Al-Ayen University
Petroleum Engineering College

Mechanics

Dynamics

Title: Principals of dynamics and rectilinear motion

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Lecture No. 7

Principals of dynamics: Newton's laws

Law I: A particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.

Law II: The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.

Law III: The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear

Newton's second law forms the basis for most of the analysis in dynamics. For a particle of mass *m* subjected to a resultant force F, the law may be stated as

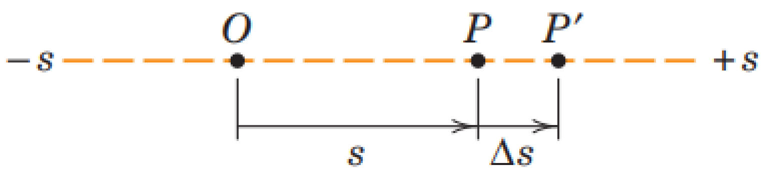
$$\boxed{\mathbf{F} = m\mathbf{a}}$$

where **a** is the resulting acceleration measured in a nonaccelerating frame of reference. Newton's first law is a consequence of the second law since there is no acceleration when the force is zero, and so the particle is either at rest or is moving with constant velocity. The third law constitutes the principle of action and reaction with which you should be thoroughly familiar from your work in statics.

Rectilinear motion

The rectilinear motion of a particle means that the direction of the motion is that of the given straight-line path.

Consider a particle P moving along a straight line as shown in the right figure. The position of P at any instant of time t can be specified by its distance s measured from some convenient reference point O fixed on the line. At time (t+Δt) the particle has moved to P′ and its coordinate becomes (s+Δs). The change in the position coordinate during the interval Δt is called the displacement Δs of the particle. The displacement would be negative if the particle moved in the negative s-direction



The instantaneous velocity of the particle, which is $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$ or $\boxed{v = \frac{ds}{dt} = \dot{s}}$ **(1)**

The instantaneous acceleration of the particle, which is $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$ or $\boxed{a = \frac{dv}{dt} = \dot{v}}$ or $\boxed{a = \frac{d^2 s}{dt^2} = \ddot{s}}$ **(2)**

From eq.(1) and first form of eq.(2) by eliminating dt, we obtain a differential equation relating displacement, velocity, and acceleration as follow:

$\boxed{v\,dv = a\,ds}$ or $\boxed{\dot{s}\,d\dot{s} = \ddot{s}\,ds}$ **(3)**

Integration to Rectilinear motion equations: Constant acceleration

At t = 0 with s= s_0 and v = v_0, the integrations of the differential equations 2 and 3 of rectilinear equations become:

$$\int_{v_0}^{v} dv = a\int_0^t dt \qquad \text{or} \qquad \boxed{v = v_0 + at}$$

$$\int_{v_0}^{v} v\,dv = a\int_{s_0}^{s} ds \qquad \text{or} \qquad \boxed{v^2 = {v_0}^2 + 2a(s - s_0)}$$

Substitution of the integrated expression for v into Eq.1 and integration with respect to t give:

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + at)\,dt \qquad \text{or} \qquad \boxed{s = s_0 + v_0 t + \frac{1}{2}at^2}$$

Important note: The above equations are necessarily restricted to the special case where the acceleration is constant. Otherwise, the given acceleration can be variable as it can be as a function of time, velocity, or displacement so that the above integrations must be re-calculated to consider the new acceleration state.

Example:

The position coordinate of a particle which is confined to move along a straight line is given by

$$s = 2t^3 - 24t + 6$$

where s is measured in meters from a convenient origin and t is in seconds. Determine:

(a) the time required for the particle to reach a velocity of 72 m/s from its initial condition at t = 0,

(b) the acceleration of the particle when v = 30 m /s, and

(c) the net displacement of the particle during the interval from t = 1 s to t = 4 s.

Solution. The velocity and acceleration are obtained by successive differentiation of s with respect to the time. Thus,

$$[v = \dot{s}] \qquad v = 6t^2 - 24 \text{ m/s}$$

$$[a = \dot{v}] \qquad a = 12t \text{ m/s}^2$$

(a) Substituting $v = 72$ m/s into the expression for v gives us $72 = 6t^2 - 24$, from which $t = \pm 4$ s. The negative root describes a mathematical solution for t before the initiation of motion, so this root is of no physical interest. Thus, the desired result is

$$t = 4 \text{ s} \qquad \textit{Ans.}$$

(b) Substituting $v = 30$ m/s into the expression for v gives $30 = 6t^2 - 24$, from which the positive root is $t = 3$ s, and the corresponding acceleration is

$$a = 12(3) = 36 \text{ m/s}^2 \qquad \textit{Ans.}$$

(c) The net displacement during the specified interval is

$$\Delta s = s_4 - s_1 \qquad \text{or}$$

$$\Delta s = [2(4^3) - 24(4) + 6] - [2(1^3) - 24(1) + 6]$$

$$= 54 \text{ m} \qquad \textit{Ans.}$$