Al-Ayen University

College of Petroleum Engineering

Numerical Methods and Reservoir Simulation

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Lecture 4: Basic Equations of Fluid Flow in Porous Media (Part 2)

Outlines

- Unsteady-State Single-Phase Flow
- Linear Unsteady-State Single-Phase Flow
- Basic Differential Equations of Single-Phase Flow in Porous Media

Unsteady-State Single-Phase Flow

- Under the *steady-state* flowing condition, the same quantity of fluid enters the flow system as leaves it. In *unsteady-state* flow condition, the flow rate into an element of volume of a porous medium may not be the same as the flow rate out of that element.

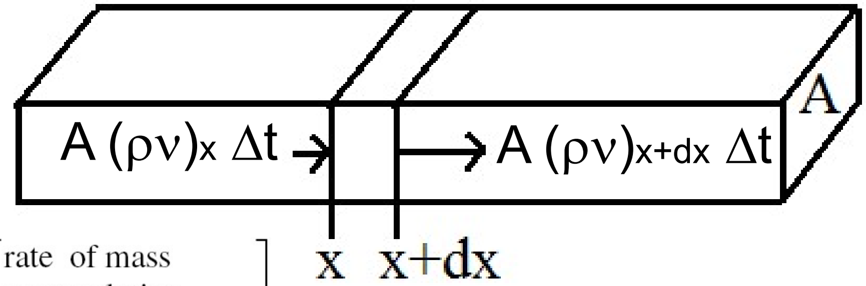
- The mathematical formulation of the diffusivity (transient-flow) equation is based on combining three independent equations:

  a. Continuity Equation

  b. Transport Equation: Basically, the transport equation is Darcy's equation in its generalized differential form.

  c. Compressibility Equation (Equation of State)

Linear Unsteady-State Single-Phase Flow



$$\begin{bmatrix}\text{mass entering}\\ \text{volume element}\\ \text{during interval }\Delta t\end{bmatrix} - \begin{bmatrix}\text{mass leaving}\\ \text{volume element}\\ \text{during interval }\Delta t\end{bmatrix} = \begin{bmatrix}\text{rate of mass}\\ \text{accumulation}\\ \text{during interval }\Delta t\end{bmatrix}$$

$$\{A(\rho v)_x\, \Delta t - A(\rho v)_{x+dx}\, \Delta t = A\, dx[(\emptyset\rho)_{t+\Delta t} - (\emptyset\rho)_t]\} \div A\Delta t dx$$

$$\left\{\frac{1}{\partial x}[(\rho v)_x - (\rho v)_{x+dx}] = \frac{1}{\partial t}[(\emptyset\rho)_{t+\Delta t} - (\emptyset\rho)_t]\right\}$$

$$-\frac{\partial(\rho v)}{\partial x} = \frac{\partial(\emptyset\rho)}{\partial t}$$

$$Darcy's\ Law:\ v = -\frac{k}{\mu}\frac{\partial p}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{k\rho}{\mu}\frac{\partial p}{\partial x}\right) = \frac{\partial(\emptyset\rho)}{\partial t}$$

The general partial differential equation used to describe the laminar flow of any fluid flowing in a linear direction in porous media.

Linear Unsteady-State Single-Phase Flow

Slightly Compressible Fluids

$$\frac{\partial}{\partial x}\left(\frac{\rho k}{\mu}\frac{\partial p}{\partial x}\right) = \frac{\partial(\emptyset\rho)}{\partial t}$$

The general partial differential equation used to describe the laminar flow of any fluid flowing in a linear direction in porous media.

$$\frac{k}{\mu}\left(\frac{\partial \rho}{\partial p}\frac{\partial p}{\partial x}\frac{\partial p}{\partial x} + \rho\frac{\partial^2 p}{dx^2}\right) = \rho\frac{\partial \emptyset}{\partial p}\frac{\partial p}{\partial t} + \emptyset\frac{\partial \rho}{\partial p}\frac{\partial p}{\partial t}$$

For a homogeneous porous medium and fluid viscosity is independent of pressure

$$\frac{\rho k}{\mu}\left(\frac{1}{\rho}\frac{\partial \rho}{\partial p}\left(\frac{\partial p}{\partial x}\right)^2 + \frac{\partial^2 p}{dx^2}\right) = \rho\emptyset\left(\frac{1}{\emptyset}\frac{\partial \emptyset}{\partial p} + \frac{1}{\rho}\frac{\partial \rho}{\partial p}\right)\frac{\partial p}{\partial t}$$

$$C_{rock} = \frac{1}{\emptyset}\frac{\partial \emptyset}{\partial p}, \qquad C_{fluid} = \frac{1}{\rho}\frac{\partial \rho}{\partial p}, \qquad C_t = C_{rock} + C_{fluid},$$

the value of $\frac{1}{\rho}\frac{\partial \rho}{\partial x}\left(\frac{\partial p}{\partial x}\right)^2$ *can be neglected for slightly compressible fluids*

$$\frac{\partial^2 p}{dx^2} = \frac{\emptyset\mu C_t}{k}\frac{\partial p}{\partial t}$$

This is the ***diffusivity equation for flow of slightly compressible fluids in linear homogeneous porous media*** with consistent units.

Basic Differential Equations of Single-Phase Flow in Porous Media

1D single phase flow of slightly compressible fluid in linear-reservoir with sources/sinks.

$$\text{Homogeneous Res.:} \quad 1.127\times10^{-3}\frac{k}{\mu}\frac{\partial^2 p}{\partial x^2}-\frac{q_{sc}(x,t)B}{V_b}=\frac{\phi c_t}{5.615}\frac{\partial p}{\partial t}$$

$$\text{Heterogeneous Res.:} \quad 1.127\times10^{-3}\frac{\partial}{\partial x}\left(\frac{k_x}{\mu}\frac{\partial p}{\partial x}\right)-\frac{q_{sc}(x,t)B}{V_b}=\frac{\phi c_t}{5.615}\frac{\partial p}{\partial t}$$

q_{sc} = surface rate (STB/D), $q_{sc} > 0$ for production (sink) well, and $q_{sc} < 0$ for source (injection) well, V_b = bulk volume (cuft), k (md), μ (cp), p (psi), t (days), x (ft), c_t(1/psi).

2D single phase flow of slightly compressible fluid in a heterogeneous linear-reservoir with sources/sinks.

$$1.127\times10^{-3}\left[\frac{\partial}{\partial x}\left(\frac{k_x}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{k_y}{\mu}\frac{\partial p}{\partial y}\right)\right]-\frac{q_{sc}(x,y,t)B}{V_b}=\frac{\phi c_t}{5.615}\frac{\partial p}{\partial t}$$

Basic Differential Equations of Single-Phase Flow in Porous Media

Similar method (as in linear flow) can be used to derive the following equations for the radial flow:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu c_t}{0.000264\,k}\frac{\partial p}{\partial t}$$

The radial diffusivity equation for slightly compressible fluids

$$\frac{\partial^2 m(p)}{\partial r^2} + \frac{1}{r}\frac{\partial m(p)}{\partial r} = \frac{\phi \mu c_t}{0.000264\,k}\frac{\partial m(p)}{\partial t}$$

$$m(p) = \int_0^p \frac{2p}{\mu z}\,dp$$

The radial diffusivity equation for compressible fluids

THANK YOU