Lecture Seven

Flow of Natural Gas through Restrictions

7.1 Introduction

There are several locations in the gas production system where the gas must pass through relatively short restrictions. Surface chokes and subsurface control equipment will be discussed in this section. The flow through restrictions may be either subcritical or critical.

In subcritical flow, the velocity of gas through the restriction is below the speed of sound in the gas. The flow rate will depend on both the upstream and downstream pressures. Subsurface control equipment is sized so that flow is subcritical.

In critical flow, the velocity of gas through the restriction is equal to the speed of sound in the gas. Since pressure disturbances travel at the speed of sound, a disturbance downstream of the restriction in critical flow will not affect the up-stream pressure or flow rate. The flow rate depends only on the upstream pressure in critical flow. Surface chokes are usually sized for critical flow.

7.2 The Objective of Surface Choke

Practically all flowing wells utilize some surface restriction in order to regulate the flow rate. Only very few produce with absolutely no restrictions in order to obtain maximum production rate. Use of a surface choke to control flow may be necessary because of the following reasons:

1. Maintaining well allowable production rate.

2. Sand control by maintaining sufficient back pressure.

3. Protection of surface equipment.

4. Prevention of water coning.

5. Reservoir management.

7.3 The Objective of Sub-surface Restriction

Subsurface control equipment comes in a variety of types and sizes. They can be grouped under three main functions:

1. Subsurface tubing safety valves, which shut in the well downhole in the event surface control equipment, becomes damaged or is completely removed.

2. Bottom-hole chokes and regulators, which reduce the wellhead flowing pressure and prevent freezing of surface controls and lines by taking pressure drop downhole.

3. Check valves, which prevent back flow of injection wells.

7.4 General Equation for Flow Through Restrictions

A general equation for flow through restrictions can be obtained by combining the Bernoulli equation with an equation of state and assuming that there are no irreversible or friction losses taking place. An empirical discharge coefficient is included to account for the simplifying assumptions used in deriving the equation. The following equation may be used for both critical (sonic) or subcritical (subsonic) flow.

$$q_{sc} = \frac{C_n (p_1)(d)^2}{\sqrt{\gamma_g (T_1) Z_1}} \sqrt{\left(\frac{k}{k-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{2/k} - \left(\frac{p_2}{p_1}\right)^{k+1/k}\right]} \quad \text{……….(7.1)}$$

where

$$C_n = \frac{C_s (C_d) T_{sc}}{P_{sc}},$$

q_{sc} — volumetric gas flow rate,
C_n — coefficient based on system of units, discharge coefficient and standard conditions,
d — ID of bore opening to gas flow,
γ_g — gas specific gravity (air = 1.0), dimensionless,
k — ratio of specific heats = C_p/C_v, dimensionless,
p_1 — upstream pressure, absolute units,
p_2 — downstream pressure, absolute units,
T_1 — upstream temperature, absolute units,
Z_1 — compressibility factor at p_1 and T_1, dimensionless,
C_s — coefficient based on system of units,
C_d — discharge coefficient (empirical), dimensionless,
T_{sc} — standard temperature base, absolute units,
p_{sc} — standard pressure base, absolute units, and
R_{pc} — critical pressure ratio, dimensionless.

Values of k can be obtained from:

$$k = \frac{C_p}{C_v} = 1 + \frac{1.987}{MC_p - 1.987}$$

where

M = molecular weight, lbm/mole,
C_p = specific heat, BTU/lbm − °R

Tables 7-1 and 7-2 give values for the constants in the equation for various systems of units.

TABLE 7-1
Coefficients and Units for Equation 7-1

Symbol	English System	Metric System	SI Metric System
q_{sc}	Mscf/d	m^3/d	m^3/d
d	in.	mm	mm
p abs	psia	kg_f/cm^2	kPa
T abs	°R	°K	°K
C_s	27.611	1.6259	1.6259

TABLE 7-2
Coefficient for Equation 7-1

System of Units	C_d	p_{sc} abs	T_{sc} abs	C_n
English	0.865	14.696 psi	491.68°R	799.06
	0.865	14.696 psi	519.68°R	844.57
Metric	0.865	1.0332 kg/cm^2	273.16°K	371.83
	0.865	1.0332 kg/cm^2	288.72°K	393.01
SI Metric	0.865	101.325 kPa	273.16°K	3.7915
	0.865	101.325 kPa	288.72°K	4.0075

The pressure ratio at which flow becomes critical depends on the k value for the flowing gas and is given by

$$R_{pc} = \left(\frac{2}{k+1}\right)^{k/k-1}$$

In calculating the values for C_n, given in Table 7-2, a discharge coefficient of 0.865 was used. The discharge coefficient actually depends on the Reynolds number, the ratio of the diameter of the pipe to the diameter of the restriction, and the geometry of the restriction.

Let's call the function of the pressure ratio, $F\ (P_2/P_1)$, where

$$F\left(\frac{p_2}{p_1}\right) = \left[\left(\frac{p_2}{p_1}\right)^{2/k} - \left(\frac{p_2}{p_1}\right)^{(k+1)/k}\right]^{1/2}$$

The function F (P2/P1) is plotted in Figure 7.1 for various values of p2/p1 and for a particular gas and upstream temperature.

The maximum value of F (p2/p1) in Figure 7.1 corresponds to the critical flow condition. The critical-flow range, over which the value of the flow rate through the restriction is proportional to the upstream pressure, occurs for values of the ratio of the downstream to upstream pressure less than the value corresponding to the maximum F (p2/p1). For this reason the left-hand arch, the curve in Figure 7.1, has been shown as a broken line, and a full horizontal line has been drawn through the maximum value.

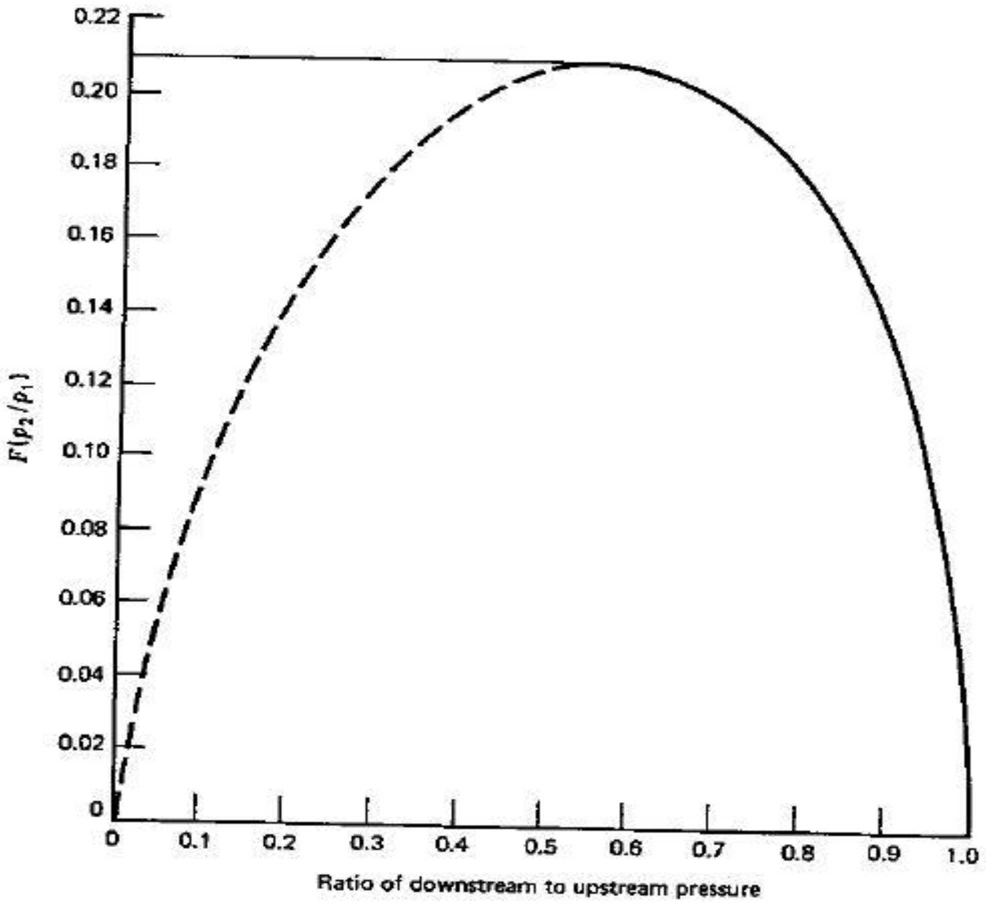

Figure 7.1: Gas flow through constriction

Example 7-1:

Using the following data, find the flow rate through the choke for:

a) $p_2 = 2837$ kPa, and
b) $p_2 = 1420$ kPa
$d = 10$ mm, $\gamma_g = 0.69$, $k = 1.25$, $C_d = 0.865$
$p_{sc} = 101.325$ kPa $T_{sc} = 288.72°K$,
$T_1 = 333°K$, $p_1 = 3546$ kPa, $Z_1 = 0.93$

Solution:

From table 7-2, $C_n = 4.0075$

$$R_{pc} = \left(\frac{2}{k+1}\right)^{k/k-1} = \left(\frac{2}{1.25+1}\right)^{1.25/.25} = 0.555$$

(a) $\frac{p_2}{p_1} = \frac{2837}{3546} = 0.80$, therefore flow is subcritical.

Using Equation 7.1

$$q_{sc} = \frac{4.0075(3546)(10)^2}{[(0.69)(333)(.93)]^{.5}} \left(5[(.8)^{1.6} - (.8)^{1.8}]\right)^{0.5}$$

$$q_{sc} = 97{,}213\ (0.391) = 38{,}000\ m^3/d$$

(b) $\frac{p_2}{p_1} = \frac{1420}{3546} = 0.4$, therefore flow is critical.

$$q_{sc} = 97{,}213\ \left(5[(.555)^{1.6} - (.555)^{1.8}]\right)^{0.5}$$

$$q_{sc} = 97{,}213\ (0.465) = 45{,}235\ m^3/d$$

Equation 7-1 has been modified for particular types of wellhead chokes. An equation which is used for the types of chokes manufactured by the **Thornhill-Craver company** is given below. This equation applies for 6 in. long chokes with rounded entrances operating in critical flow. The equation is

$$q_{sc} = \frac{605.4\, A\, p_1 C_d}{(T\, \gamma_g)^{0.5}} \quad \text{.................... (7.1a)}$$

where

q_{sc} = flow rate, Mscfd,
A = area of choke opening, in.^{2},
p_1 = upstream pressure, psia,
C_d = discharge coefficient, usually = 0.82,
T = upstream temperature, °R, and
γ_g = gas specific gravity.

Example 7.2:

Recalculate the flow rate in Example 7-1 using Equation 7.1a.

d = 10 mm = 0.394 in. p_1 = 3546 kPa = 514 psia
T_1 = 333°K = 600°R γ_g = 0.69

Solution:

$$A = .7854(0.394)^2 = 0.122 \text{ in.}^2$$

$$q_{sc} = \frac{605.4(0.122)(514)(0.82)}{[(600)(0.69)]^{0.5}} = 1530 \text{ Mscfd}$$

$$= 43{,}337\ m^3/d$$

An equation for calculating the pressure drop across a **subsurface safety valve** operating in **subcritical** flow was presented by the **API in 1974**. For the English System of units given in Table 7-1, the equation is

$$p_1 - p_2 = \frac{2.7\, \gamma_g\, p_1}{Z_1\, T_1}(1 - \beta^4) \left[\frac{6.23 \times 10^{-4}\, Z_1\, T_1\, q_{sc}}{p_1\, d^2\, C_d\, Y}\right]^2 \quad \text{........... (7.2)}$$

where

$\beta = d/d_p$,
d_p = pipe diameter,
C_d = discharge coefficient (API suggests using 0.9), and
Y = expansion factor,

$$Y = 1 - [0.41 + 0.35\beta^4]\left(\frac{p_1 - p_2}{k p_1}\right).$$

The solution of Equation 7-2 is iterative since Y is a function of Δp = P1 - P2. The value for Y ranges from about 0.67 to 1.0. For quick estimates of Δp, a value of 0.85 can be used.

Example 7-3:

A subsurface safety valve having a bean diameter of 1.0 in. is installed in a gas well equipped with 3.5 in. tubing (2.992 in. ID). The well is flowing at a rate of 20 MMscfd. Calculate the pressure drop across the SSSV if the pressure upstream of the SSSV is 2000 psia. The temperature is 180°F. Assume C_d=0.9, Y = 0.85. Gas gravity is 0.70.

Solution:

$$\beta = \frac{d}{d_p} = \frac{1.0}{2.992} = 0.334 \quad Z_1 = 0.84$$

$$\Delta p = \frac{2.7(0.7)(2000)}{0.84(640)}(1 - (.334)^4) \cdot \left[\frac{6.23 \times 10^{-4}(.84)(640)(20000)}{(2000)(1.0)^2(.9)(.85)}\right]^2$$

$$\Delta p = 6.944\ (4.378)^2 = 134 \text{ psi}$$