Al-Ayen University
Petroleum Engineering College

Mechanics

Dynamics

Title: Work and Energy

Dr. Mohaimen Al-Thamir

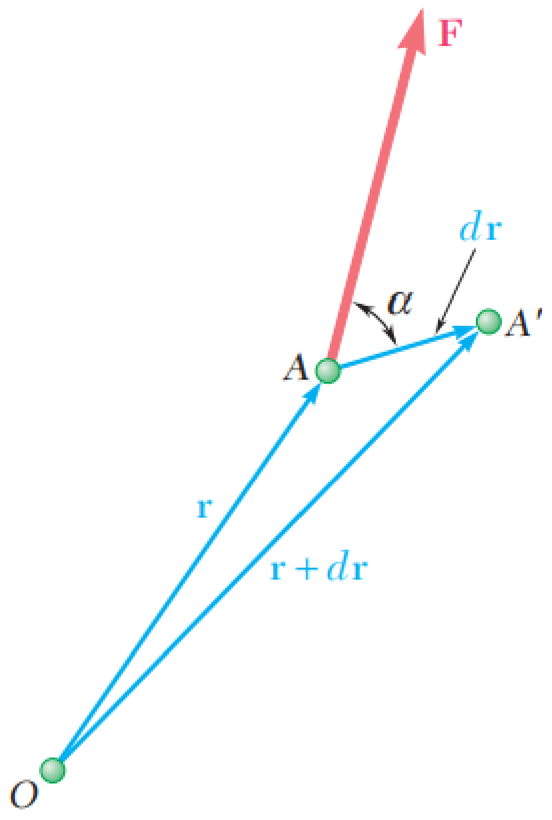
Lecture No. 10

Work of a force

Consider a particle which moves from a point A to a neighboring point A′ as shown in the figure below.

If **r** denotes the position vector corresponding to point A, the small vector joining A and A′ can be denoted by the differential d**r**; the vector d**r** is called the displacement of the particle. Now, let us assume that a force **F** is acting on the particle. The work of the force **F** corresponding to the displacement d**r** is defined as

$$dU = \mathbf{F} \cdot d\mathbf{r} \quad \Rightarrow \quad U_{A-A'} = \int_{A}^{A'} F \,.\, dr$$



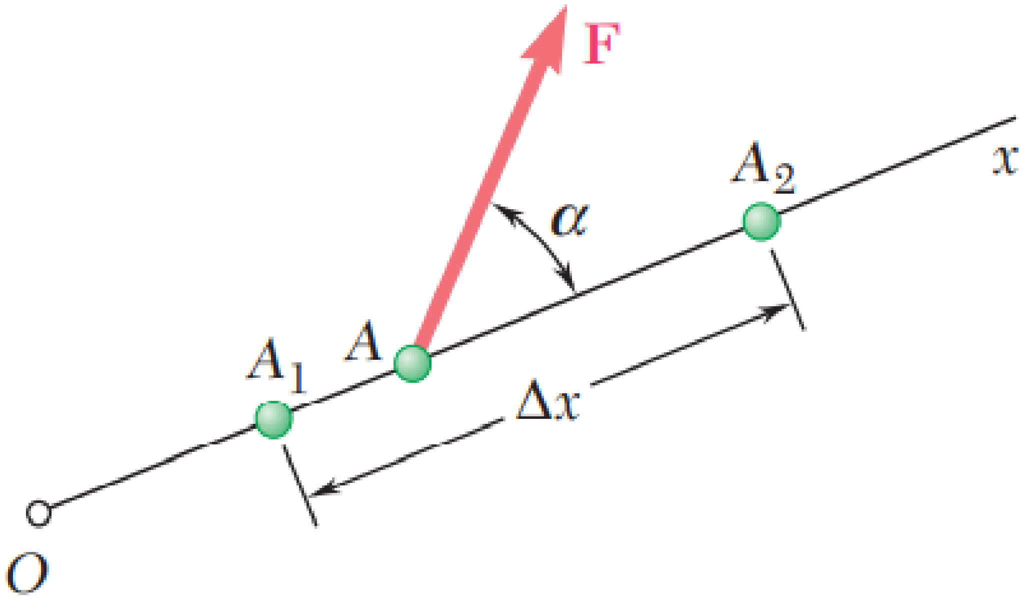
we can also express the work dU in terms of the rectangular components of the force and of the displacement:

$$dU = F_x\,dx + F_y\,dy + F_z\,dz$$

Work of a Constant Force in Rectilinear Motion

When a particle moving in a straight line is acted upon by a force F of constant magnitude and of constant direction as shown in the figure below, the work will be given as:

$$\boxed{U_{1\to2} = (F \cos \alpha)\, \Delta x}$$



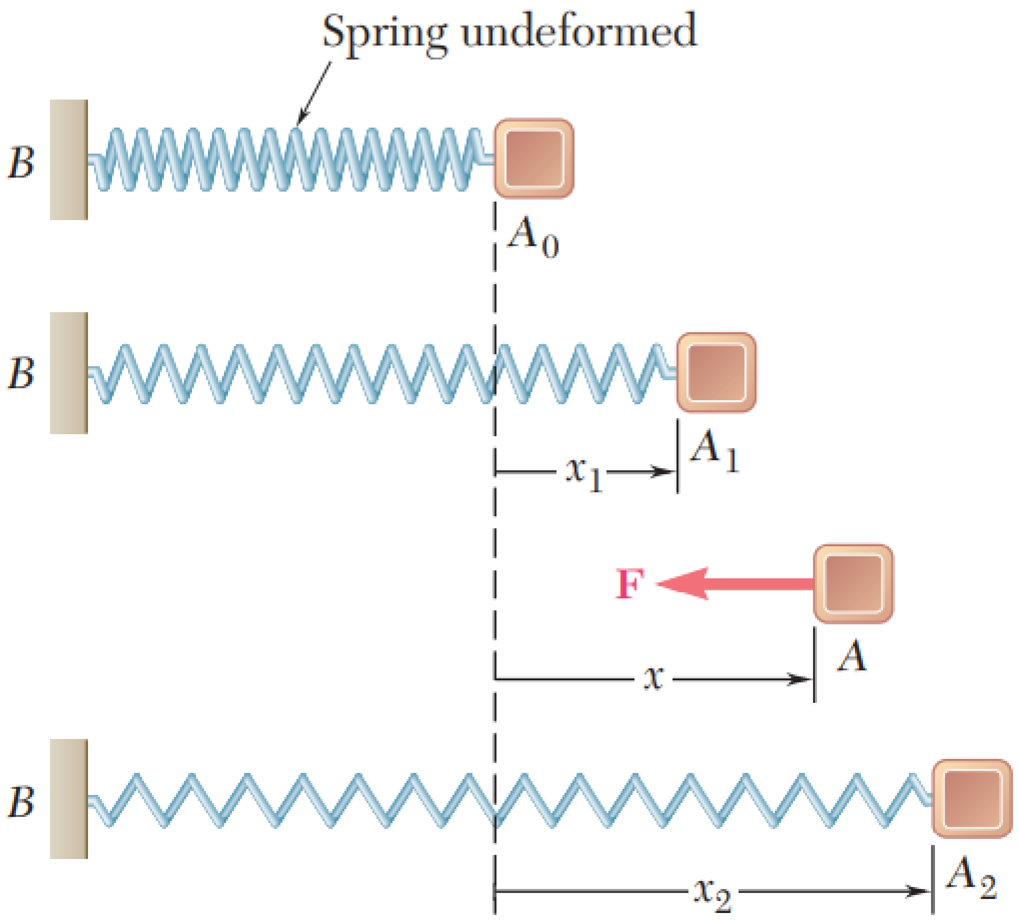
This implies that:

1- If the force F has the same direction as Δx, the work U reduces to F Δx.
2- If F has a direction opposite to that of Δx, the work is U = − F Δx.
3- Finally, if F is perpendicular to Δx, the work U is zero.

Work of the Force Exerted by a Spring

Consider a body A attached to a fixed point B by a spring; it is assumed that the spring is undeformed when the body is at A_o as shown in the figure below. Experimental evidence shows that the magnitude of the force **F** exerted by the spring on body A is proportional to the deflection x of the spring measured from the position A_o. We have

$$\boxed{F = Kx}$$



where k is the spring constant, expressed in N/m or kN/m if SI units are used and in lb/ft or lb/in if U.S. customary units are used. The work of the force F exerted by the spring during a finite displacement of the body from A_1(x = x_1) to A_2(x = x_2) is obtained by writing

$$dU = -F\,dx = -kx\,dx$$

$$\boxed{U_{1\rightarrow 2} = -\int_{x_1}^{x_2} kx\,dx = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2}$$

In other words, the energy stored in the spring (potential energy) (E_s) due to the force F can be expressed as:

$$\boxed{E_s = \frac{1}{2}\,k\,(x_2^2 - x_1^2)}$$

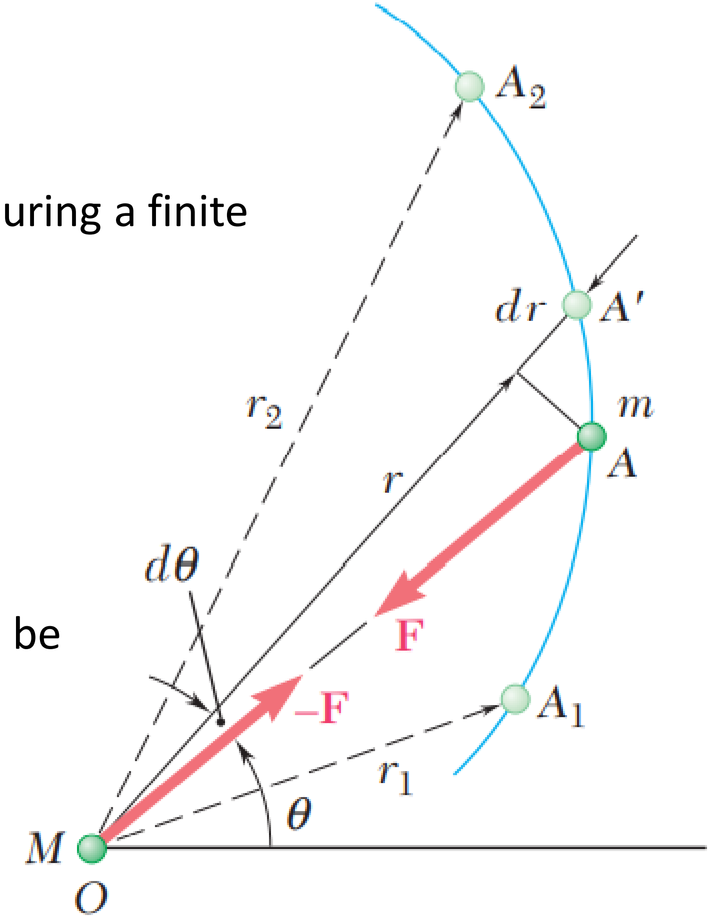
Work of a Gravitational Force (between bodies)

According to the Newton's law of gravitation, when two particles of mass M and m at a distance r from each other, as shown in the figure below, attract each other with equal and opposite forces **F** and − **F**, directed along the line joining the particles and of magnitude

$$F = G\frac{Mm}{r^2}$$

where G is the constant of gravitation. The work of the gravitational force F during a finite displacement from $A_1(r = r_1)$ to $A_2(r = r_2)$ is therefore

$$U_{1\to2} = -\int_{r_1}^{r_2} \frac{GMm}{r^2}dr = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

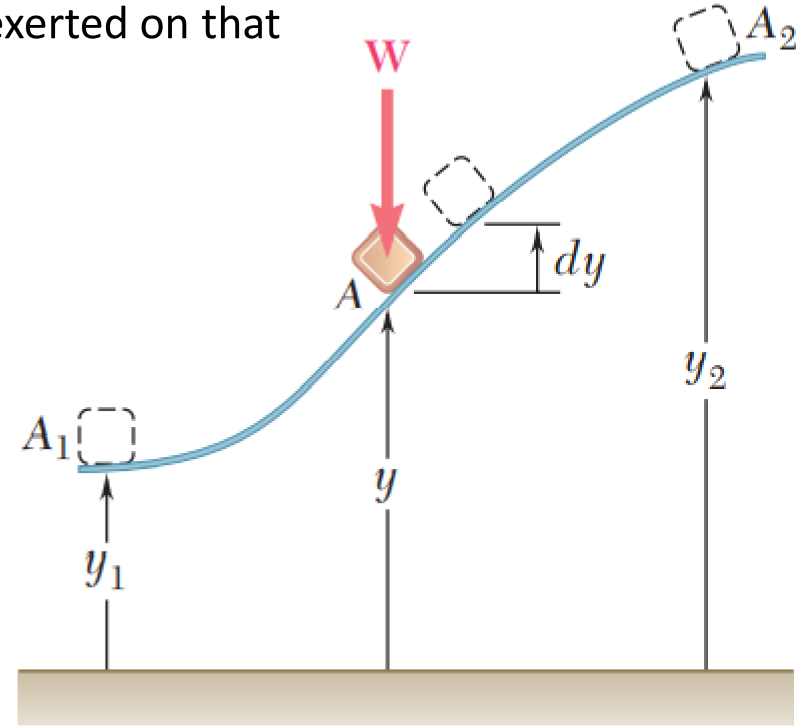


Thus, the potential energy (v_g) when the variation in the gravity force cannot be neglected can be expressed as

$$\boxed{V_g = -\frac{GMm}{r}}$$

Work of the Force of Gravity

The work of the weight W of a body A, i.e., of the force of gravity exerted on that body as shown in the figure, is obtained by



$$dU = -W\,dy$$

$$U_{1\to 2} = -\int_{y_1}^{y_2} W\,dy = Wy_1 - Wy_2$$

or

$$U_{1\to 2} = -W(y_2 - y_1) = -W\,\Delta y$$

or $\boxed{U_{1-2} = Wy_1 - Wy_2 = V_1 - V_2}$

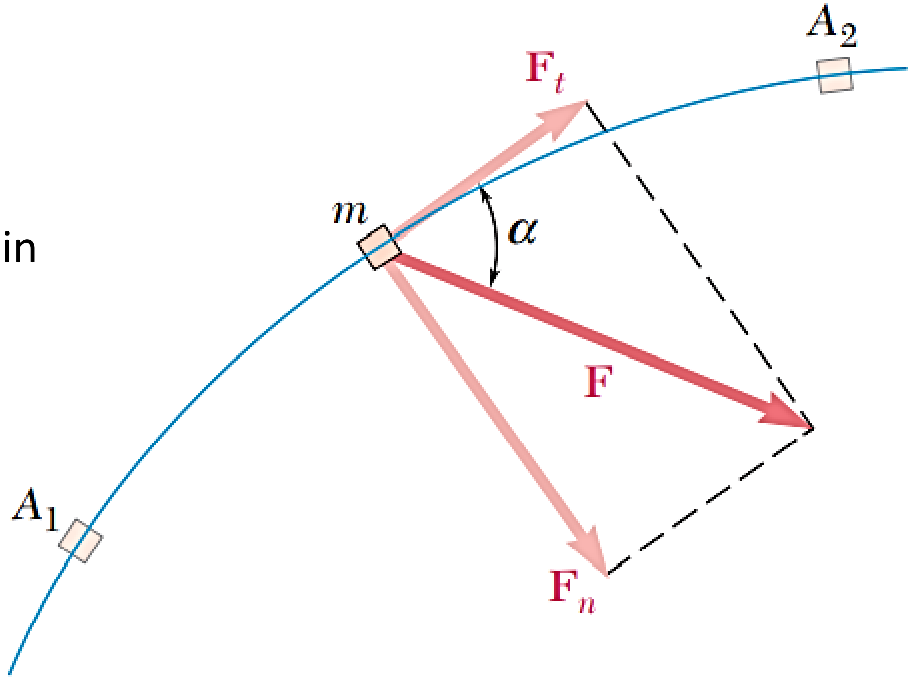
where Δy is the vertical displacement from A_1 to A_2. The work of the weight W is thus equal to the product of W and the vertical displacement of the center of gravity of the body. The work is positive when $\Delta y < 0$, that is, when the body moves down. This can also be expressed as the potential energy (P_E) as follow:

$$\boxed{P_E = W\,\Delta y = mg\,\Delta y}$$

Kinetic Energy of A Particle

Consider a particle of mass m acted upon by a force $\mathbf{F}$ and moving along a path which is either rectilinear or curved as shown in the figure below. Expressing Newton's second law in terms of the tangential components of the force and of the acceleration, we write

$$F_t = ma_t \qquad \text{or} \qquad F_t = m\frac{dv}{dt}$$



where v is the speed of the particle. Recalling that v = ds/dt, we obtain

$$F_t = m\frac{dv}{ds}\frac{ds}{dt} = mv\frac{dv}{ds}$$

$$F_t\,ds = mv\,dv$$

Integrating from A_1, where s = s_1 and v = v_1, to A_2, where s = s_2 and v = v_2, we write

$$U_{1-2} = \int_{s_1}^{s_2} F_t\ ds = \mathrm{m}\int_{v_1}^{v_2} v\ dv = \frac{1}{2}\,m\,v_2^2 - \frac{1}{2}\,\mathrm{m}\,v_1^2 = T_2 - T_1$$

where U_{1-2} is the work of the force F exerted on the particle during the displacement from A_1 to A_2; the corresponding kinetic energy values of the particle are T_1 and T_2, respectively. We can write

$$\boxed{T = \tfrac{1}{2}mv^2} \quad \text{and} \quad \boxed{U_{1\to 2} = T_2 - T_1} \Rightarrow \boxed{T_1 + U_{1\to 2} = T_2}$$

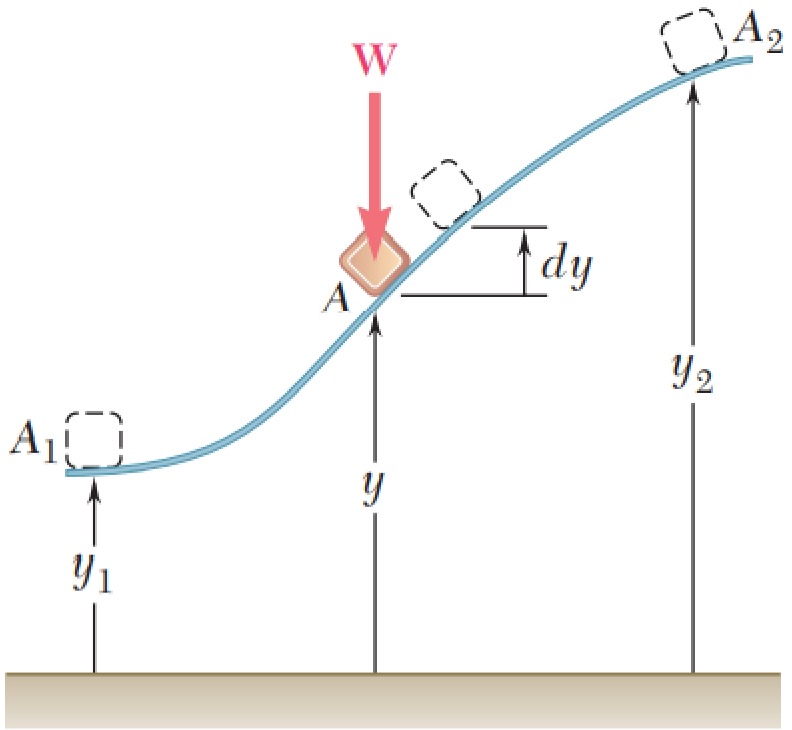
Conservation of energy

Consider a particle of mass m acted upon by a force **F** and moving along a path which is either rectilinear or curved as shown in the figure below.

From the law of work of gravity force:

$$U_{1-2} = Wy_1 - Wy_2 = V_1 - V_2 \qquad (1)$$

where $V_1 = Wy_1$, and $V_2 = Wy_2$



From the law of kinetic energy:

$$U_{1-2} = T_2 - T_1 \qquad (2)$$

From (1) and (2), the conservation of energy will be as follow:

$$V_1 - V_2 = T_2 - T_1$$

or

$$\boxed{V_1 + T_1 = T_2 + V_2}$$

Power and Efficiency

Power is defined as the time rate at which work is done. If ΔU is the work done during the time interval Δt, then the power during that time interval is

$$\text{Power} = \frac{dU}{dt} = \frac{\mathbf{F} \cdot d\mathbf{r}}{dt}$$

and, recalling that dr/dt represents the velocity **v** of the point of application of **F**,

$$\boxed{\text{Power} = \mathbf{F} \cdot \mathbf{v}}$$

The mechanical efficiency (η) of a machine can be defined as

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

Because of energy losses due to friction, the output work is always smaller than the input work, and consequently the power output is always smaller than the power input. The mechanical efficiency of a machine is therefore always less than 1.

Example 1

An automobile weighing 4000 lb is driven down a 5° incline at a speed of 60 mi/h when the brakes are applied, causing a constant total braking force (applied by the road on the tires) of 1500 lb. Determine the distance traveled by the automobile as it comes to a stop.

Solution

Kinetic Energy

Position 1: $v_1 = \left(60 \frac{\text{mi}}{\text{h}}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 88 \text{ ft/s}$

$$T_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(4000/32.2)(88)^2 = 481{,}000 \text{ ft} \cdot \text{lb}$$

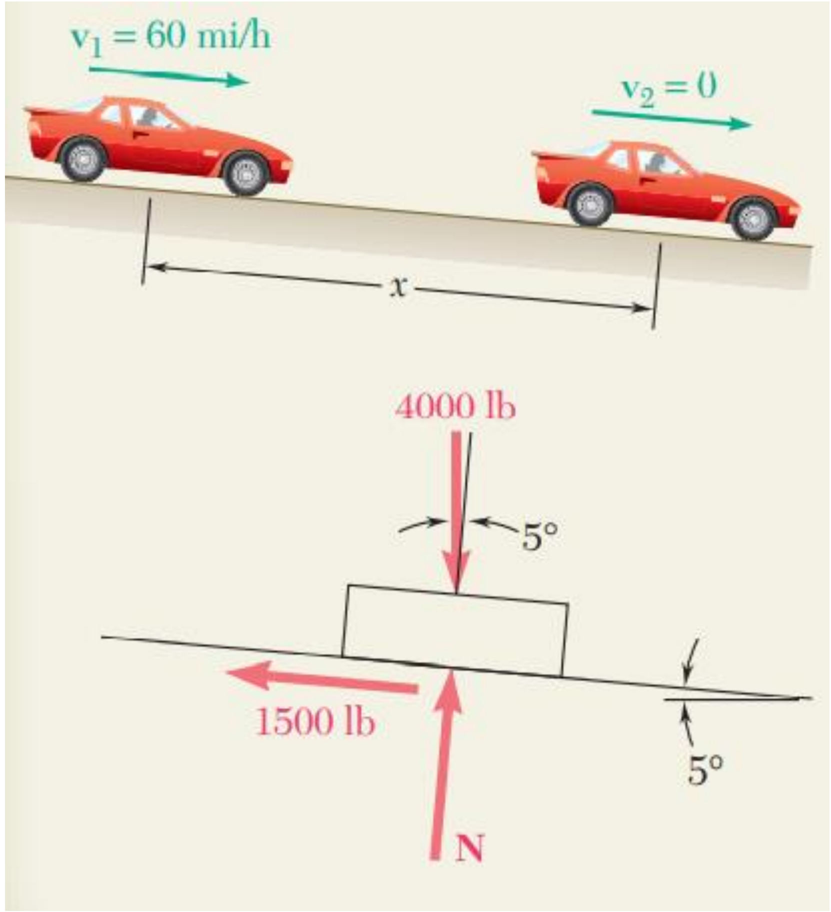
Position 2: $v_2 = 0 \qquad T_2 = 0$

Work $U_{1\rightarrow 2} = -1500x + (4000 \sin 5°)x = -1151x$

Principle of Work and Energy

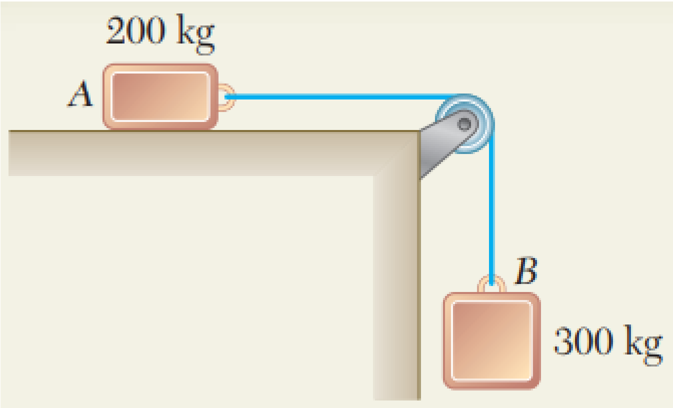
$$T_1 + U_{1\rightarrow 2} = T_2$$

$$481{,}000 - 1151x = 0 \qquad x = 418 \text{ ft}$$



Example 2

Two blocks are joined by an inextensible cable as shown. If the system is released from rest, determine the velocity of block A after it has moved 2 m. Assume that the coefficient of kinetic friction between block A and the plane is $\mu_k = 0.25$ and that the pulley is weightless and frictionless.



Solution

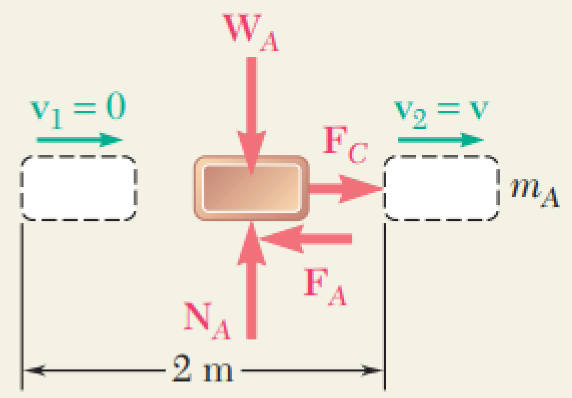
Work and Energy for Block A. We denote the friction force by $\mathbf{F}_A$ and the force exerted by the cable by $\mathbf{F}_C$, and write

$$m_A = 200 \text{ kg} \qquad W_A = (200 \text{ kg})(9.81 \text{ m/s}^2) = 1962 \text{ N}$$

$$F_A = \mu_k N_A = \mu_k W_A = 0.25(1962 \text{ N}) = 490 \text{ N}$$

$$T_1 + U_{1\to 2} = T_2: \qquad 0 + F_C(2 \text{ m}) - F_A(2 \text{ m}) = \tfrac{1}{2} m_A v^2$$

$$F_C(2 \text{ m}) - (490 \text{ N})(2 \text{ m}) = \tfrac{1}{2}(200 \text{ kg})v^2 \qquad (1)$$

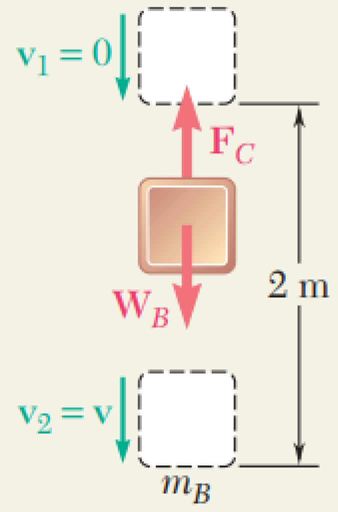


Work and Energy for Block B. We write

$$m_B = 300 \text{ kg} \qquad W_B = (300 \text{ kg})(9.81 \text{ m/s}^2) = 2940 \text{ N}$$

$$T_1 + U_{1\to 2} = T_2: \qquad 0 + W_B(2 \text{ m}) - F_C(2 \text{ m}) = \tfrac{1}{2} m_B v^2$$

$$(2940 \text{ N})(2 \text{ m}) - F_C(2 \text{ m}) = \tfrac{1}{2}(300 \text{ kg})v^2 \qquad (2)$$



Adding the left-hand and right-hand members of (1) and (2), we observe that the work of the forces exerted by the cable on A and B cancels out:

$$(2940 \text{ N})(2 \text{ m}) - (490 \text{ N})(2 \text{ m}) = \tfrac{1}{2}(200 \text{ kg} + 300 \text{ kg})v^2$$

$$4900 \text{ J} = \tfrac{1}{2}(500 \text{ kg})v^2 \qquad v = 4.43 \text{ m/s}$$