Al-Ayen University
Petroleum Engineering College

Mechanics

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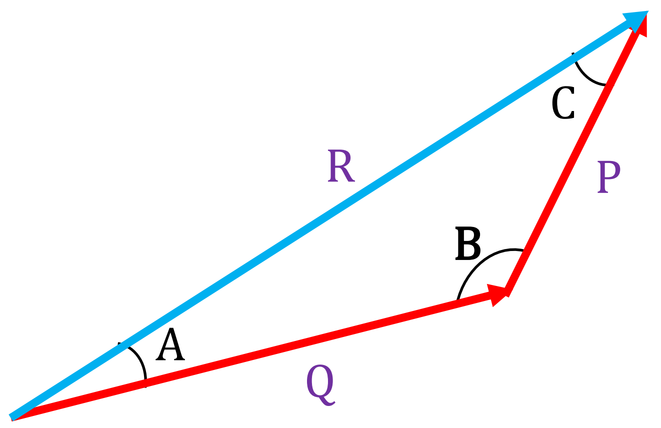
Title: Vectors; Trigonometric solution

Lec. Number: 2

Vectors

Scalar resultant: Trigonometric Solution



Not right angle

Law of cosines: The triangle rule is used; two sides and the included angle are known:

$$R^2 = P^2 + Q^2 - 2PQ \cos B$$

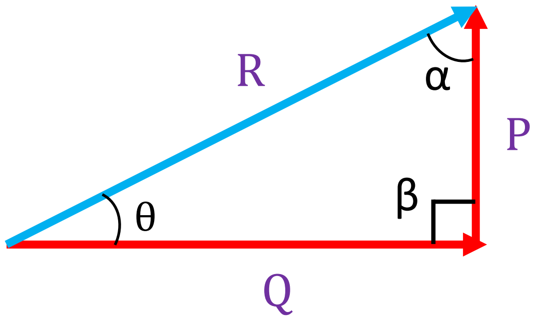
Law of sines:

$$\frac{R}{\sin B} = \frac{P}{\sin A} = \frac{Q}{\sin C}$$

Internal angles: A + B + C = 180˚

Scalar resultant: Trigonometric Solution

Right angle



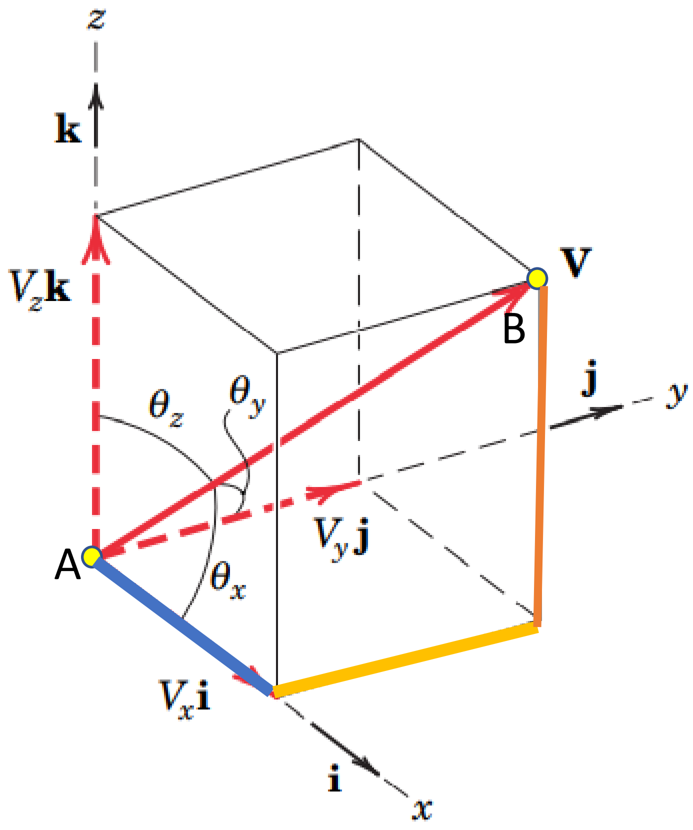
Pythagorean theorem: $R^2 = P^2 + Q^2$

Angle: $\tan\theta = \frac{P}{Q}$ ⇒ $\theta = \tan^{-1}(\frac{P}{Q})$

$Q = R\cos\theta$ $P = R\sin\theta$

Internal angles: $\alpha + \beta + \theta = 180^\circ$

3D resultant



Vector resultant

$$\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}$$

<u>Example</u>

V = 5i + 7j + 4k

Scalar resultant

$$V^2 = {V_x}^2 + {V_y}^2 + {V_z}^2$$

Direction cosines

$$l = \cos\theta_x \qquad m = \cos\theta_y \qquad n = \cos\theta_z$$

$$l^2 + m^2 + n^2 = 1$$

$$V_x = lV \qquad V_y = mV \qquad V_z = nV$$

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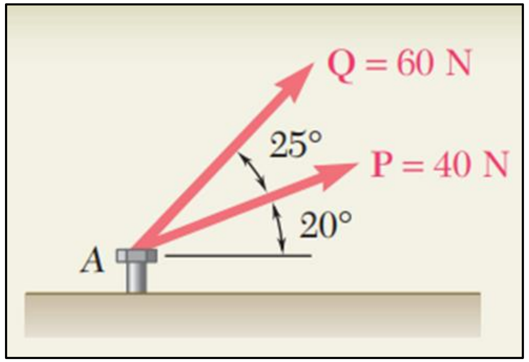
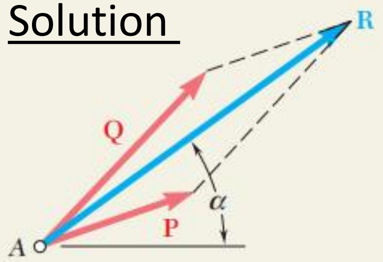
Example



Two forces P and Q act on a bolt. Determine their resultant and its direction.

Solution

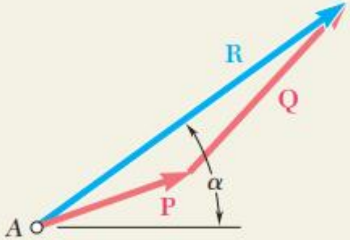


Graphical Solution. A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant are measured and found to be

$$R = 98\ \text{N} \qquad \alpha = 35^\circ \qquad \mathbf{R} = 98\ \text{N} \measuredangle 35^\circ$$

The triangle rule may also be used. Forces **P** and **Q** are drawn in tip-to-tail fashion. Again the magnitude and direction of the resultant are measured.

$$R = 98\ \text{N} \qquad \alpha = 35^\circ \qquad \mathbf{R} = 98\ \text{N} \measuredangle 35^\circ$$



Trigonometric Solution. The triangle rule is again used; two sides and the included angle are known. We apply the law of cosines.

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$R^2 = (40\ \text{N})^2 + (60\ \text{N})^2 - 2(40\ \text{N})(60\ \text{N})\cos 155^\circ$$
$$R = 97.73\ \text{N}$$

Now, applying the law of sines, we write

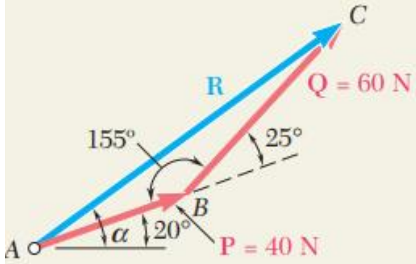
$$\frac{\sin A}{Q} = \frac{\sin B}{R} \qquad \frac{\sin A}{60\ \text{N}} = \frac{\sin 155^\circ}{97.73\ \text{N}} \qquad (1)$$

Solving Eq. (1) for sin A, we have

$$\sin A = \frac{(60\ \text{N})\sin 155^\circ}{97.73\ \text{N}}$$

Using a calculator, we first compute the quotient, then its arc sine, and obtain

$$A = 15.04^\circ \qquad \alpha = 20^\circ + A = 35.04^\circ$$



We use 3 significant figures to record the answer (cf. Sec. 1.6):

$$\mathbf{R} = 97.7\ \text{N} \measuredangle 35.0^\circ$$

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Homework

Find R and α in the previous example by considering ACD triangle in the figure below.