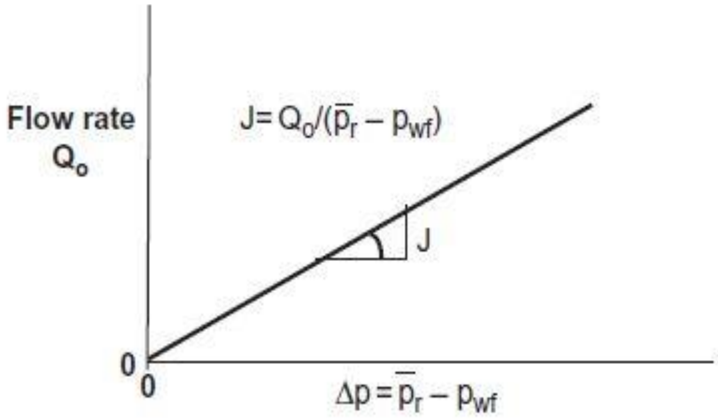

FIGURE 2-2: Oil rate vs. ΔP.

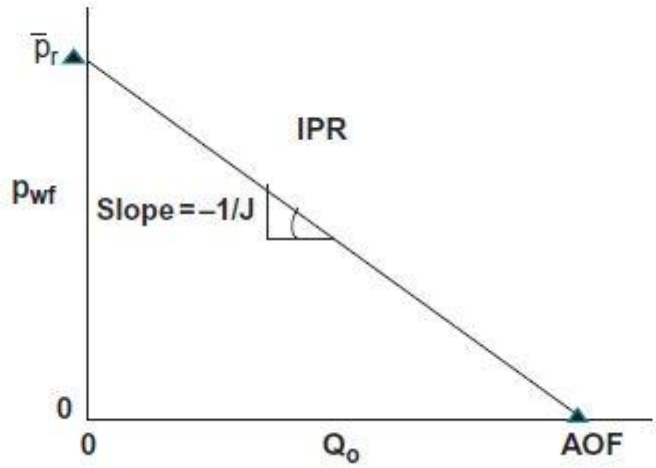

FIGURE 2-3: Inflow performance relationship.

Alternatively, Equation 2-1 can be written as:

$$p_{wf} = \bar{p}_r - \left(\frac{1}{J}\right) Q_o \qquad \text{……………….. (2-7)}$$

The above expression shows that the plot p_{wf} against Q_o is a straight line with a slope of (- 1/J) as shown schematically in Figure 2-3. This graphical representation of the relationship that exists between the oil flow rate and bottom-hole flowing pressure is called the inflow performance relationship and referred to as IPR. Several important features of the straight-line IPR can be seen in Figure 2-3:

- ❖ When p_{wf} equals average reservoir pressure, the flow rate is zero due to the absence of any pressure drawdown.

- Maximum rate of flow occurs when pwf is zero. This maximum rate is called absolute open flow and referred to as AOF. Although in practice this may not be a condition at which the well can produce, it is a useful definition that has widespread applications in the petroleum industry (e.g., comparing flow potential of different wells in the field). The AOF is then calculated by:

$$AOF = J\, p_r$$

- The slope of the straight line equals the reciprocal of the productivity index.

Example 2-1:

A productivity test was conducted on a well. The test results indicate that the well is capable of producing at a stabilized flow rate of 110 STB/day and a bottom-hole flowing pressure of 900 psi. After shutting the well for 24 hours, the bottom-hole pressure reached a static value of 1300 psi.

Calculate: Productivity index, AOF, Oil flow rate at a bottom-hole flowing pressure of 600 psi, Wellbore flowing pressure required to produce 250 STB/day

Solution:

a. Calculate J from Equation 2-1:

$$J = \frac{110}{1300 - 900} = 0.275 \text{ STB/psi}$$

b. Determine the AOF from:

$$AOF = J(\bar{p}_r - 0)$$

$$AOF = 0.275(1300 - 0) = 375.5 \text{ STB/day}$$

c. Solve for the oil-flow rate by applying Equation 2-1:

$$Q_o = 0.275(1300 - 600) = 192.5 \text{ STB/day}$$

d. Solve for p_{wf} by using Equation 2-7:

$$p_{wf} = 1300 - \left(\frac{1}{0.275}\right)250 = 390.9 \text{ psi}$$

Equation 2-6 suggests that the inflow into a well is directly proportional to the pressure drawdown and the constant of proportionality is the productivity index.

Muskat and Evinger (1942) and Vogel (1968) observed that when the pressure drops below the bubble-point pressure, the IPR deviates from that of the simple straight-line relationship as shown in Figure 2-4. Recalling Equation 2-4:

$$J = \left[\frac{0.00708\,h\,k}{\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s}\right]\left(\frac{k_{ro}}{\mu_o B_o}\right)$$

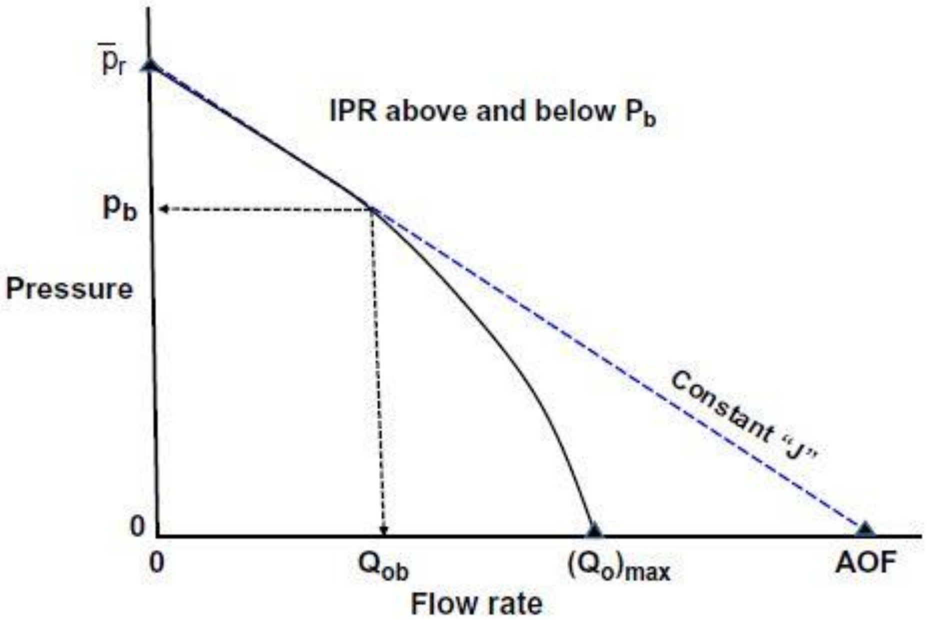

FIGURE 2-4: Inflow performance relationship below bubble point pressure.

Treating the term between the two brackets as a constant c, the above equation can be written in the following form:

$$J = c\left(\frac{k_{ro}}{\mu_o B_o}\right) \quad \text{……………. (2-8)}$$

With the coefficient c as defined by:

$$c = \frac{0.00708\,k\,h}{\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s}$$

Equation 2-8 reveals that the variables affecting the productivity index are essentially those that are pressure dependent, i.e.:

- ❖ Oil viscosity μ_o
- ❖ Oil formation volume factor Bo
- ❖ Relative permeability to oil kro

Figure 2-5 schematically illustrates the behavior of those variables as a function of pressure. Figure 2-6 shows the overall effect of changing the pressure on the term ($k_{ro}/\mu_o B_o$). Above the bubble-point pressure p_b, the relative oil permeability k_{ro} equals unity (kro = 1) and the term ($k_{ro}/\mu_o B_o$) is almost constant. As the pressure declines below p_b, the gas is released from solution, which can cause a large decrease in both k_{ro} and ($k_{ro}/\mu_o B_o$). Figure 2-7 shows qualitatively the effect of reservoir depletion on the IPR.

There are several empirical methods that are designed to predict the nonlinearity behavior of the IPR for solution gas drive reservoirs. Most of these methods require at least one stabilized flow test in which Q_o and p_{wf} are measured. All the methods include the following two computational steps:

- ❖ Using the stabilized flow test data, construct the IPR curve at the current average reservoir pressure pr.

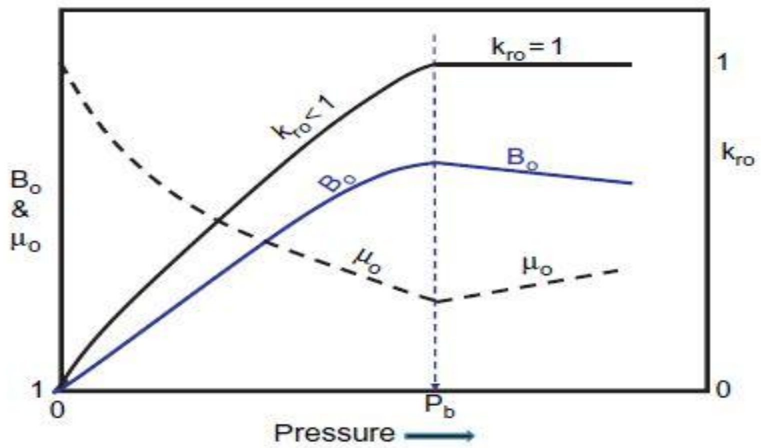

FIGURE 2-5: Effect of pressure on B_o, μ_o, and k_{ro}.

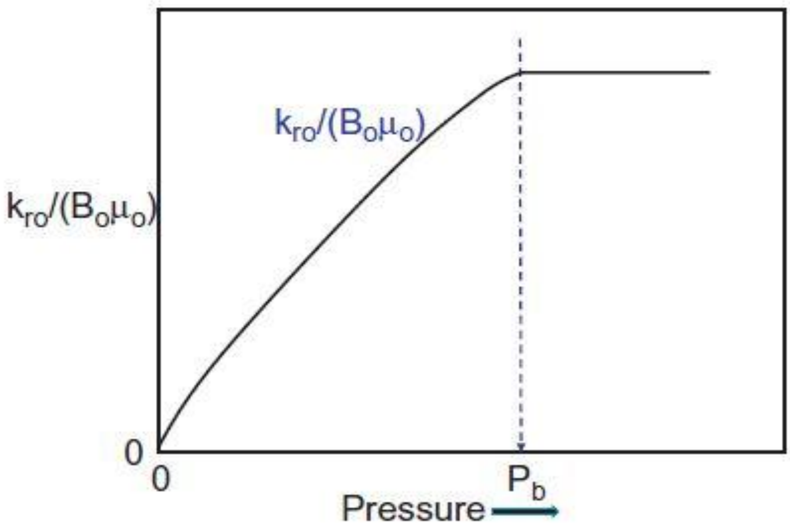

FIGURE 2-6: $k_{ro}/(B_o\mu_o)$ as a function of pressure.

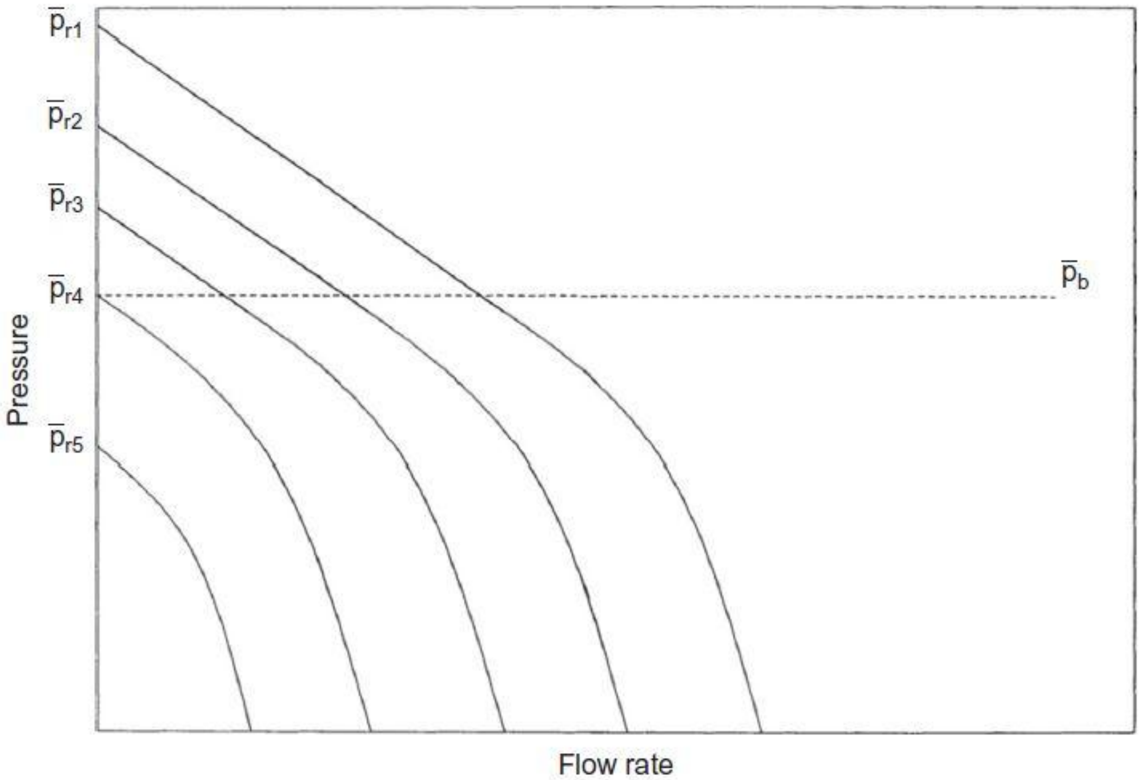

FIGURE 2-7: Effect of reservoir pressure on IPR.

- ❖ Predict future inflow performance relationships as to the function of average reservoir pressures.

The following empirical methods that are designed to generate the current and future inflow performance relationships:

- ❖ Vogel's Method
- ❖ Wiggins' Method
- ❖ Standing's Method
- ❖ Fetkovich's Method
- ❖ The Klins-Clark Method

Vogel's Method

Vogel (1968) used a computer model to generate IPRs for several hypothetical saturated-oil reservoirs that are producing under a wide range of conditions. Vogel normalized the calculated IPRs and expressed the relationships in a dimensionless form. He normalized the IPRs by introducing the following dimensionless parameters:

$$\text{dimensionless pressure} = \frac{p_{wf}}{\overline{p}_r}$$

$$\text{dimensionless } \boxed{\text{Flow rate}} = \frac{Q_o}{(Q_o)_{max}}$$

Where $(Q_o)_{max}$ is the flow rate at zero wellbore pressure, i.e., AOF.

Vogel plotted the dimensionless IPR curves for all the reservoir cases and arrived at the following relationship between the above dimensionless parameters:

$$\frac{Q_o}{(Q_o)_{max}} = 1 - 0.2\left(\frac{p_{wf}}{\overline{p}_r}\right) - 0.8\left(\frac{p_{wf}}{\overline{p}_r}\right)^2 \quad \text{………………..} \ (2\text{-}9)$$

Where:

Q_o = oil rate at p_{wf}

$(Q_o)_{max}$ = maximum oil flow rate at zero wellbore pressure, i.e., AOF

P_r = current average reservoir pressure, psig

p_{wf} = wellbore pressure, psig

Notice: that p_{wf} and p_r must be expressed in psig.

Vogel's method can be extended to account for water production by replacing the dimensionless rate with $Q_L/(Q_L)_{max}$ where $Q_L = Q_o + Q_w$. This has proved to be valid for wells producing at water cuts as high as 97%. The method requires the following data:

- Current average reservoir pressure p_r
- Bubble-point pressure p_b
- Stabilized flow test data that include Q_o at p_{wf}

Vogel's methodology can be used to predict the IPR curve for the following two types of reservoirs:

- Saturated oil reservoirs p_r < or = p_b
- Under saturated oil reservoirs $p_r > p_b$

Saturated Oil Reservoirs

When the reservoir pressure equals the bubble-point pressure, the oil reservoir is referred to as a saturated-oil reservoir. The computational procedure of applying Vogel's method in a saturated oil reservoir to generate the IPR curve for a well with a stabilized flow data point, i.e., a recorded Q_o value at p_{wf}, is summarized below:

Step 1: Using the stabilized flow data, i.e., Q_o and p_{wf}, calculate:

$(Q_o)_{max}$ from Equation 2-9, or

$$(Q_o)_{max} = Q_o / \left[1 - 0.2\left(\frac{P_{wf}}{\bar{P}_r}\right) - 0.8\left(\frac{P_{wf}}{\bar{P}_r}\right)^2\right]$$

Step 2: Construct the IPR curve by assuming various values for p_{wf} and calculating the corresponding Q_o from:

$$\boxed{Q_o} = (Q_o)_{max}\left[1 - 0.2\left(\frac{P_{wf}}{\bar{P}_r}\right) - 0.8\left(\frac{P_{wf}}{\bar{P}_r}\right)^2\right]$$

Example 2-2:

A well is producing from a saturated reservoir with an average reservoir pressure of 2500 psia. Stabilized production test data indicated that the stabilized rate and wellbore pressure are 350 STB/day and 2000 psia, respectively.

Calculate: Oil flow rate at p_{wf} = 1850 psia, Calculate oil flow rate assuming constant J, Construct the IPR by using Vogel's method and the constant productivity index approach.

Solution:

Part A

Step 1: Calculate $(Q_o)_{max}$:

$$(Q_o)_{max} = 350 / \left[1 - 0.2\left(\frac{2000}{2500}\right) - 0.8\left(\frac{2000}{2500}\right)^2\right]$$
$$= 1067.1 \text{ STB/day}$$

Step 2: Calculate Q_o at p_{wf} = 1850 psig by using Vogel's equation

$$Q_o = (Q_o)_{max}\left[1 - 0.2\left(\frac{P_{wf}}{\bar{P}_r}\right) - 0.8\left(\frac{P_{wf}}{\bar{P}_r}\right)^2\right]$$
$$= 1067.1\left[1 - 0.2\left(\frac{1850}{2500}\right) - 0.8\left(\frac{1850}{2500}\right)^2\right] = 441.7 \text{ STB/day}$$

Part B

Calculating oil flow rate by using the constant J approach

Step 1: Apply Equation 2-1 to determine J

$$J = \frac{350}{2500 - 2000} = 0.7 \text{ STB/day/psi}$$

Step 2: Calculate Q_o

$$Q_o = J(\overline{p}_r - p_{wf}) = 0.7(2500 - 1850) = 455 \text{ STB/day}$$

Part C

Generating the IPR by using the constant J approach and Vogel's method:
Assume several values for p_{wf} and calculate the corresponding Q_o.

P_{wf}	Vogel's	$Q_o = J(\overline{p}_r - p_{wf})$
2500	0	0
2200	218.2	210
1500	631.7	700
1000	845.1	1050
500	990.3	1400
0	1067.1	1750

Under saturated Oil Reservoirs

Beggs (1991) pointed out that in applying Vogel's method for under-saturated reservoirs, there are two possible outcomes to the recorded stabilized flow test data that must be considered, as shown schematically in Figure 2-8:

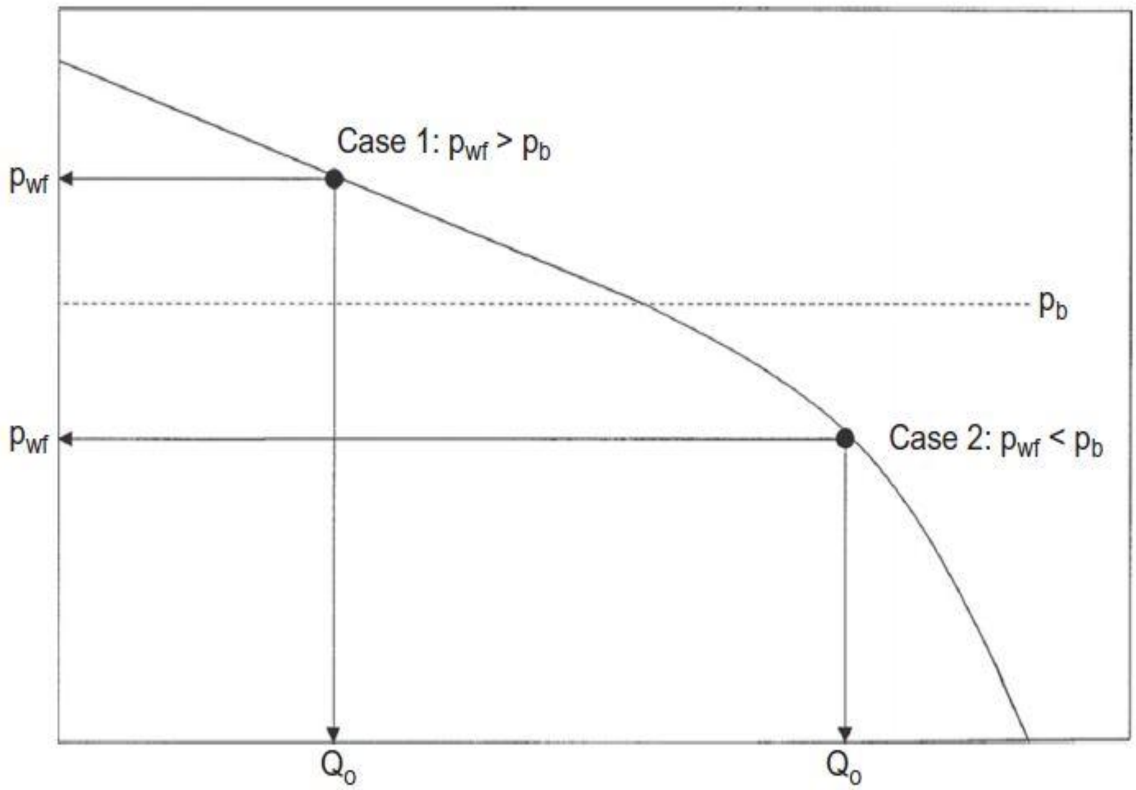

FIGURE 2-8: Stabilized flow test data.

- The recorded stabilized bottom-hole flowing pressure is greater than or equal to the bubble-point pressure, i.e. p_{wf} > or = p_b
- The recorded stabilized bottom-hole flowing pressure is less than the bubble-point pressure $p_{wf} < p_b$

Case 1: The Value of the Recorded Stabilized p_{wf} > or = p_b

Beggs outlined the following procedure for determining the IPR when the stabilized bottom-hole pressure is greater than or equal to the bubble point pressure (Figure 2-8):

Step 1: Using the stabilized test data point (Q_o and p_{wf}) calculate the productivity index J:

$$J = \frac{Q_o}{\overline{p}_r - p_{wf}}$$

Step 2: Calculate the oil flow rate at the bubble-point pressure:

$$Q_{ob} = J(\overline{p}_r - P_b) \qquad \text{.................. (2-10)}$$

Where Q_{ob} is the oil flow rate at p_b

Step 3: Generate the IPR values below the bubble-point pressure by assuming different values of $p_{wf} < p_b$ and calculating the corresponding oil flow rates by applying the following relationship:

$$Q_o = Q_{ob} + \frac{Jp_b}{1.8}\left[1 - 0.2\left(\frac{p_{wf}}{p_b}\right) - 0.8\left(\frac{p_{wf}}{p_b}\right)^2\right] \qquad \text{......... (2-11)}$$

The maximum oil flow rate (Q_o max or AOF) occurs when the bottom-hole flowing pressure is zero, i.e. $p_{wf} = 0$, which can be determined from the above expression as:

$$Q_{o\ max} = Q_{ob} + \frac{Jp_b}{1.8}$$

It should be pointed out that when p_{wf} > or = p_b, the IPR is linear and is described by:

$$Q_o = J(\overline{p}_r - p_{wf})$$

Example 2-3:

An oil well is producing from an undersaturated reservoir that is characterized by a bubble-point pressure of 2130 psia. The current average reservoir pressure is 3000 psig. Available flow test data show that the well produced 250 STB/day at a stabilized p_{wf} of 2500 psia. Construct the IPR data.

Solution:

The problem indicates that the flow test data were recorded above the bubble point pressure; therefore, the **Case 1** procedure for undersaturated reservoirs as outlined previously must be used.

Step 1: Calculate J using the flow test data.

$$J = \frac{250}{3000 - 2500} = 0.5 \text{ STB/day/psi}$$

Step 2: Calculate the oil flow rate at the bubble-point pressure by applying Equation 2-10.

$$Q_{ob} = 0.5(3000 - 2130) = 435 \text{ STB/day}$$

Step 3: Generate the IPR data by applying the constant J approach for all pressures above p_b and Equation 2-11 for all pressures below p_b.

P_{wf}	Equation	Q_o
3000	(2-6)	0
2800	(2-6)	100
2600	(2-6)	200
2130	(2-6)	435
1500	(2-11)	709
1000	(2-11)	867
500	(2-11)	973
0	(2-11)	1027

Case 2: The Value of the Recorded Stabilized pwf < pb

When the recorded p_{wf} from the stabilized flow test is below the bubble-point pressure, as shown in Figure 2-8, the following procedure for generating the IPR data is proposed:

Step 1: Using the stabilized well flow test data and combining Equation 2-10 with 2-11, solve for the productivity index J to give:

$$J = \frac{Q_o}{(\bar{p}_r - p_b) + \frac{p_b}{1.8}\left[1 - 0.2\left(\frac{p_{wf}}{p_b}\right) - 0.8\left(\frac{p_{wf}}{p_b}\right)^2\right]} \quad \text{..........} \quad (2\text{-}12)$$

Step 2: Calculate Q_{ob} by using Equation 2-10, or:

$$Q_{ob} = J(\overline{p}_r - p_b)$$

Step 3: Generate the IPR for p_{wf} > or = p_b by assuming several values for p_{wf} above the bubble point pressure and calculating the corresponding Q_o from:

$$Q_o = J(\overline{p}_r - p_{wf})$$

Step 4: Use Equation 2-11 to calculate Q_o at various values of p_{wf} below p_b, or:

$$Q_o = Q_{ob} + \frac{Jp_b}{1.8}\left[1 - 0.2\left(\frac{p_{wf}}{p_b}\right) - 0.8\left(\frac{p_{wf}}{p_b}\right)^2\right]$$

Example 2-4:

The well described in Example 2-3 was retested and the following results obtained: P_{wf} = 1700 psia, Q_o = 630.7 STB/day, Generate the IPR data using the new test data.

Solution:

Notice that the stabilized p_{wf} is less than p_b.

Step 1: Solve for J by applying Equation 2-12.

$$J = \frac{630.7}{(3000 - 2130) + \frac{2130}{1.8}\left[1 - \left(\frac{1700}{2130}\right) - \left(\frac{1700}{2130}\right)^2\right]}$$

Step 2:

$$Q_{ob} = 0.5\ (3000 - 21300) = 435\ \text{STB/day}$$

Step 3: Generate the IPR data.

P_{wf}	Equation	Q_o
3000	(2-6)	0
2800	(2-6)	100
2600	(2-6)	200
2130	(2-6)	435
1500	(2-11)	709
1000	(2-11)	867
500	(2-11)	973
0	(2-11)	1027

Quite often it is necessary to predict the well's inflow performance for future times as the reservoir pressure declines. Future well performance calculations require the development of a relationship that can be used to predict future maximum oil flow rates.