Al-Ayen University
College of Petroleum Engineering

Reservoir Engineering II

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Lecture 18: Water Influx (Part 2)

Ref.: Reservoir Engineering Handbook by Tarek Ahmed

Outline

- Schilthuis' Steady-state Model
  - Example 1
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- Hurst's Modified Steady-State Model
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Schilthuis' Steady-State Model

Schilthuis (1936) proposed that for an aquifer that is flowing under the steady-state flow regime, the flow behavior could be described by Darcy's equation. The rate of water influx e_w can then be determined by applying Darcy's equation:

$$\frac{dW_e}{dt} = e_w = \left[\frac{0.00708\, kh}{\mu_w \ln\left(\frac{r_a}{r_e}\right)} \right] (p_i - p)$$

The above relationship can be more conveniently expressed as:

$$\frac{dW_e}{dt} = e_w = C\,(p_i - p)$$

where e_w = rate of water influx, bbl/day
k = permeability of the aquifer, md
h = thickness of the aquifer, ft
r_a = radius of the aquifer, ft
r_e = radius of the reservoir
t = time, days

- The parameter C is called ***the water influx constant*** and is expressed in bbl/(Day.psi). This water influx constant C may be calculated from the reservoir historical production data.

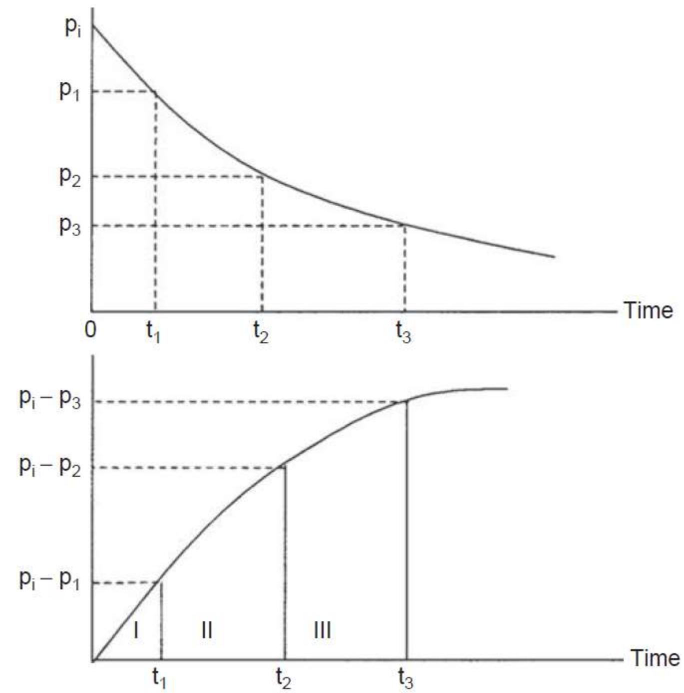
$$\int_0^{W_e} dW_e = \int_0^t C\ (p_i - p)\ dt$$

$$W_e = C \int_0^t (p_i - p)\ dt$$

W_e = cumulative water influx, bbl
C = water influx constant, bbl/(day.psi)
t = time, days
p_i = initial reservoir pressure, psi
p = pressure at the oil-water contact at time t, psi

When the pressure drop (p_i – p) is plotted versus the time t, as shown in Figure, the area under the curve represents the integral $\int_0^t (p_i - p) = dt$.

This area at time t can be determined numerically by using the trapezoidal rule (or any other numerical integration method), as:



Calculating the area under the curve numerically

$$\int_0^t (p_i - p) = dt = \text{area}_I + \text{area}_{II} + \text{area}_{III} + \text{etc.} = \left(\frac{p_i - p_1}{2}\right)(t_1 - 0) + \frac{(p_i - p_1) + (p_i - p_2)}{2}(t_2 - t_1) + \frac{(p_i - p_2) + (p_i - p_3)}{2}(t_3 - t_2) + \text{etc.}$$

$$W_e = C \sum_0^t (\Delta p)\ \Delta t$$

Example 1

Calculate Schilthuis' water influx constant in a reservoir whose pressure is stabilized at 3000 psi.

Given: initial reservoir pressure = 3500 psi

$dN_p/dt = 32{,}000$ STB/day

$B_o = 1.4$ bbl/STB

GOR = 900 scf/STB

$R_s = 700$ scf/STB

$B_g = 0.00082$ bbl/scf

$dW_p/dt = 0$

$B_w = 1.0$ bbl/STB

Solution

$$e_w = \frac{dW_e}{dt} = B_o \frac{dN_p}{dt} + (GOR - R_s)\frac{dN_p}{dt} B_g + \frac{dW_p}{dt} B_w$$

$$e_w = (1.4)\ (32{,}000) + (900 - 700)\ (32{,}000)\ (0.00082) + 0 = 50{,}048 \text{ bbl/day}$$

$$e_w = C\ (p_i - p)$$ → **Schilthuis' Steady-State Model**

$$C = \frac{e_w}{(p_i - p)} = \frac{50{,}048}{(3500 - 3000)} = 100 \text{ bbl/(Day.psi)}$$

Example 2

The pressure history of a water-drive oil reservoir is given in the Table below. The aquifer is under a steady-state flowing condition with an estimated water influx constant of 130 bbl/day/psi. Calculate the cumulative water influx after 100, 200, 300, and 400 days using the steady-state model.

t, days	p, psi
0	3500 (p_i)
100	3450
200	3410
300	3380
400	3340

Solution

$$W_e = C \sum_{o}^{t} (\Delta p)\, \Delta t$$

C =130 bbl/day/psi

t, days	p	$p_i - p$
0	3500	0
100	3450	50
200	3410	90
300	3380	120
400	3340	160

The cumulative water influx, We, after 100 days: $W_e = 130\left(\frac{50}{2}\right)(100-0) = 325,000 \text{ bbl}$

W_e after 200 days: $W_e = 130\left[\left(\frac{50}{2}\right)(100-0) + \left(\frac{50+90}{2}\right)(200-100)\right] = 1,235,000 \text{ bbl}$

We after 300 days: $W_e = 130\left[\left(\frac{50}{2}\right)(100) + \left(\frac{50+90}{2}\right)(200-100) + \left(\frac{120+90}{2}\right)(300-200)\right] = 2,600,000 \text{ bbl}$

We after 400 days: $W_e = 130\left[2500 + 7000 + 10,500 + \left(\frac{160+120}{2}\right)(400-300)\right] = 4,420,000 \text{ bbl}$

Hurst's Modified Steady-State Model

- One of the problems associated with the Schilthuis' steady-state model is that as the water is drained from the aquifer, the dimensionless radius r_a/r_e will increase as the time increases.
- Hurst (1943) proposed that the dimensionless radius r_a/r_e may be replaced with a time dependent function, as: $r_a/r_e = at$

Substituting this Equation into Equation of Schilthuis' steady-state model gives:

$$e_w = \frac{dW_e}{dt} = \frac{0.00708\, kh\, (p_i - p)}{\mu_w \ln(at)}$$

or

$$e_w = \frac{dW_e}{dt} = \frac{C\,(p_i - p)}{\ln(at)}$$

and in terms of the cumulative water influx

$$W_e = C \int_0^t \left[\frac{p_i - p}{\ln(at)} \right] dt$$

$$W_e = C\int_0^t \left[\frac{p_i - p}{\ln(at)}\right] dt$$

or

$$W_e = C\sum_0^t \left[\frac{\Delta p}{\ln(at)}\right] \Delta t$$

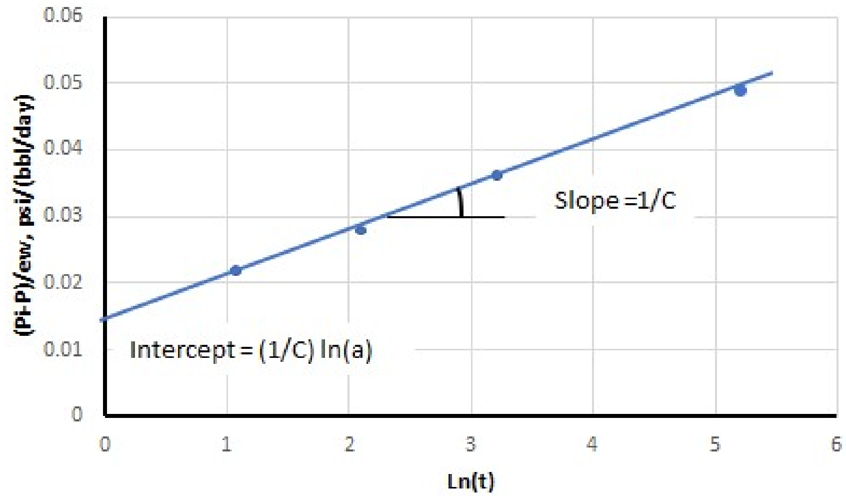
To determine the constants a and C, from the equation: $e_w = \frac{dW_e}{dt} = \frac{C(p_i - p)}{\ln(at)}$

$$\left(\frac{p_i - p}{e_w}\right) = \frac{1}{C}\ln(at)$$

or

$$\frac{p_i - p}{e_w} = \left(\frac{1}{C}\right)\ln(a) + \left(\frac{1}{C}\right)\ln(t)$$

This Equation indicates that a plot of $(p_i - p)/e_w$ versus ln(t) will be a straight line with a slope of 1/C and intercept of (1/C)ln(a), as shown schematically in Figure.



Example 3

The following data document the reservoir pressure as a function of time for a water-drive reservoir. Using the reservoir historical data, the water influx was calculated by applying the material balance equation. The rate of water influx was also calculated numerically at each time period.

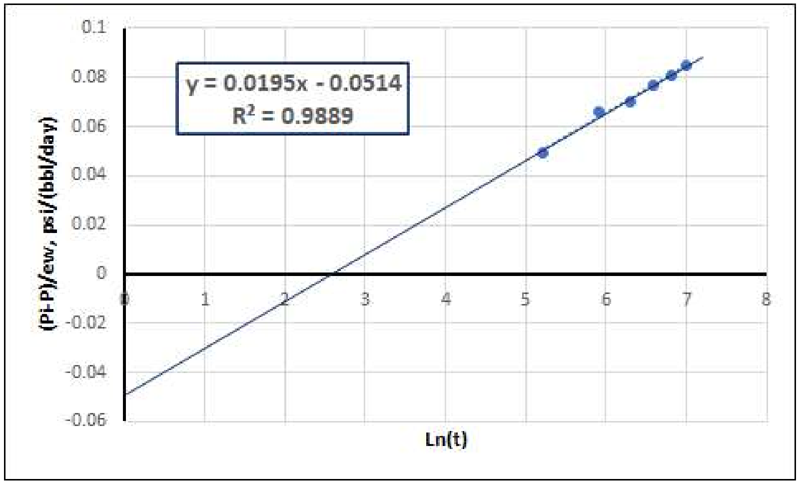
Time days	Pressure psi	W_e M bbl	e_w bbl/day	$p_i - p$ psi
0	3793	0	0	0
182.5	3774	24.8	389	19
365.0	3709	172.0	1279	84
547.5	3643	480.0	2158	150
730.0	3547	978.0	3187	246
912.5	3485	1616.0	3844	308
1095.0	3416	2388.0	4458	377

Assuming that the boundary pressure would drop to 3379 psi after 1186.25 days of production, calculate cumulative water influx at that time.

Solution

Construct the following table:

t, days	ln(t)	$p_i - p$	e_w, bbl/day	$(p_i - p)/e_w$
0	—	0	0	—
182.5	5.207	19	389	0.049
365.0	5.900	84	1279	0.066
547.5	6.305	150	2158	0.070
730.0	6.593	246	31.87	0.077
912.5	6.816	308	3844	0.081
1095.0	6.999	377	4458	0.085



Plot the term (pi – p)/ew versus ln(t) and draw the best straight line, and determine the slope and intercept of the line to give:

$$C = 1/\text{slope} = 1/0.0195 = 51.282 \text{ bbl/day/psi}$$

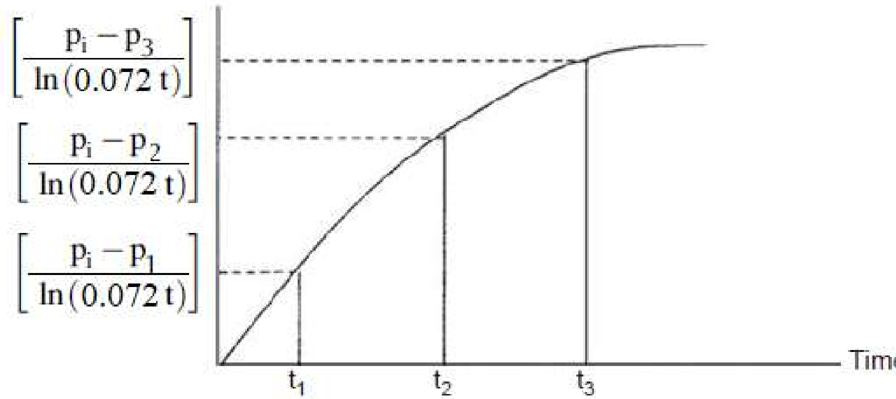
$$\text{intercept} = \left(\frac{1}{C}\right) \ln(a)$$

a = exp(C × intercept)= exp(51.282 × (-0.0514)) = 0.072

The Hurst equation is represented by:

$$W_e = C\int_0^t \left[\frac{p_i - p}{\ln(at)}\right] dt$$

$$W_e = 51.282\int_0^t \left[\frac{p_i - p}{\ln(0.072\ t)}\right] dt$$



Calculating the area under the curve numerically

Calculate the cumulative water influx after 1186.25 days from:

$$W_e = 2388 \times 10^3 + 51.282\int_{1095}^{1186.25} \left[\frac{p_i - p}{\ln(0.072\ t)}\right] dt$$

$$W_e = 2388 \times 10^3 + 51.282\left[\frac{\dfrac{3793 - 3379}{\ln(0.072 \times 1186.25)} + \dfrac{3793 - 3416}{\ln(0.072 \times 1095)}}{2}\right] \times (1186.25 - 1095)$$

$$W_e = 2388 \times 10^3 + 419.768 \times 10^3 = 2807.8 \text{ Mbbl}$$

THANK YOU