Al-Ayen University
Petroleum Engineering College

Mechanics

Dynamics

Title: Rotation

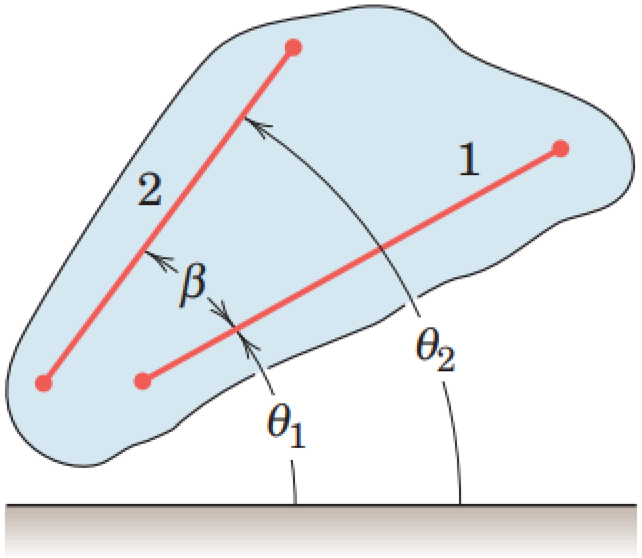
Dr. Mohaimen Al-Thamir

Lecture No. 9

Rotation

The rotation of a rigid body is described by its angular motion. The figure below shows a rigid body which is rotating as it undergoes plane motion in the plane of the figure. The angular positions of any two lines 1 and 2 attached to the body are specified by $\theta 1$ and $\theta 2$ measured from any convenient fixed reference direction. Because the angle β is invariant, the relation $\theta 2 = \theta 1 + \beta$ upon differentiation with respect to time gives $\dot{\theta} 2 = \dot{\theta} 1$ and $\ddot{\theta} 2 = \ddot{\theta} 1$ or, during a finite interval, $\Delta\theta 2 = \Delta\theta 1$.

Thus, all lines on a rigid body in its plane of motion have the same angular displacement, the same angular velocity, and the same angular acceleration.



Angular-motion relations

The angular velocity ω and angular acceleration α of a rigid body in plane rotation are, respectively, the first and second time derivatives of the angular position coordinate θ of any line in the plane of motion of the body. These definitions give

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} \quad \text{or} \quad \alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta} \qquad (1)$$

$$\omega\, d\omega = \alpha\, d\theta \quad \text{or} \quad \dot{\theta}\, d\dot{\theta} = \ddot{\theta}\, d\theta$$

For rotation with constant angular acceleration, the integrals of Eqs.1 becomes

$$\omega = \omega_0 + \alpha t$$

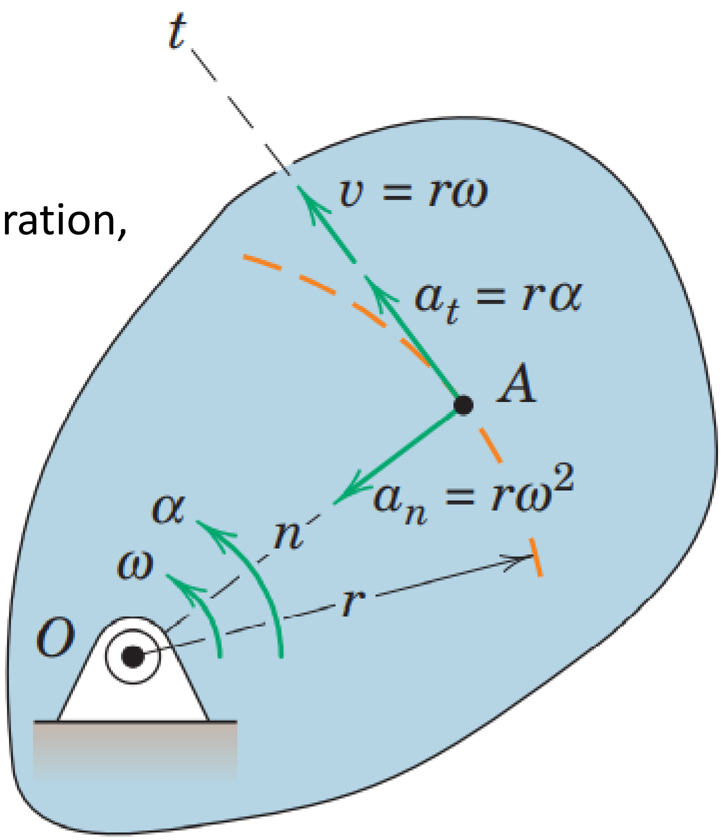
$$\omega^2 = {\omega_0}^2 + 2\alpha(\theta - \theta_0)$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Here θ_o and ω_o are the values of the angular position coordinate and angular velocity, respectively, at t = 0, and t is the duration of the motion considered.

Rotation about a Fixed Axis

When a rigid body rotates about a fixed axis, all points other than those on the axis move in concentric circles about the fixed axis. Thus, for the rigid body in the figure below rotating about a fixed axis normal to the plane of the figure through O, any point such as A moves in a circle of radius r.



With the notation $\omega = \dot{\theta}$ and $\alpha = \dot{\omega} = \ddot{\theta}$ for the angular velocity and angular acceleration, respectively, of the body we have

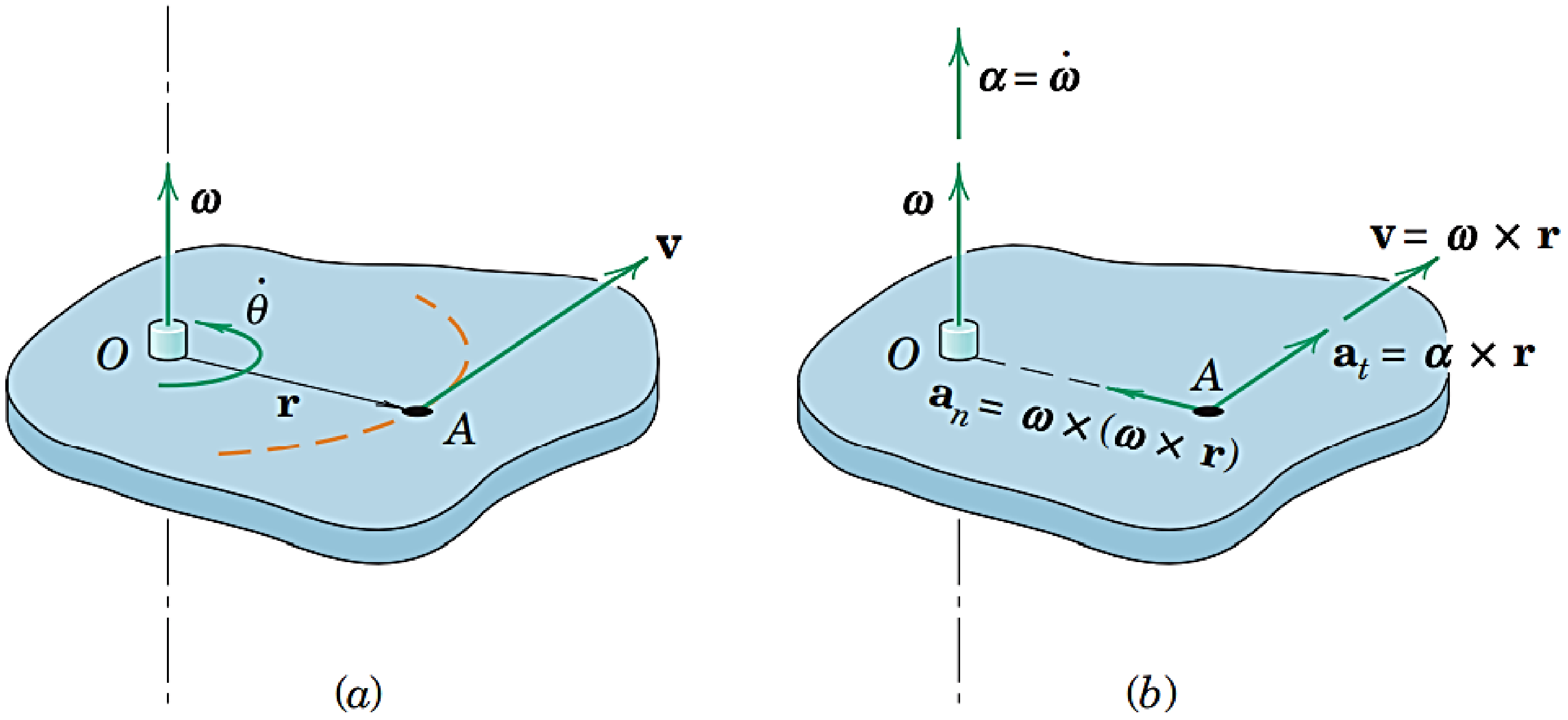
$$v = r\omega$$
$$a_n = r\omega^2 = v^2/r = v\omega$$
$$a_t = r\alpha$$

Vectors expressions

The angular velocity of the rotating body may be expressed by the vector ω normal to the plane of rotation and having a sense governed by the right-hand rule. The aforementioned quantities may be expressed alternatively using the cross product relationship of vector notation as explained below:

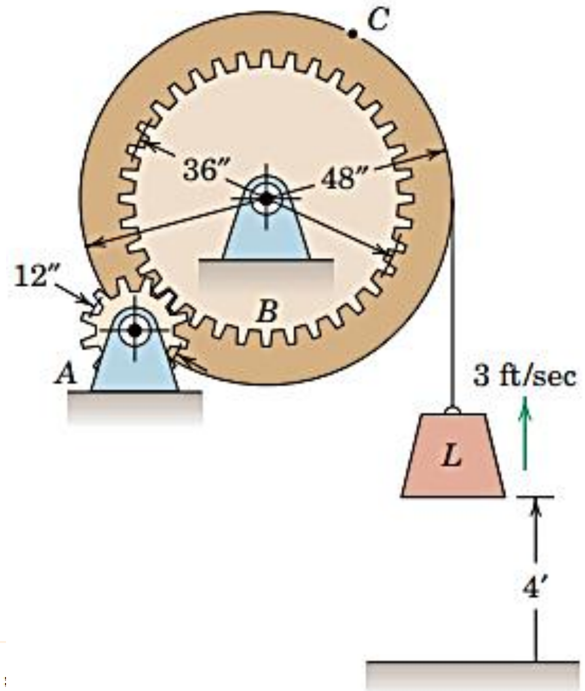
$$\mathbf{v} = \dot{\mathbf{r}} = \boldsymbol{\omega} \times \mathbf{r}$$

The order of the vectors to be crossed must be retained. The reverse order gives $\mathbf{r} \times \boldsymbol{\omega} = -\mathbf{v}$.



Example 1

The pinion A of the hoist motor drives gear B, which is attached to the hoisting drum. The load L is lifted from its rest position and acquires an upward velocity of 3 ft /sec in a vertical rise of 4 ft with constant acceleration. As the load passes this position, compute (a) the acceleration of point C on the cable in contact with the drum and (b) the angular velocity and angular acceleration of the pinion A.



Solution

(a)

$[v^2 = 2as]$ $\qquad a = a_t = v^2/2s = 3^2/[2(4)] = 1.125 \text{ ft/sec}^2$

$[a_n = v^2/r]$ $\qquad a_n = 3^2/(24/12) = 4.5 \text{ ft/sec}^2$

$[a = \sqrt{a_n{}^2 + a_t{}^2}]$ $\qquad a_C = \sqrt{(4.5)^2 + (1.125)^2} = 4.64 \text{ ft/sec}^2$ $\qquad$ *Ans.*

(b)

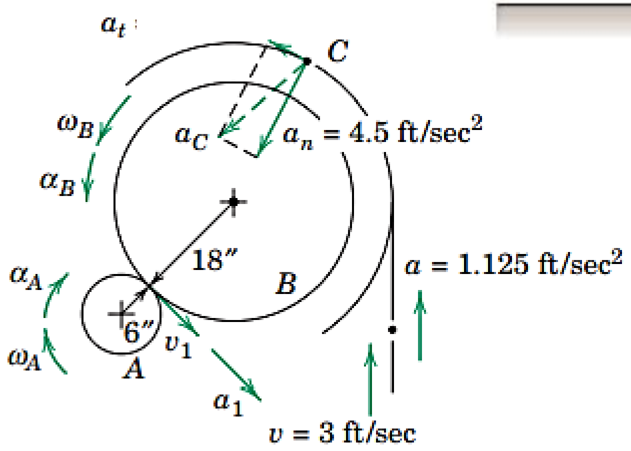
$[v = r\omega]$ $\qquad \omega_B = v/r = 3/(24/12) = 1.5 \text{ rad/sec}$

$[a_t = r\alpha]$ $\qquad \alpha_B = a_t/r = 1.125/(24/12) = 0.562 \text{ rad/sec}^2$

Then from $v_1 = r_A\omega_A = r_B\omega_B$ and $a_1 = r_A\alpha_A = r_B\alpha_B$, we have

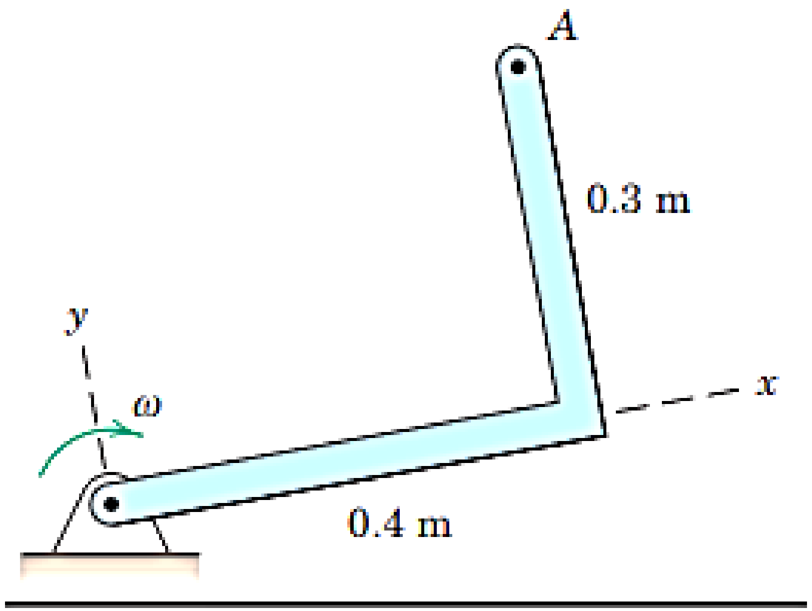
$$\omega_A = \frac{r_B}{r_A}\omega_B = \frac{18/12}{6/12}1.5 = 4.5 \text{ rad/sec CW} \qquad \textit{Ans.}$$

$$\alpha_A = \frac{r_B}{r_A}\alpha_B = \frac{18/12}{6/12}0.562 = 1.688 \text{ rad/sec}^2 \text{ CW} \qquad \textit{Ans.}$$



Example 2

The right-angle bar rotates clockwise with an angular velocity which is decreasing at the rate of 4 rad/s^2. Write the vector expressions for the velocity and acceleration of point A when $\omega = 2$ rad/s.



Solution Using the right-hand rule gives

$$\boldsymbol{\omega} = -2\mathbf{k} \text{ rad/s} \qquad \text{and} \qquad \boldsymbol{\alpha} = +4\mathbf{k} \text{ rad/s}^2$$

The velocity and acceleration of A become

$[\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}]$ $\quad \mathbf{v} = -2\mathbf{k} \times (0.4\mathbf{i} + 0.3\mathbf{j}) = 0.6\mathbf{i} - 0.8\mathbf{j}$ m/s *Ans.*

$[\mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})]$ $\quad \mathbf{a}_n = -2\mathbf{k} \times (0.6\mathbf{i} - 0.8\mathbf{j}) = -1.6\mathbf{i} - 1.2\mathbf{j}$ m/s^2

$[\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}]$ $\quad \mathbf{a}_t = 4\mathbf{k} \times (0.4\mathbf{i} + 0.3\mathbf{j}) = -1.2\mathbf{i} + 1.6\mathbf{j}$ m/s^2

$[\mathbf{a} = \mathbf{a}_n + \mathbf{a}_t]$ $\quad \mathbf{a} = -2.8\mathbf{i} + 0.4\mathbf{j}$ m/s^2 *Ans.*

The magnitudes of **v** and **a** are

$$v = \sqrt{0.6^2 + 0.8^2} = 1 \text{ m/s} \qquad \text{and} \qquad a = \sqrt{2.8^2 + 0.4^2} = 2.83 \text{ m/s}^2$$