Chapter Two

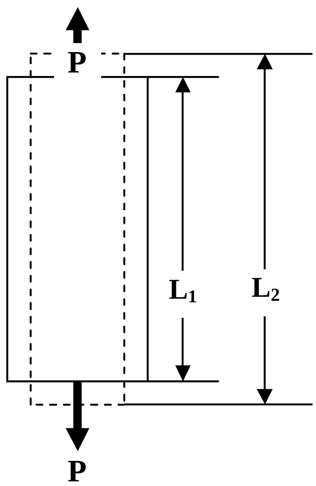
Simple Strain

Strain : Is a measure of the deformation of the material which is subjected to an external load , and its non-dimensional .

The strain may divided into : 1) Normal strain . 2) Shear strain

1) Normal Strain : It is occur due to normal stresses (tensile causes +ve strain and compressive stress causes –ve strain).

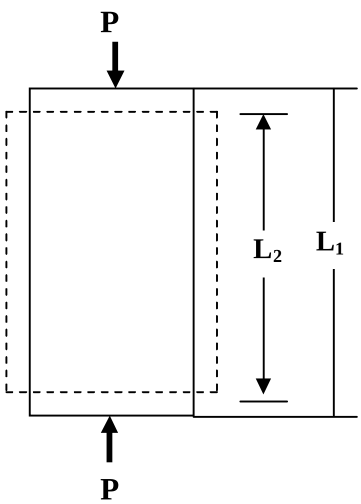
$$\varepsilon = \frac{\Delta L}{L} = \frac{L_2 - L_1}{L_1}$$



In tension :

$$\varepsilon_t = \frac{\Delta L}{L_1} = \frac{L_2 - L_1}{L_1} \quad \textbf{(+ve strain)}$$

In compression :

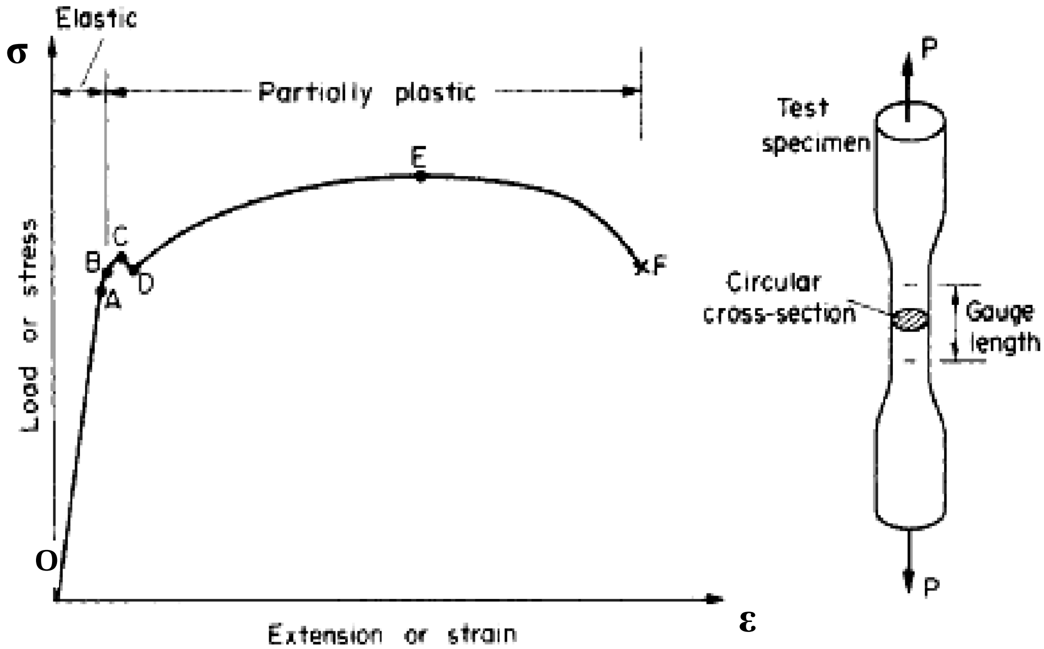


$$\varepsilon_c = \frac{\Delta L}{L_1} = \frac{L_2 - L_1}{L_1} \quad \textbf{(-ve strain)}$$

as L_1 **larger than** L_2

Stress-Strain Diagram :

In order to compare the strength of various materials to select the better material for using , it is necessary to carry out some standard form of tests to establish their relative properties . The most important of these tests is (tensile tests) , in which a circular steel bar of uniform cross-sectional area is subjected to gradually increasing tensile load until failure occurs , and measuring the change in selected length (gauge length) of bar simultaneously , and then plot the relation between the tensile stress and tensile strain for bar graph .



Stress-Strain diagram for mild steel

1-(O----A) : Straight line where the strain is linear proportional with stress $(\sigma \propto \varepsilon)$

$$\therefore \sigma = (Constant) * \varepsilon$$

$$\therefore \sigma = E * \varepsilon \qquad \textbf{(Hook's law)}$$

***E*-----Young modulus (modulus of elasticity) .**

$$\sigma = E * \varepsilon \; , \quad n\sigma = \frac{P}{A} \quad \text{and} \quad \varepsilon = \frac{\Delta L}{L} = \frac{\delta}{L}$$

$$\therefore \frac{P}{A} = E * \frac{\delta}{L} \qquad \Rightarrow \delta = \frac{P * L}{A * E}$$

A-----Refer to proportional limit .

2- (A----B) : The material may still be elastic but Hook's law not valid .

B-----Elastic limit

3-(B----C) : Beyond point (B) strain are not removed (permanent strain).

C-----Upper yield point .

4-(C----D) : Increasing in deformation without increasing in load .

D-----Lower yield point .

5- (D----E) :The reduction in cross-sectional area of specimen will occur (nicked specimen) .

E-----Ultimate strength .

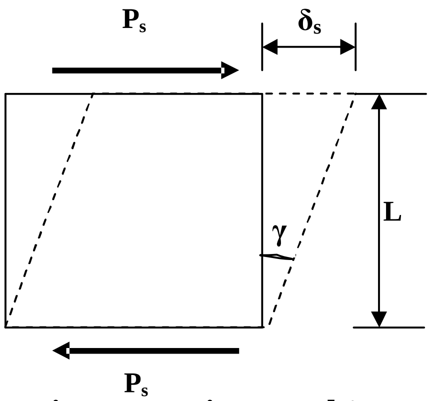
6- (E----F) : The failure will occur in specimen .

F----- Failure point .

<u>2) Shear Strain :</u> It is occur due to shear stress and its represented by (γ) in radians , and its is defined as ″ *The angular change between two faces of differential element* ″ . The shearing force cause shearing deformation , the element that subjected to shear dose not change in length of its sides , but undergoes change in shape from rectangle to parallelogram .

The average shearing strain is found by :

$$\tan(\gamma) = \frac{\delta_s}{L}$$



since the angle (γ) is usually very small , then

tan (γ) ≈ (γ)

$$\therefore \gamma = \frac{\delta_s}{L}$$

The relation between shearing stress and shearing strain , assuming Hook's law apply to shear .

$$\therefore \tau = Constant * \gamma$$

$$\therefore \tau = G * \gamma$$

G------Modulus of elasticity in shear (Modulus of rigidity)

Shearing deformation is expressed as :

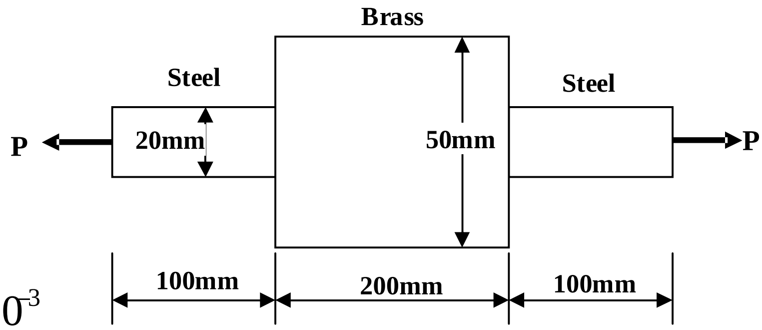
$$\tau = \frac{V}{A} \quad , \quad \tau = G * \gamma \quad , \quad \gamma = \frac{\delta_s}{L}$$

$$\therefore \frac{V}{A} = G * \frac{\delta_s}{L}$$

$$\therefore \delta_s = \frac{V * L}{A * G}$$

Ex :-6- During a tensile test of the bar shown in figure , the overall extension is (0.15mm) . Find the stress in each part of the bar .

E_{st}=210GPa , E_{Br}=120GPa



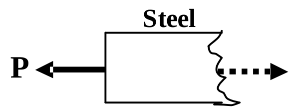
Sol:

$$2\delta_{st} + \delta_{Br} = \delta_{Total}$$

$$2\delta_{st} + \delta_{Br} = 0.15 * 10^{-3}$$

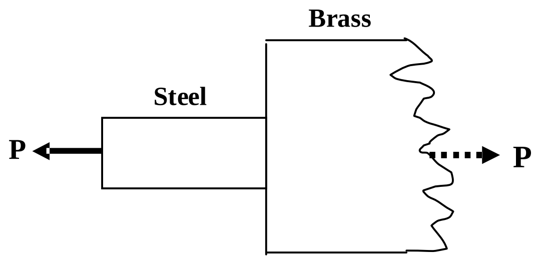
$$\therefore 2\frac{P_{st} * L_{st}}{A_{st} * E_{st}} + \frac{P_{Br} * L_{Br}}{A_{Br} * E_{Br}} = 0.15 * 10^{-3}$$

$$\therefore P_{st} = P_{Br} = P'$$



$$\therefore 2\frac{P_{st} * 0.1}{\frac{\pi}{4}(0.02)^2 * 210 * 10^9} + \frac{P_{Br} * 0.2}{\frac{\pi}{4}(0.05)^2 * 120 * 10^9} = 0.15 * 10^{-3}$$

$$\therefore P_{st} = P_{Br} = P' = 38.66kN$$

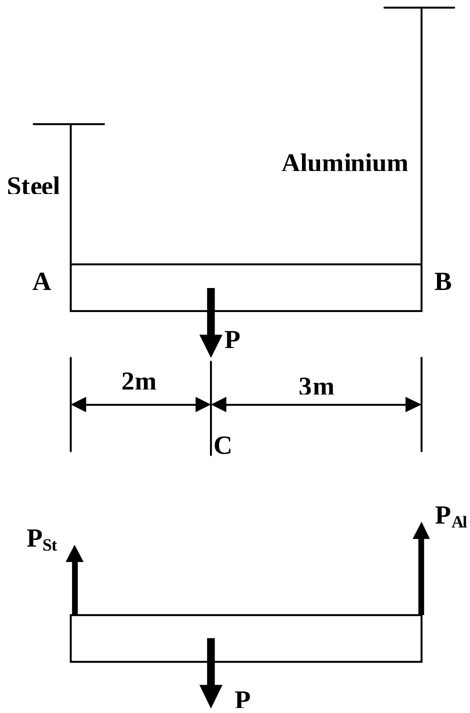


$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{38.66 * 10^3}{\frac{\pi}{4}(0.02)^2} = 196.6MPa$$

$$\sigma_{Br} = \frac{P_{Br}}{A_{Br}} = \frac{38.66 * 10^3}{\frac{\pi}{4}(0.05)^2} = 123MPa$$

Ex:-7- The rigid bar (AB) attached to two vertical rods as shown in figure , horizontally before the load (P) applied . If the load (P=50kN) , determine its vertical movement .

	St.	**Al.**
L (m)	**3**	**4**
Area (mm^2)	**300**	**500**
E (GPa)	**200**	**70**



F.B.D

Sol :

From F.B.D

$$\therefore \sum F_y = 0 \Rightarrow P_{St} + P_{Al} = P = 50kN \quad \text{------(1)}$$

$$\therefore \sum M_C = 0 \Rightarrow 2 * P_{St} = 3 * P_{Al} \Rightarrow P_{St} = 1.5 P_{Al} \quad \text{-----(2)}$$

Sub. (1) into (2)

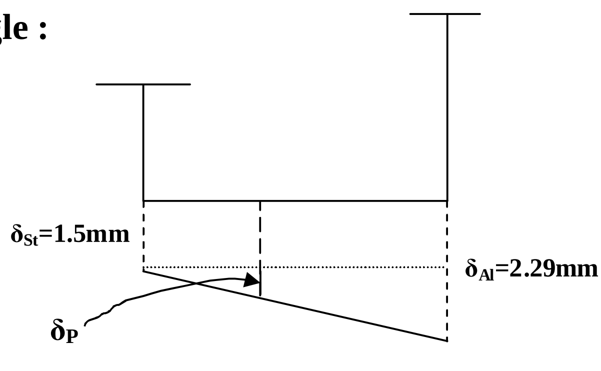
$$\therefore 1.5 P_{Al} + P_{Al} = 50kN \Rightarrow P_{Al} = 20kN$$

$$\therefore P_{St} = 30kN$$

$$\therefore \delta_{St} = \frac{P_{St} * L_{St}}{A_{St} * E_{St}} = \frac{30 * 10^3 * 3}{300 * 10^{-6} * 200 * 10^9} = 1.5 * 10^{-3} m = 1.5 mm$$

$$\therefore \delta_{Al} = \frac{P_{Al} * L_{Al}}{A_{Al} * E_{Al}} = \frac{20 * 10^3 * 4}{500 * 10^{-6} * 70 * 10^9} = 2.29 * 10^{-3} m = 2.29 mm$$

For triangle shown by similarity of triangle :



$$\therefore \frac{\delta_P - 1.5}{2} = \frac{2.29 - 1.5}{5}$$

$$\delta_P = \frac{2(0.79)}{5} + 1.5$$

$$\delta_P = 1.816 \ mm$$