Petroleum engineering

Third stage

Engineering analysis

Lecture -3-

③ exact Diff equation

$$\boxed{M(x,y)\,dx + N(x,y)\,dy = 0}$$ الصيغة العام

if

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ ∴ the equ is exact

$$\int_a^x M(x,y)\,dx + \int_b^y N(x,y)\,dy = \int 0$$

* قبل اجراء التكامل نعوض بدل (X) بـ (a) في ص $\int N(x,y)\,dy$ ثم نجري

Ex: Show that the following equation are exact & solve ~~each one~~.

$$(2x + 3y - 2)\,dx + (3x - 4y + 1)\,dy = 0$$

sol

$M = 2x + 3y - 2 \implies \frac{\partial M}{\partial y} = 3$

$N = 3x - 4y + 1 \implies \frac{\partial N}{\partial x} = 3$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 3 \implies$ the equation is exact

$$\int (2x + 3y - 2)\,dx + \int_b^y (3a - 4y + 1)\,dy = c$$

$$\Big[x^2 + 3yx - 2x\Big]_a^x + \Big[3ay - 2y^2 + y\Big]_b^y = c$$

$$[(x^2+3yx-2x)-(a^2+3ya-2a)]+[(3ay-2y^2+y)-(3ab-2b^2+b)] = c$$

$$+3yx-2x-a^2-\cancel{3y}a+2a+\cancel{3y}a-2y^2+y-3ab+2b^2+b=c$$

$$+3yx-2x-2y^2+y=c+a^2-2a+3ab-2b^2-b$$

$$+3yx-2x-2y^2+y=K$$

$$\{2xy\cos^2 x-2xy+1\}dx+\{\sin x^2-x^2+3\}dy=0$$

$$= 2xy\cos^3 x-2xy+1 \Longrightarrow \frac{\partial M}{\partial y} = 2x\cos^3 x-2x$$

$$= \sin x^2 - x^2+3 \Longrightarrow \frac{\partial N}{\partial x} = 2x\cos x^2 - 2x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$ the equ. is exact

Ex-2. $(x+y)\,dx + (x+y^2)\,dy = 0$

Sol

$M = (x+y)\,dx \Longrightarrow \dfrac{\partial M}{\partial y} = 1$

$N = (x+y^2)\,dy \Longrightarrow \dfrac{\partial N}{\partial x} = 1$

$\therefore$ the equation is exact

$$\int_a^x (x+y)\,dx + \int_b^y (a+y^2)\,dy = c$$

$$\left[\frac{x^2}{2} + xy\right]_a^x + \left[ay + \frac{y^3}{3}\right]_b^y = c$$

$$\left[\left(\frac{x^2}{2} + xy\right) - \left(\frac{a^2}{2} + ay\right)\right] + \left[\left(ay + \frac{y^3}{3}\right) - \left(ab + \frac{b^3}{3}\right)\right] = c$$

$$\frac{x^2}{2} + xy - \frac{a^2}{2} - ay + ay + \frac{y^3}{3} - ab - \frac{b^3}{3} = c$$

$$\frac{x^2}{2} + xy + \frac{y^3}{3} = c + ab + \frac{b^3}{3}$$

$$\frac{x^2}{2} + xy + \frac{y^3}{3} = K$$

Ex-3- $(2x e^{y} + e^{x})\,dx + (x^2+1)e^{y}\,dy = 0$

$M = 2xe^{y} + e^{x} \Longrightarrow \dfrac{\partial M}{\partial y} = 2xe^{y}$

$N = x^2 e^{y} + e^{y} \Longrightarrow \dfrac{\partial N}{\partial x} = 2xe^{y}$

$\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x} \Longrightarrow$ the equ is exact

$$\int_a^x (2x e^{y} + e^{x})\,dx + \int_b^y (a^2+1)e^{y}\,dy = c$$

$$\left[x^2 e^{y} + e^{x}\right]_a^x + \left[(a^2+1)e^{y}\right]_b^y = c$$

$$\left[(x^2 e^{y} + e^{x}) - (a^2 e^{y} + e^{a})\right] + \left[(a^2+1)e^{y} - (a^2+1)e^{b}\right] =$$

$$x^2 e^{y} + e^{x} - a e^{y} - e^{a} + a^2 e^{y} + e^{y} - a^2 e^{b} - e^{b} = c$$

$$x^2 e^{y} + e^{x} + e^{y} = c + a^2 e^{b} - e^{b}$$

$$x^2 e^{y} + e^{x} + e^{y} = K$$

H.W

1) $dy + \dfrac{y - \sin x}{x}\,dx = 0$

2) $(2x - y)\,dx = (x - y)\,dy$

Integeration factor

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$ the equation is n't exact

I.f نضرب كل المعادلة

$$I.f = e^{\int f(x)\, dx}$$

where $f(x) = \dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$

or

$$I.f = e^{-\int f(y)\, dy}$$

where $f(y) = \dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M}$

EX $(x^2+y^2+x)\,dx + xy\,dy = 0$