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## POWER LAW MODEL

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## Determine $\mathbf{n}$ and $\mathbf{K}$ from equations

- $n=3.32 \log \frac{\theta_{600}}{\theta_{300}}$
- $K=\frac{\theta_{300}}{511^{\mathrm{n}_{\mathrm{p}}}}$
- Where:
- $\theta_{600}=2 \mathrm{PV}+\mathrm{YP}$
- $\theta_{300}=\mathbf{P V}+\mathbf{Y P}$


## Pipe Flow

- $\hat{\mathbf{V}}=\frac{24.5 \times \mathbf{Q}}{\mathrm{D}^{2}}$
- $\mathbf{V}_{\mathbf{c}}=\left(\frac{5.82 \times 10^{4} \times K}{\rho}\right)^{\left(\frac{1}{2-n}\right)}\left(\frac{1.6 \times(3 n+1)}{D \times 4 n}\right)^{\left(\frac{n}{2-n}\right)}$
- If $\hat{V}>V_{c}$, flow is turbulent; use $\quad \mathbf{P}=\frac{8.91 \times 10^{-5} \times \rho^{0.8} \times \mathbf{Q}^{1.8} \times \mathbf{P V}^{0.2} \times \mathrm{L}}{(\mathrm{D})^{4.8}}$
- If $V$ V $<V_{c}$, flow is laminar; use

$$
P=\left(\frac{K L}{300 \mathrm{D}}\right)\left(\frac{1.6 \dot{\mathbf{V}} \times(3 \mathrm{n}+1)}{\mathrm{D} \times 4 \mathrm{n}}\right)^{\mathrm{n}}
$$

## Annular Flow

- $\dot{\mathbf{V}}=\frac{24.5 \times \mathrm{Q}}{\mathrm{D}_{\mathrm{h}}^{2}-0 \mathrm{D}^{2}}$
- $\mathbf{V}_{\mathbf{c}}=\left(\frac{3.878 \times 10^{4} \times K}{\rho}\right)^{\left(\frac{1}{2-n}\right)}\left(\frac{2.4 \times(2 n+1)}{D_{e} \times 3 n}\right)^{\left(\frac{n}{2-n}\right)}$
- If V́ $>V_{c}$, flow is turbulent; use $\quad \mathbf{P}=\frac{8.91 \times 10^{-5} \times \boldsymbol{\rho}^{0.8} \times \mathbf{Q}^{1.8} \times \mathbf{P V}^{0.2} \times \mathbf{L}}{\left(\mathrm{D}_{\mathrm{h}}-\mathbf{O D}\right)^{3}\left(\mathrm{D}_{\mathrm{h}}+\mathrm{OD}\right)^{1.8}}$
- If $\hat{V}<\mathrm{V}_{\mathrm{c}}$, flow is laminar; use

$$
\mathbf{P}=\left(\frac{\mathrm{K} * \mathrm{~L}}{300 \mathrm{D}_{\mathrm{e}}}\right)\left(\frac{2.4 \hat{\mathbf{V}} \times(3 \mathrm{n}+1)}{\mathrm{D}_{\mathrm{e}} \times 3 \mathrm{n}}\right)^{\mathbf{n}}
$$

Ex.17: Using the Bingham plastic and power-law models, determine the various pressure drops, nozzle velocity and nozzle sizes for a section of $\mathbf{1 2 . 2 5}$ in ( $\mathbf{3 1 1 m m}$ ) hole. Two pumps are used to provide $\mathbf{7 0 0} \mathrm{gpm}(\mathbf{2 6 5 0} 1 / \mathrm{min})$. The Data:

- Plastic viscosity $=\mathbf{1 2} \mathrm{cp}, \quad$ Yield point $=\mathbf{1 2} \mathrm{lb} / 100 \mathrm{ft}^{2} \quad$ Mud weight $=\mathbf{8 . 8} \mathrm{lb} / \mathrm{gal}$
- Drillpipe $(\mathrm{ID}=\mathbf{4 . 2 7 6} \mathrm{in}, \mathrm{OD}=\mathbf{5} \mathrm{in}$, length $=\mathbf{6 4 8 0} \mathrm{ft})$
- Drill collars $(\mathrm{ID}=\mathbf{2 . 8 7 5} \mathrm{in}, \mathrm{OD}=\mathbf{8}$ in, length $=\mathbf{6 2 0} \mathrm{ft}(\mathbf{1 8 9} \mathrm{m}))$
- Last casing was $\mathbf{1 3 . 3 7 5}$ in with an ID of $\mathbf{1 2 . 5 6 5} \mathbf{i n . ~} \mathbf{1 3 . 3 7 5}$ in casing was set at $\mathbf{2 5 5 0} \mathbf{f t}$.
- The two pumps are to be operated at a maximum standpipe pressure of $\mathbf{2 2 0 0} \mathrm{psi}$.
- Assume a surface equipment type of 4.



## Bingham Plastic Model

- Solution:

Surface losses

- $\mathrm{P}_{1}=\operatorname{Ex} \rho^{0.8} \times \mathrm{Q}^{1.8} \times \mathrm{PV}^{0.2}=52 \mathrm{psi}$
- $\mathrm{P}_{1}=4.2 \times 10^{-5} \times 8.8^{0.8} \times 700^{1.8} \times 12^{0.2}=\mathbf{5 2} \mathbf{~ p s i}$

Pipe losses

## Pressure losses inside drillpipe

- $\hat{V}=\frac{24.5 \times \mathrm{Q}}{\mathrm{D}^{2}}=\frac{24.5 \times 700}{(4.276)^{2}}=937.97 \mathrm{ft} / \mathrm{min}$
- $\mathrm{V}_{\mathrm{C}}=\frac{97 \mathrm{PV}+97 \sqrt{\mathrm{PV}^{2}+8.2 \rho \mathrm{D}^{2} \mathrm{Y}_{\mathrm{P}}}}{\rho \mathrm{D}}=\frac{97 * 12+97 \sqrt{12^{2}+8.2 * 8.8 * 4.276^{2} * 12}}{8.8 * 4.276}=356.76 \mathrm{ft} / \mathrm{min}$
- Since $V>V_{c}$, flow is turbulent and pressure drop inside drill pipe is calculated from:
- $\mathrm{P}_{2}=\frac{8.91 \times 10^{-5} \rho^{0.8} \mathrm{Q}^{1.8} \mathrm{PV}^{0.2} \mathrm{~L}}{\mathrm{D}^{4.8}}=\frac{8.91 \times 10^{-5} \times 8.8^{0.8} \times 700^{1.8} \times 12^{0.2} \times 6480}{4.276^{4.8}}=\mathbf{6 6 8 . 4 5} \mathbf{~ p s i}$


## Pressure losses inside drill collars

- $\hat{V}=\frac{24.5 \times \mathrm{Q}}{\mathrm{D}^{2}}=\frac{24.5 \times 700}{(2.875)^{2}}=2074.85 \mathrm{ft} / \mathrm{min}$
- $\mathrm{V}_{\mathrm{C}}=\frac{97 \mathrm{PV}+97 \sqrt{\mathrm{PV}^{2}+8.2 \rho \mathrm{D}^{2} Y_{\mathrm{P}}}}{\rho \mathrm{D}}=\frac{97 * 12+97 \sqrt{12^{2}+8.2 * 8.8 * 2.875^{2} * 12}}{8.8 * 3}=373.61 \mathrm{ft} / \mathrm{min}$
- Since $V$ $>\mathrm{V}_{\mathrm{c}}$, flow is turbulent and pressure drop inside drill collar is calculated from:
- $\mathrm{P}_{3}=\frac{8.91 \times 10^{-5} \rho^{0.8} \mathrm{Q}^{1.8} \mathrm{PV}^{0.2} \mathrm{~L}}{\mathrm{D}^{4.8}}=\frac{8.91 \times 10^{-5} \times 8.8^{0.8} \times 700^{1.8} \times 12^{0.2} \times 620}{2.875^{4.8}}=\mathbf{4 2 9 . 9 3} \mathbf{~ p s i}$


## Pressure losses around drillcollar: Open hole section

- $\hat{V}=\frac{24.5 \times \mathrm{Q}}{\mathrm{Dh}^{2}-\mathrm{OD}^{2}}=\frac{24.5 \times 350}{12.25^{2}-8^{2}}=199.27 \mathrm{ft} / \mathrm{min}$
- $\mathrm{V}_{\mathrm{C}}=\frac{97 \mathrm{PV}+97 \sqrt{\mathrm{PV}^{2}+6.2 \rho \mathrm{D}_{\mathrm{e}}^{2} \mathrm{Y}_{\mathrm{P}}}}{\rho \mathrm{D}}=\frac{97 * 12+97 \sqrt{12^{2}+6.2 * 8.8 *(12.25-8)^{2} * 12}}{8.8 *(12.25-8)}=314.87 \mathrm{ft} / \mathrm{min}$
- Since $\hat{V}<\mathrm{V}_{\mathrm{c}}$, flow is laminar and the pressure loss around the drillcollar is determined from:
- $\mathrm{P}_{4}=\frac{\mathrm{LPV} \mathrm{V}}{}{ }^{\prime} \operatorname{L0000~D}_{\mathrm{e}}^{2}+\frac{\mathrm{LY}}{225 \mathrm{D}_{\mathrm{e}}}=\frac{620 \times 12 \times 199.27}{60000(12.25-8)^{2}}+\frac{620 \times 12}{225(12.25-8)}=\mathbf{9 . 1 4 \mathbf { p s i }}$


## Pressure losses around drillpipe: Open hole section

- $\hat{V}=\frac{24.5 \times \mathrm{Q}}{\mathrm{Dh}^{2}-\mathrm{OD}^{2}}=\frac{24.5 \times 350}{12.25^{2}-5^{2}}=137.13 \mathrm{ft} / \mathrm{min}$
- $\mathrm{V}_{\mathrm{C}}=\frac{97 \mathrm{PV}+97 \sqrt{\mathrm{PV}^{2}+6.2 \rho \mathrm{D}_{\mathrm{e}}^{2} \mathrm{Y}_{\mathrm{P}}}}{\rho \mathrm{D}}=\frac{97 * 12+97 \sqrt{12^{2}+6.2 * 8.8 *(12.25-5)^{2} * 12}}{8.8 *(12.656-5)}=300.87 \mathrm{ft} / \mathrm{min}$
- Since $V$ V́ $<$ Vc, flow is laminar and the pressure loss around the drillpipe is determined from:
- $\mathrm{P}_{5}=\frac{\mathrm{LPV} \mathrm{V}}{60000 \mathrm{D}_{\mathrm{e}}^{2}}+\frac{\mathrm{L} \mathrm{Y}_{\mathrm{p}}}{225 \mathrm{D}_{\mathrm{e}}}=\frac{3930 \times 12 \times 137.13}{60000(12.25-5)^{2}}+\frac{3930 \times 12}{225(12.25-5)}=\mathbf{3 0 . 9 6} \mathbf{~ p s i}$


## Pressure losses around drill pipe: Cased hole section

- $\hat{V}=\frac{24.5 \times \mathrm{Q}}{\mathrm{Dh}^{2}-\mathrm{OD}^{2}}=\frac{24.5 \times 700}{12.565^{2}-5^{2}}=129 \mathrm{ft} / \mathrm{min}$
- $\mathrm{V}_{\mathrm{C}}=\frac{97 \mathrm{PV}+97 \sqrt{\mathrm{PV}^{2}+6.2 \rho \mathrm{D}_{\mathrm{e}}^{2} \mathrm{Y}_{\mathrm{P}}}}{\rho \mathrm{D}}=\frac{97 * 12+97 \sqrt{12^{2}+6.2 * 8.8 *(12.565-5)^{2} * 12}}{8.8 *(12.565-5)}=300 \mathrm{ft} / \mathrm{min}$
- Since $V$ V́Vc, flow is laminar and the pressure loss around the drillpipe is determined from:
- $\mathrm{P}_{6}=\frac{\mathrm{LPV} \mathrm{V}^{\prime}}{60000 \mathrm{D}_{\mathrm{e}}^{2}}+\frac{\mathrm{L} \mathrm{Y}_{\mathrm{p}}}{225 \mathrm{D}_{\mathrm{e}}}=\frac{2550 \times 12 \times 129}{60000(12.565-5)^{2}}+\frac{2550 \times 12}{225(12.565-5)}=19.12 \mathrm{psi}$


## Pressure drops across bit

- Total pressure loss in circulating system, except bit.
- $\Delta \mathrm{P}_{\text {Total }}=\mathrm{P}_{1}+\mathrm{P}_{2}+\mathrm{P}_{3}+\mathrm{P}_{4}+\mathrm{P}_{5}+\mathrm{P}_{6}$
$\cdot=52+668.45+429.93+19.12+30.96+9.14=\mathbf{1 2 0 9 . 6} \mathbf{~ p s i}$
- Therefore, pressure drop available for bit $\left(\mathrm{P}_{\mathrm{bit}}\right)$
- $\Delta \mathrm{P}_{\text {bit }}=2200-1209.6=990.4 \mathbf{~ p s i}$

Determine nozzle velocity (ft/s)

- $\mathrm{V}_{\mathrm{n}}=33.36 \sqrt{\frac{\mathrm{P}_{\mathrm{bit}}}{\rho}}$
- $\mathrm{V}_{\mathrm{n}}=33.36 \sqrt{\frac{990.4}{8.8}}=353.9 \mathrm{ft} / \mathrm{s}$

Determine total area of nozzles (in ${ }^{2}$ )

- $\mathrm{A}_{\mathrm{n}}=\frac{0.32 * \mathrm{Q}}{\mathrm{V}_{\mathrm{n}}}=\frac{0.32 * 700}{353.9}=0.632 \mathrm{in}^{2}$
- Nozzle size (in multiples of ( $1 / 32$ )
- $\mathrm{d}_{\mathrm{n}}=32 \sqrt{\frac{4 * \mathrm{~A}_{\mathrm{n}}}{3 * \pi}}=32 * \sqrt{\frac{4 * 0.632}{3 * \pi}}=16.577^{\prime \prime}$
- Hence, select two nozzles of size 17 and one of size 16.


## B.H.C.P.

- B.H.C.P. $=0.052 \rho \mathrm{D}+\Delta \mathrm{P}_{\mathrm{ap}}+\Delta \mathrm{P}_{\mathrm{ac}}$
- $=0.052 * 8.8 * 7100+50+9.14=\mathbf{3 3 0 8} \mathbf{~ p s i}$
- E. C. D. $=\mathrm{IMW}+\frac{\Delta_{\mathrm{ap}}+\Delta_{\mathrm{ac}}}{0.052 \times \mathrm{D}}$
- E.C. D. $=8.8+\frac{50+9.14}{0.052 \times 7100}=\mathbf{8 . 9 6 ~ I b} / \mathbf{g a l}$


