<u>The Material Balance As an Equation of a Straight Line</u>

Refereeing to the general material balance equation

$$N = \frac{N_pB_o + N_p\left(R_p - R_s\right)B_g - (W_e - W_pB_w)}{(B_o - B_{oi}) + (R_{si} - R_s)\,B_g + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) + (1+m)B_{oi}\frac{(c_wS_w + c_f)}{(1 - S_{wi})}\Delta p} \quad eq11$$

There are essentially **<u>three unknown</u>** in the above equation:

1. The original oil in place (**N**)
2. The cumulative water influx (**We**)
3. The original size of the gas cap as compared to the oil zone size (**m**)

<u>Havlena and Odeh 1963</u> expressed the above equation in the following form;

$$N_p\left(B_o + \left(R_p - R_s\right)B_g\right) + W_pB_w = N(B_o - B_{oi}) + N(R_{si} - R_s)\,B_g + mNB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) + (1+m)NB_{oi}\frac{(c_wS_w + c_f)}{(1 - S_{wi})}\Delta p + W_e \qquad eq12$$

and they put equation **12** in the following form:

$$\boldsymbol{F = N\left[E_o + mE_g + E_{f,w}\right]} \qquad eq13$$

<u>Where:</u>

F represents the underground withdrawals

$$F = N_p\left[B_o + \left(R_p - R_s\right)B_g\right] + W_pB_w \qquad (a)$$

and in terms of two phase formation volume factor $\boldsymbol{B_t}$,

$$F = N_p\left[B_t + \left(R_p - R_{si}\right)B_g\right] + W_pB_w$$

$$B_t = B_o + (R_{si} - R_s)B_g$$

- $\boldsymbol{E_o}$: describes the expansion of oil and its originally dissolved gas

$$E_o = (B_o - B_{oi}) + (R_{si} - R_s)B_g \quad \text{or}$$

$$E_o = B_t - B_{ti} \qquad (b)$$

- $\boldsymbol{E_g}$: describes the expansion of the gas cap

$$E_g = B_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)$$

<u>or</u>

$$E_g = B_{ti}\left(\frac{B_g}{B_{gi}} - 1\right) \qquad (c)$$

- $\boldsymbol{E_{f,w}}$: describes the expansion of the initial water and the reduction in the (pv)

$$E_{f,w} = (1+m)B_{oi}\frac{(c_w S_w + c_f)}{(1-S_{wi})}\Delta p \qquad (d)$$

Equation 13 is a straight line equation, **Havlena** and **Odeh** examined several cases with equation 13

This significant observation will provide the engineer with valuable information that can be used in determining the following unknowns:

- **Initial oil in place (N)**
- **Size of the gas cap (m)**
- **Water influx (We)**
- **Drive mechanism**
- **Average reservoir pressure**

Six cases will present with the application of the straight line form of the MBE

Case1: Determination of **N** in volumetric under saturated reservoirs ($R_{si} = R_s$, We = 0)

Case 2: Determination of **N** in volumetric saturated reservoir ($R_{si} \neq R_s$)

Case 3:Determination of **N** and **m** in gas cap drive reservoirs

Case 4: Determination of **N** , **m** and **We** in water drive reservoirs

<u>Case 5</u>: Determination of **N** , **m** and **We** in combination drive reservoirs

<u>Case 6</u>: Determination of average reservoir pressure $\boldsymbol{p}^{-}$

<u>Case1: Volumetric Undersaturated Oil Reservoirs</u>

The general linear MBE

$$F = N\left[E_o + mE_g + E_{f,w}\right] + \ W_e \qquad\qquad eq14$$

Assume $W_e = 0$, m=0, $R_{si} = R_s = R_p$

Applying the above conditions to equation 14

$$\boldsymbol{F = N\left[E_o + E_{f,w}\right]}$$

$$\boldsymbol{F = N_p B_o + W_p B_w}$$

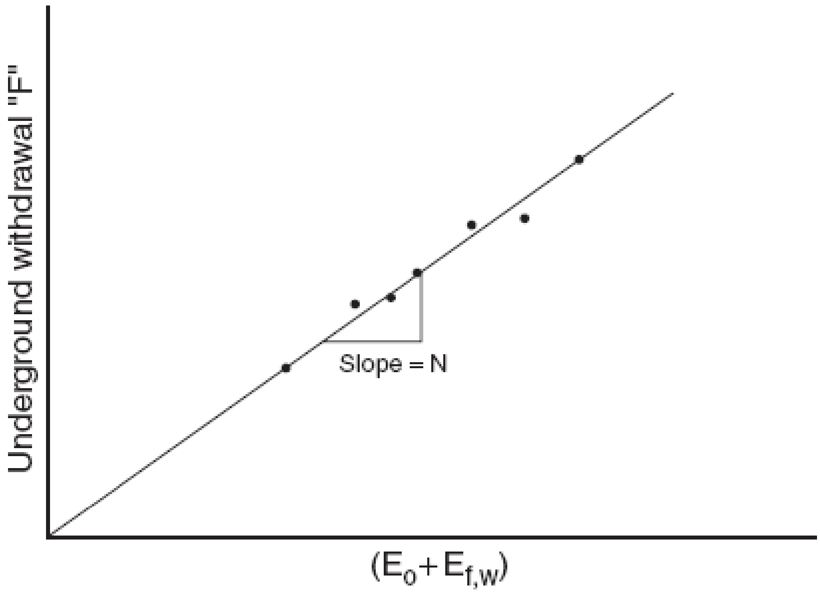
$$\boldsymbol{E_o = B_t - B_{ti}}$$

$$\boldsymbol{E_{f,w} = B_{oi}\frac{(c_w S_w + c_f)}{(1 - S_{wi})}\Delta p}\ , \quad \boldsymbol{\Delta p = \quad p_i - p_r^{-}}$$

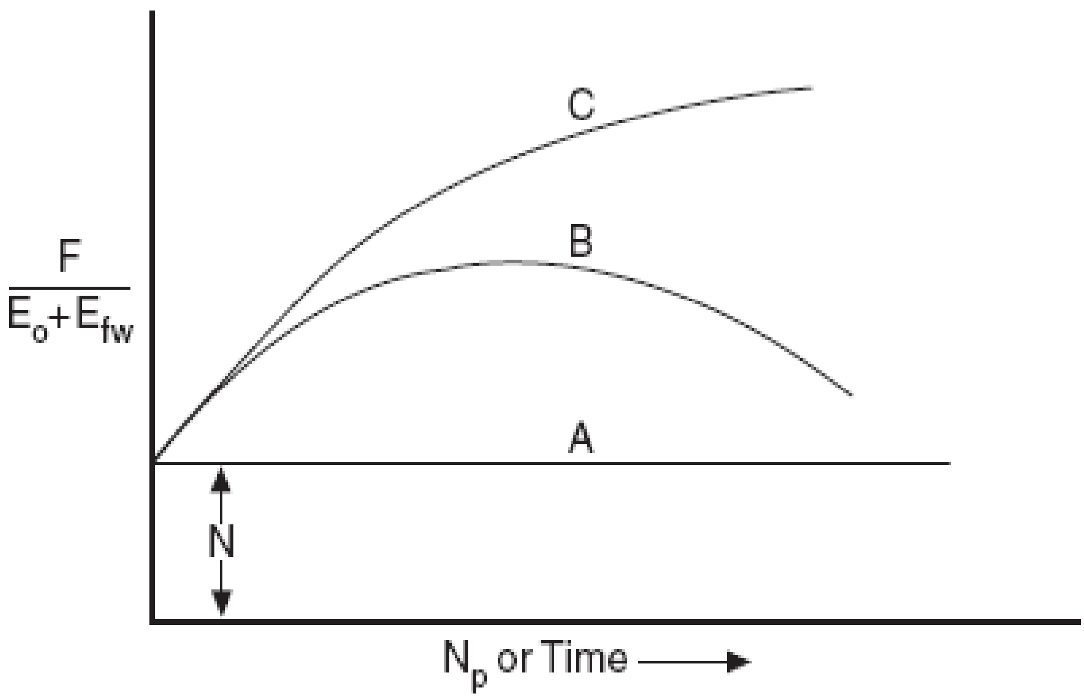
<u>Then</u>

Plot of F Vs $\boldsymbol{(E_o + E_{f,w})}$ will yield a straight line with a slope = N as shown in the figure below

Above the P_b $\boldsymbol{B_t = \ B_o}$



Plot of $\frac{F}{E_o+E_{f,w}}$ for each pressure and time observation versus cumulative production N_p or **time** as shown in the figure below.(**Campbell plot**)



Dake 1994 suggested that such a plot can assume two various shapes;

1. If all the calculated point of $\frac{F}{E_o+E_{f,w}}$ lie on a horizontal straight line (line **A**), then the reservoir is a volumetric reservoir and the ordinate value of plateau determines the initial oil in place **N**.
2. If the calculated values of $\frac{F}{E_o+E_{f,w}}$ rise, as illustrated by curves (**B**) and (**C**), it indicated that the reservoir has been energized by **water influx**, **abnormal pore completion** or **combination of these two**. curve **C** might be for a strong water drive field in which the aquifer is displacing an infinite acting behaviour , whereas curve **B** represents an aquifer whose outer boundary had been felt.
3. If the withdrawal > **We** then calculated values of $\frac{F}{E_o+E_{f,w}}$ will dip downward plus if the withdrawal < **We** reverse happens and the points are elevated.

Example:

The x-field is a volumetric undersaturated reservoir. volumetric calculation indicates the reservoir contains 270.6 MMSTB of oil initially in place. The initial reservoir pressure is 3685 psia.

The following additional data is available:

$S_{wi} = 24\%$, $B_w = 1$, $c_w = 3.6 \times 10^{-6}$ psia $^{-1}$, $c_f = 4.95 \times 10^{-6}$ psia $^{-1}$,

$p_b = 1500$ psia

The field production and PVT data are summarized below.

P psi	Well Num.	Bo Bbl/stb	Np, MSTB	Wp, MSTB
3685	1	1.3102	0	0
3680	2	1.3104	20.481	0
3676	2	1.3104	34.75	0
3667	3	1.3105	78.557	0
3664	4	1.3105	101.846	0
3640	19	1.3109	215.681	0
3605	25	1.3116	364.613	0
3567	36	1.3122	542.981	0.159
3515	48	1.3128	841.591	0.805
3448	59	1.313	1273.53	2.579
3360	59	1.315	1691.887	5.008
3275	61	1.316	2127.077	6.5
3188	61	1.317	2575.33	8

<u>Calculate</u> the initial oil in place by using the **MBE** and compare with the volumetric estimate of N.

<u>Solution:</u>

1. Calculate $\boldsymbol{E_{f,w}}$

$$E_{f,w} = B_{oi}\frac{(c_w S_w + c_f)}{(1 - S_{wi})}\Delta p = \quad 10 \times 10^{-6}\ \ (3685 - \bar{p_r})$$

2. Calculate :**F**

$$F = N_p B_o + W_p B_w$$

$$E_o = B_o - B_{oi}$$

And construct the following table

p	F, MSTB	E_o	Δp	$E_{f,w}$	$E_o+E_{f,w}$
3685	0	0	0	0	0
3680	26.8383	0.0002	5	5.015652E-05	0.00025
3676	45.5364	0.0002	9	9.028174E-05	0.00029
3667	102.9489	0.0003	18	1.805635E-04	0.000481
3664	133.4692	0.0003	21	2.106574E-04	0.000511
3640	282.7362	0.0007	45	4.514087E-04	0.001151
3605	478.2264	0.0014	80	8.025044E-04	0.002203

3567	712.6587	0.002	118	1.183694E-03	0.003184
3515	1105.646	0.0026	170	1.705322E-03	0.004305
3448	1674.724	0.0028	237	2.377419E-03	0.005177
3360	2229.839	0.0048	325	3.260174E-03	0.00806
3275	2805.733	0.0058	410	4.112835E-03	0.009913
3188	3399.71	0.0068	497	4.985559E-03	0.011786

3. Plot **F against (E_o+$E_{f,w}$)** on a Cartesian scale as shown in the figure below

4. Draw the best straight line through the points and determine the slop N

N = 257 MMSTB (**effective oil or active**)This value usually smaller than that of the volumetric due to oil being trapped in un drained fault compartments or low permeability regions of the reservoir.