Al-Ayen University
College of Petroleum Engineering

Numerical Methods and Reservoir Simulation

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L11: Methods of Solving Systems of Linear Equations (Part 4)

(Matrices and Determinants-Part 1)

Outline

- Scalar Matrix
- Identity Matrix or Unit Matrix
- Operations on Matrices
  - Multiplication of a Matrix by a Scalar
  - Addition and Subtraction of Matrices
  - Product of Matrices
- The Transpose of a Matrix
- The Determinant

Scalar Matrix:

A diagonal matrix in which all the diagonal elements are same, is called a scalar matrix i.e.

Thus $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ and $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$ are scalar matrices

Identity Matrix or Unit Matrix :

A scalar matrix in which each diagonal element is 1(unity) is called a unit matrix. An identity matrix of order n is denoted by I_n.

Thus $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are the identity matrices of order 2 and 3 .

Operations on matrices:

Multiplication of a Matrix by a Scalar:

For example, if $A = \begin{bmatrix} 4 & -3 \\ 8 & -2 \\ -1 & 0 \end{bmatrix}$ then for a scalar k, $kA = \begin{bmatrix} 4k & -3k \\ 8k & -2k \\ -k & 0 \end{bmatrix}$

Addition and subtraction of Matrices:

If $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 0 \end{bmatrix}$

Then $C = A + B = \begin{bmatrix} 3+1 & 1+0 & 2+2 \\ 2-1 & 1+3 & 4+0 \end{bmatrix} = \begin{bmatrix} 4 & 1 & 4 \\ 1 & 4 & 4 \end{bmatrix}$

Thus if $A = \begin{bmatrix} 6 & 2 \\ 7 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 1 \\ 3 & 4 \end{bmatrix}$

then $A - B = \begin{bmatrix} 6 & 2 \\ 7 & -5 \end{bmatrix} - \begin{bmatrix} 8 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6-8 & 2-1 \\ 7-3 & -5-4 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 4 & -9 \end{bmatrix}$

Product of Matrices:

Two matrices A and B are said to be conformable for the product AB if the number of columns of A is equal to the number of rows of B. Then the product matrix AB has the same number of rows as A and the same number of columns as B.

$$A_{2x2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{and} \quad B_{2x2} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Example : If $\mathbf{A} = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ **and** $\mathbf{B} = \begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$**, find AB**

Solution: $AB = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3+2+6 & -3+1+2 \\ 1+0+3 & -1+0+1 \end{bmatrix} = \begin{bmatrix} 11 & 0 \\ 4 & 0 \end{bmatrix}$

The Transpose of a Matrix:

The transpose of an $m \times n$ matrix $\mathbf{A}$, written $\mathbf{A}^T$, is the $n \times m$ matrix formed by interchanging the rows and columns of A. For example, if

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \Rightarrow \mathbf{A}^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$$

Example if $\mathrm{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, then $\mathrm{A}^{t} = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

The Determinant:

The determinant is denoted by det A or |A| for a square matrix A.

The determinant of a matrix is a scalar (number), obtained from the elements of a matrix by specified, operations, which is characteristic of the matrix.

The determinants are defined only for square matrices.

The determinant of the (2 x 2) matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given by $\det A = |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\, a_{22} - a_{12}\, a_{21}$

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Example : If $A = \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$ find |A|

Solution: $|A| = \begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix} = 9 - (-2) = 9 + 2 = 11$

The Determinant:

The determinant of the (3 x 3) matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, denoted by

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

The value of the determinant can be found by expanding it from any row or column.

The Determinant:

Example : If $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 4 \end{bmatrix}$ find det A by expansion about (a) the first row (b) the first column.

Solution (a) : $|A| = \begin{vmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 4 \end{vmatrix} = 3\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} - 2\begin{vmatrix} 0 & -2 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} = 3(4+6) - 2(0+2) + 1\,(0-1) = 25$

Solution (b) : $|A| = \begin{vmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 4 \end{vmatrix} = 3\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} - 0\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = 3\,(4+6) + 1\,(-4-1) = 25$

THANK YOU