Graphing Polar Coordinate Equations

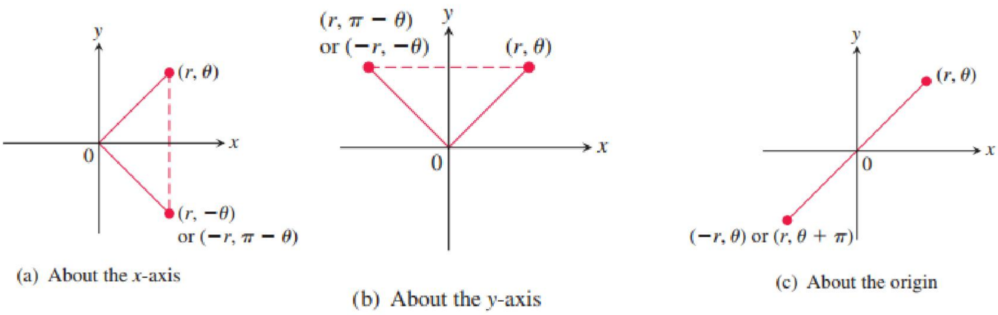
It is often helpful to graph an equation expressed in polar coordinates in the Cartesian xy-plane. This section describes some techniques for graphing these equations using symmetries and tangents to the graph.

Symmetry

Figure illustrates the standard polar coordinate tests for symmetry. The following summary says how the symmetric points are related.

Symmetry Tests for Polar Graphs in the Cartesian *xy*-Plane

1. *Symmetry about the x-axis:* If the point (r, θ) lies on the graph, then the point $(r, -\theta)$ or $(-r, \pi - \theta)$ lies on the graph (Figure 11.27a).
2. *Symmetry about the y-axis:* If the point (r, θ) lies on the graph, then the point $(r, \pi - \theta)$ or $(-r, -\theta)$ lies on the graph (Figure 11.27b).
3. *Symmetry about the origin:* If the point (r, θ) lies on the graph, then the point $(-r, \theta)$ or $(r, \theta + \pi)$ lies on the graph (Figure 11.27c).



Graphing in polar coordinates

① symmetry about x-axis

$(r, -\theta)$ or $(-r, \pi-\theta)$

② symmetry about y-axis

$(r, \pi-\theta)$ or $(-r, -\theta)$

③ symmetry about origion

$(-r, \theta)$ or $(r, \pi+\theta)$

$\cos(a+b) = \cos a * \cos b - \sin b * \sin a$

$\cos(a-b) = \cos a * \cos b + \sin a * \sin b$

$\sin(a+b) = \cos b * \sin a + \cos a * \sin b$

$\sin(a-b) = \sin a * \cos b - \cos a * \sin b$

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Example/ Sketch the graph of $r = 2 - 2\cos\theta$ in polar coordinat.

Solution /

① symmetry about x - axis

$(r, -\theta)$ or $(-r, \pi - \theta)$

$2 - 2\cos(-\theta) = 2 - 2\cos\theta = r$

have symmetry about x - axis

② symmetry about y - axis

$(r, \pi - \theta)$ or $(-r, -\theta)$

$2 - 2\cos(\pi - \theta) = 2 - 2[\cos\pi \cos\theta + \sin\pi \sin\theta]$

$= 2 - 2[-\cos\theta + 0] = 2 + 2\cos\theta \neq r$

or

$2 - 2\cos(-\theta) = 2 - 2\cos\theta \neq -r$

not have symmetry about y-axis

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③ symmetry about origion

$(-r, \theta)$ or $(r, \theta+\pi)$

$2-2(\cos\theta) \neq -r$

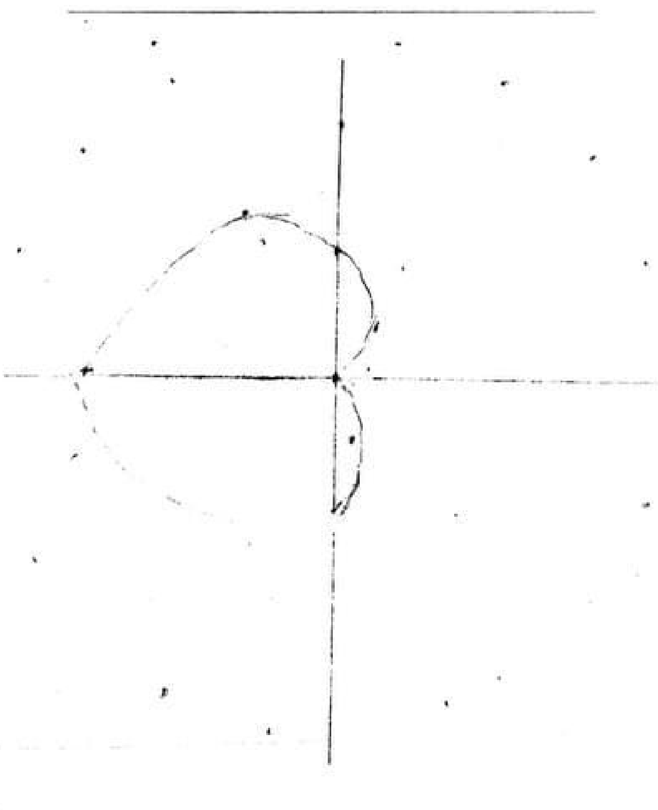
or

$2-2\cos(\theta+\pi) = 2-2\left[\cos\theta\cos\pi - \sin\theta\sin\pi\right] =$

$2-2(-\cos\theta) = 2+2\cos\theta \neq r$

not have symmetry about origion

θ	$r = 2-2\cos\theta$
0	0
20	0.12
40	0.46
60	1
80	1.65
90	2
120	3
140	3.53
160	3.87
180	4
210	3.73
240	3
260	2.34
270	2
290	1.31
300	1
320	0.46
240	[illegible]



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Example 2 / Identify the symmetries of the curves

1) $r = 1 + \cos\theta$, ② $r^2 = \cos\theta$

Solution /

1) $r = 1 + \cos\theta$

① symmetry about x-axis

$1 + \cos(-\theta) = 1 + \cos\theta = r$

have symmetry about x-axis

② symmetry about y-axis

$1 + \cos(\pi - \theta) = 1 + [\cos\pi \cos\theta + \sin\pi \sin\theta]$

$= 1 + [-\cos\theta + 0] = 1 - \cos\theta \neq r$

or

$1 + \cos(-\theta) = 1 + \cos\theta = r \neq -r$

not have symmetry about y-axis

③ symmetry about origion

$1 + \cos(\pi + \theta) = 1 + [\cos\pi \cos\theta - \sin\pi \sin\theta]$

$= 1 + [-\cos\theta - 0] = 1 - \cos\theta \neq r$

or

$-r \neq 1 + \cos\theta$

not have symmetry about origion

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Example / sketch the curve in polar coordinate

$r = 2\sin 3\theta$?

Solution /

① symmetry about x-axis

$(r, -\theta)$ or $(-r, \pi-\theta)$

$2\sin 3(-\theta) = -2\sin 3\theta \neq r$

not have symmetry about x-axis

② symmetry about y-axis

$(r, \pi-\theta)$ or $(-r, -\theta)$

$2\sin 3(\pi-\theta) = 2[\cancel{\sin 3\pi}^{0} * \cos\theta - \cos 3\pi \sin 3\theta]$

$= 2(\sin 3\theta) = r$

or

$2\sin 3(-\theta) = -2\sin 3\theta = -r$

have symmetry about y-axis

③ symmetry about origion

$(-r, \theta)$ or $(r, \pi+\theta)$

$2\sin 3\theta = 2\sin 3(\pi+\theta) = 2[\cancel{\sin 3\pi}^{0} * \cos 3\theta + \sin 3\theta \cos 3\pi]$

$= -2\sin 3\theta \neq -r$ not have symmetry about origion

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Example / sketch the curve in polar coordinate

$r = 2\sin 3\theta$?

Solution /

① symmetry about x-axis

$(r, -\theta)$ or $(-r, \pi - \theta)$

$2\sin 3(-\theta) = -2\sin 3\theta \neq r$

not have symmetry about x-axis

② symmetry about y-axis

$(r, \pi - \theta)$ or $(-r, -\theta)$

$2\sin 3(\pi - \theta) = 2[\cancel{\sin 3\pi}^{0} * \cos\theta - \cos 3\pi \sin 3\theta]$

$= 2(\sin 3\theta) = r$

or

$2\sin 3(-\theta) = -2\sin 3\theta = -r$

have symmetry about y-axis

③ symmetry about origion

$(-r, \theta)$ or $(r, \pi + \theta)$

$2\sin 3\theta = 2\sin 3(\pi + \theta) = 2[\cancel{\sin 3\pi}^{0} * \cos 3\theta + \sin 3\theta \cos 3\pi]$

$= -2\sin 3\theta \neq -r$ not have symmetry about origion

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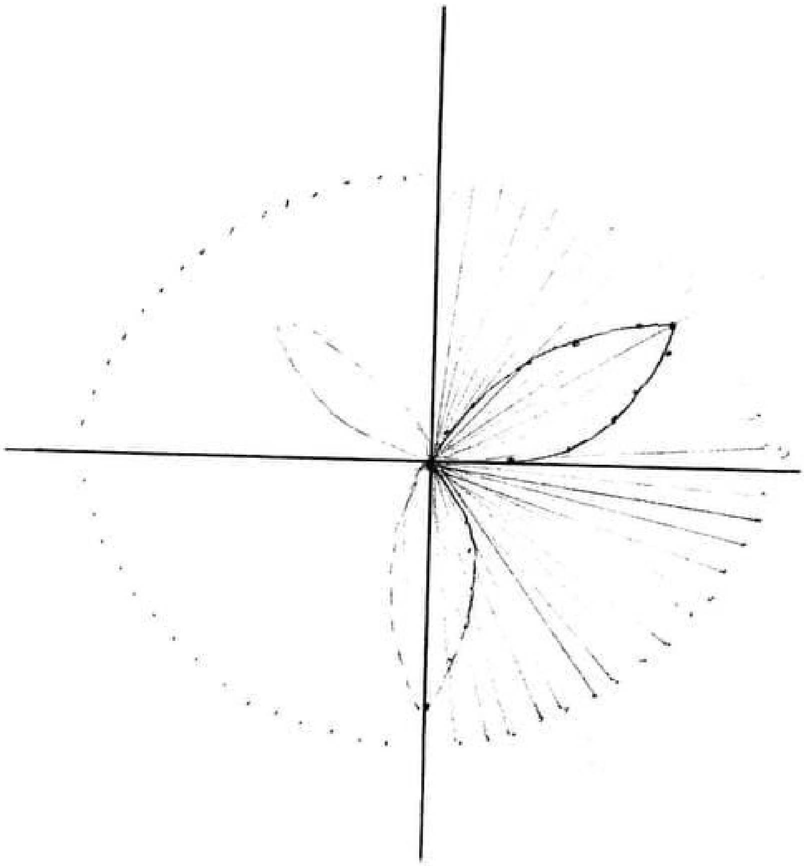
θ	$r = 2\sin 3\theta$
0	0
5	0.52
10	1
15	1.4
20	1.73
25	1.93
30	2
35	1.93
40	1.73
45	1.414
50	1
55	0.52
60	0
65	-0.5
70	-1
75	-1.41
80	-1.73
85	-1.93
90	-2
~~95~~	
180	0
210	-2
240	0
270	2
300	0
330	-2
360	0



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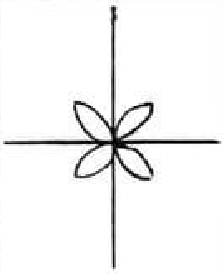
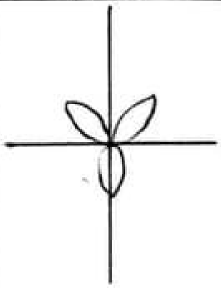
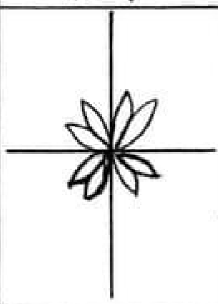
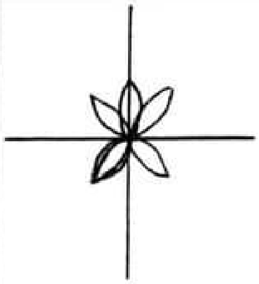
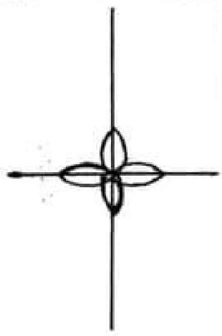
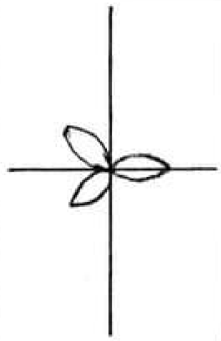
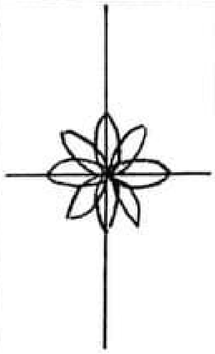
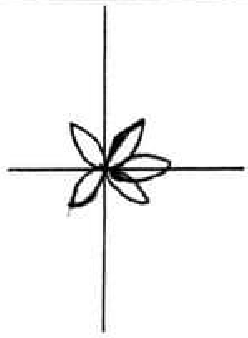
Families of Rose Curves

$r = a \sin n\theta$

$r = a \cos n\theta$

$a > 0$

$n \neq 1$

	$n=2$	$n=3$	$n=4$	$n=5$
$= a \sin n\theta$				
$a \cos n\theta$				

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Families of cardioids

$r = a + b \sin\theta$

$r = a - b \sin\theta$

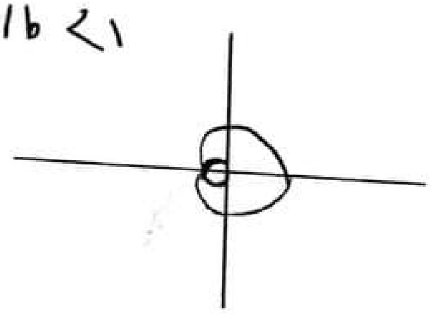
$r = a + b \cos\theta$

$r = a - b \cos\theta$

$a > 0 \quad , \; b > 0$

① Limacon with inner loop $a/b < 1$

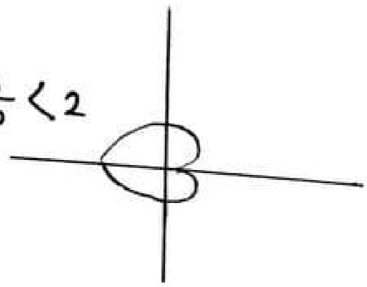
$r = a + b \cos\theta$



② cardioid $a/b = 1$

$r = a - b \cos\theta$

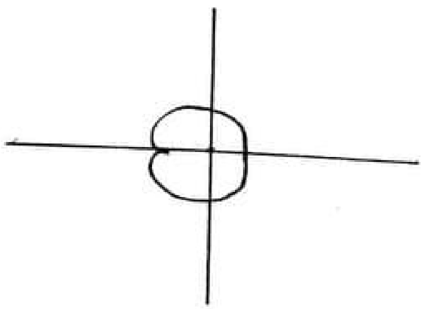
③ Dimpled limacon $1 < \frac{a}{b} < 2$

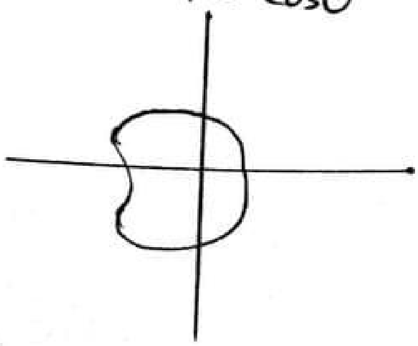


$r = a + b \cos\theta$

④ convex limacon $\frac{a}{b} \geqslant 2$

$r = a + b \cos\theta$





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