Al-Ayen University
College of Petroleum Engineering

Reservoir Engineering II

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Lecture 6: Steady-State Flow of Reservoir Fluids (Part 2)

Ref.: ***Reservoir Engineering Handbook*** by **Tarek Ahmed**

Outlines

- **Steady-State Flow**
  - Radial Flow of Compressible Fluids (Gases)
    - Pressure-Squared Method
  - Horizontal Multiple-Phase Flow

Steady-State Flow

Radial Flow of Compressible Fluids (Gases)

For a radial gas flow, the Darcy's equation takes the form:

$$q_{gr} = \frac{0.001127(2\pi rh)k}{\mu_g} \frac{dp}{dr} \qquad \text{................ (1)}$$

q_{gr} = gas flow rate at radius r, bbl/day
r = radial distance, ft
h = zone thickness, ft
μ_g = gas viscosity, cp
p = pressure, psi
0.001127 = conversion constant from Darcy units to field units

Referring to the gas flow rate at standard condition as Q_g, the gas flow rate q_{gr} under pressure and temperature can be converted to that of standard condition by applying the real gas equation of state to both conditions, or:

$$\frac{5.615\, q_{gr}\, p}{zRT} = \frac{Q_g\, p_{sc}}{z_{sc}\, R\, T_{sc}}$$

$$\left(\frac{p_{sc}}{5.615\, T_{sc}}\right)\left(\frac{zT}{p}\right) Q_g = q_{gr} \qquad \text{................ (2)}$$

p_{sc} = standard pressure, psia
T_{sc} = standard temperature, °R
Q_g = gas flow rate, scf/day
q_{gr} = gas flow rate at radius r, bbl/day
p = pressure at radius r, psia
T = reservoir temperature, °R
z = gas compressibility factor at p and T
z_{sc} = gas compressibility factor at standard condition ≅ 1.0

Combining Equations 1 and 2 yields:

$$\left(\frac{p_{sc}}{5.615\ T_{sc}}\right)\left(\frac{zT}{p}\right)Q_g = \frac{0.001127(2\pi rh)k}{\mu_g}\frac{dp}{dr}$$

Assuming that $T_{sc} = 520\ °R$ and $p_{sc} = 14.7$ psia:

$$\left(\frac{T\ Q_g}{k\ h}\right)\frac{dr}{r} = 0.703\left(\frac{2p}{\mu_g\ z}\right)dp \quad \text{.................. (3)}$$

Integrating Equation 3 from the wellbore conditions (r_w and p_{wf}) to any point in the reservoir (r and p) to give:

$$\int_{r_w}^{r}\left(\frac{T\,Q_g}{k\,h}\right)\frac{dr}{r} = 0.703\int_{p_{wf}}^{p}\left(\frac{2p}{\mu_g\,z}\right)dp \quad \text{................ (4)}$$

Imposing Darcy's Law conditions on Equation 4, i.e.:

- Steady-state flow, which requires that Q_g is constant at all radii.
- Homogeneous formation, which implies that k and h are constant.

gives:

$$\left(\frac{T\,Q_g}{k\,h}\right)\ln\left(\frac{r}{r_w}\right) = 0.703\int_{p_{wf}}^{p}\left(\frac{2p}{\mu_g\,z}\right)dp$$

The term $\int_{p_{wf}}^{p} \left(\frac{2p}{\mu_g z}\right) dp$ can be expanded to give:

$$\int_{p_{wf}}^{p} \left(\frac{2p}{\mu_g z}\right) dp = \int_{o}^{p} \left(\frac{2p}{\mu_g z}\right) dp - \int_{o}^{p_{wf}} \left(\frac{2p}{\mu_g z}\right) dp$$

Combining the above relationships yields:

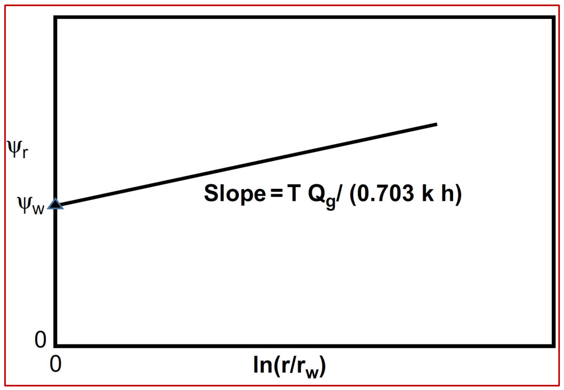
$$\left(\frac{TQ_g}{kh}\right) \ln\left(\frac{r}{r_w}\right) = 0.703 \left[\int_{0}^{p} \left(\frac{2p}{\mu_g z}\right) dp - \int_{0}^{p_{wf}} \left(\frac{2p}{\mu_g z}\right) dp\right]$$

$$m(p) = \psi = \int_{o}^{p} \left(\frac{2p}{\mu_g z}\right) dp = \textit{real gas potential} \text{ or } \textit{real gas pseudopressure}$$

$$\left(\frac{T\,Q_g}{k\,h}\right) \ln\frac{r}{r_w} = 0.703(\psi - \psi_w)$$

$$m(P) = \psi = \psi_w + \frac{Q_g T}{0.703\ kh} \ln\frac{r}{r_w}$$

This Equation indicates that a graph of ψ versus ln(r/rw) is a straight line.



$$Q_g = \frac{0.703\,kh(\psi_e - \psi_w)}{T\left(\ln\frac{r_e}{r_w}\right)}$$

ψ_e = real gas potential as evaluated from 0 to p_e, psi^2/cp
ψ_w = real gas potential as evaluated from 0 to P_{wf}, psi^2/cp
k = permeability, md
h = thickness, ft
r_e = drainage radius, ft
r_w = wellbore radius, ft
Q_g = gas flow rate, scf/day

Example

PVT data from a gas well in a Gas Field are given below:

p (psi)	μ_g(cp)	z
0	0.0127	1.000
400	0.01286	0.937
800	0.01390	0.882
1200	0.01530	0.832
1600	0.01680	0.794
2000	0.01840	0.770
2400	0.02010	0.763
2800	0.02170	0.775
3200	0.02340	0.797
3600	0.02500	0.827
4000	0.02660	0.860
4400	0.02831	0.896

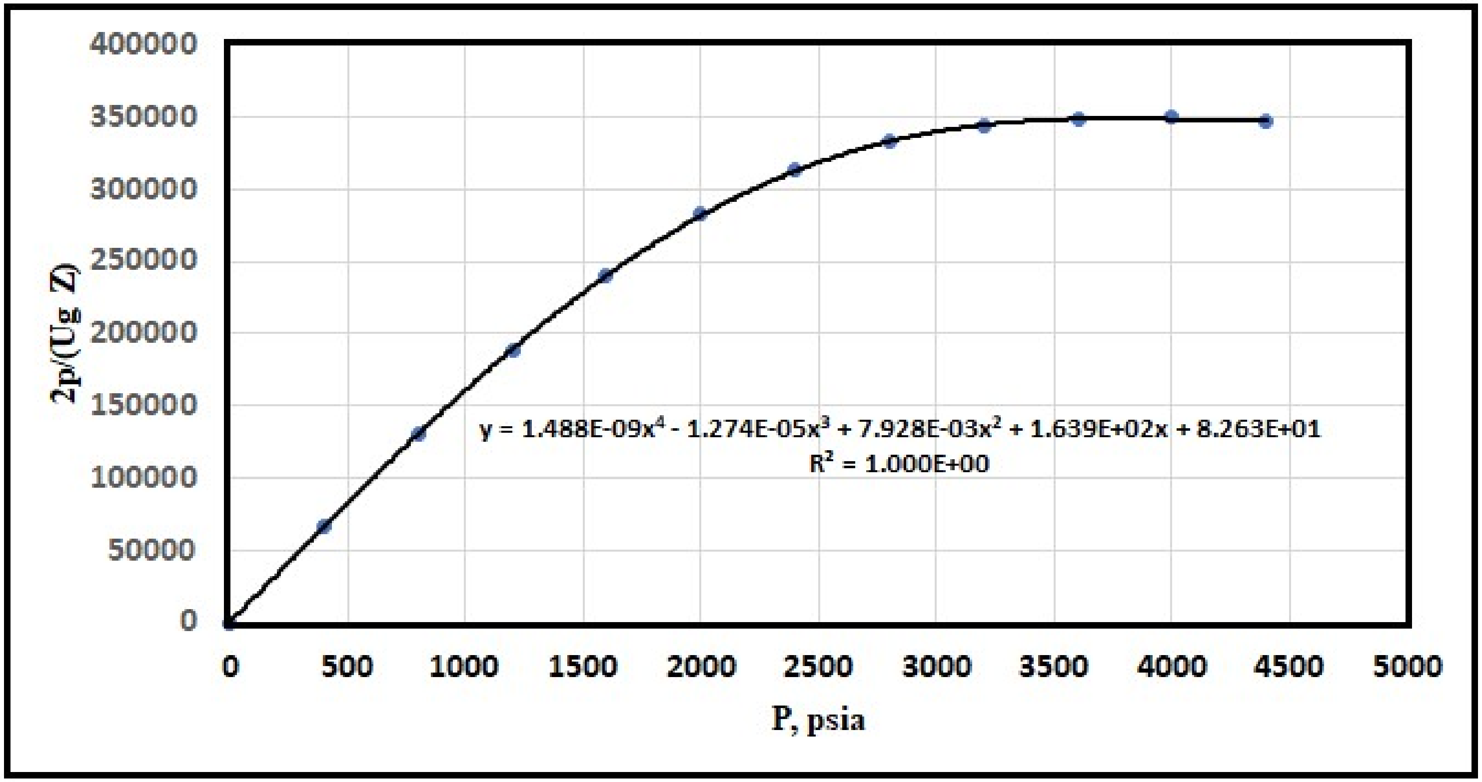
The well is producing at a stabilized bottom-hole flowing pressure of 3600 psi. The wellbore radius is 0.3 ft and the average reservoir pressure is 4400 psi. The following additional data are available: k= 65 mD, h=15 ft, re=1000 ft, and T= 600 °R. Calculate the gas flow rate.

Solution:

Step 1. Calculate the term $\left(\frac{2p}{\mu_g z}\right)$ for each pressure as shown below:

P, psia	U, cp	Z	2p/UZ, psia/cp
0	0.0127	1	0
400	0.01286	0.937	66391
800	0.0139	0.882	130508
1200	0.0153	0.832	188537
1600	0.0168	0.794	239894
2000	0.0184	0.77	282326
2400	0.0201	0.763	312983
2800	0.0217	0.775	332986
3200	0.0234	0.797	343167
3600	0.025	0.827	348247
4000	0.0266	0.86	349711
4400	0.02831	0.896	346924

Step 2. Plot the term $\left(\frac{2p}{\mu_g z}\right)$ versus pressure

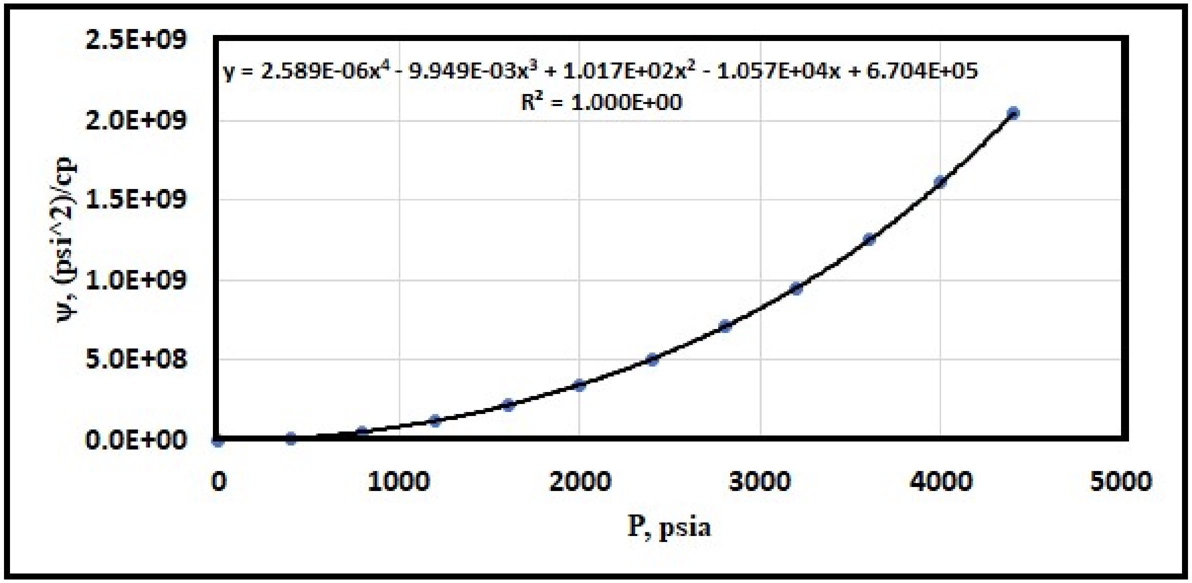


Note: Area under the curve represents $m(p) = \psi = \int_{o}^{p} \left(\frac{2p}{\mu_g z}\right) dp = \boxed{real\ \ gas\ \ potential}$ or $\boxed{real\ \ gas\ \ pseudopressure}$

Step 3. Calculate numerically the area under the curve for each value of p.

$$m(p) = \psi = \int_o^p \left(\frac{2p}{\mu_g z}\right) dp = \boxed{real\ \ gas\ \ potential} \text{ or } \boxed{real\ \ gas\ \ pseudopressure}$$

P, psia	ψ, psi2/cp
0	0
400	1.330E+07
800	5.368E+07
1200	1.220E+08
1600	2.194E+08
2000	3.477E+08
2400	5.097E+08
2800	7.098E+08
3200	9.541E+08
3600	1.251E+09
4000	1.610E+09
4400	2.046E+09
3600	1.251E+09



Step 4. Calculate the flow rate

$$Q_g = \frac{0.703\,kh(\psi_e - \psi_w)}{T\left(\ln\frac{r_e}{r_w}\right)}$$

$$Q_g = \frac{0.703\ (65)(15)\ (2.046 \times 10^9 - 1.251 \times 10^9)}{600\ (\ln\left(\frac{1000}{0.3}\right))} = 111.96\ MMscf/day$$

Pressure-Squared Method

This method is limited to flow calculations when the reservoir pressure is less that 2000 psi. It is considered that (z μ_g) is almost constant under a pressure range of < 2000 psia. The flow rate can be calculated from the following equation:

$$Q_g = \frac{kh\left(p_e^2 - p_{wf}^2\right)}{1422\,T\,\left(\mu_g\,z\right)_{avg}\,\ln\left(\frac{r_e}{r_w}\right)}$$

where

Q_g = gas flow rate, Mscf/day
k = permeability, md

The term $(\mu_g\ z)_{avg}$ is evaluated at an average pressure $\bar{p}$ that is defined by the following expression:

$$\bar{p} = \sqrt{\frac{p_{wf}^2 + p_e^2}{2}}$$

Example

Using the data given in the previous Example, re-solve for the gas flow rate by using the pressure-squared method. Compare with the exact method (i.e., real gas potential solution).

Solution

$$\bar{p} = \sqrt{\frac{p_{wf}^2 + p_e^2}{2}}$$

$$\bar{p} = \left[\frac{4400^2 + 3600^2}{2}\right]^{0.5} = 4020 \text{ psia}$$

μ_g= 0.02618 cp

Z= 0.8778

$$Q_g = \frac{kh\left(p_e^2 - p_{wf}^2\right)}{1422\,T\left(\mu_g\, z\right)_{avg}\ \ln\left(\frac{r_e}{r_w}\right)}$$

$$Q_g = \frac{(65)(15)(4400^2 - 3600^2)}{1422\ (600)(0.02618)(0.8778)\ \ln(\frac{1000}{0.3})} = 39{,}233.33\ Mscf/day$$

$$Absolute\ error = \frac{|39{,}233.33 \times 10^3 - 111.96 \times 10^6|}{111.96 \times 10^6} \times 100 = 65\ \%$$

Horizontal Multiple-Phase Flow

- **Oil Phase**

$$Q_o = \frac{0.00708(kh)(k_{ro})(p_e - p_{wf})}{\mu_o B_o \ln(r_e/r_w)}$$

- **Water Phase**

$$Q_w = \frac{0.00708(kh)(k_{rw})(p_e - p_{wf})}{\mu_w B_w \ln(r_e/r_w)}$$

The water-oil ratio is defined as:

$$WOR = \frac{Q_w}{Q_o}$$

- **Gas Phase**

In terms of the real gas potential:

$$Q_g = \frac{(kh)\, k_{rg}\, (\psi_e - \psi_w)}{1422\, T \ln(r_e/r_w)}$$

In terms of the pressure-squared:

$$Q_g = \frac{(kh)\, k_{rg}\, (p_e^2 - p_{wf}^2)}{1422\, (\mu_g z)_{avg}\, T \ln(r_e/r_w)}$$

where Q_g = gas flow rate, Mscf/day
k = absolute permeability, md
T = temperature, °R

The ***instantaneous GOR***, as expressed in *scf/STB*, is defined as:

$$GOR = R_s + \frac{Q_g\,(scf/day)}{Q_o(STB/day)}$$

THANK YOU