

**FIGURE 14-9** Flood patterns. (Permission to publish by the Society of Petroleum Engineers).

The patterns termed **inverted** have only one injection well per pattern. This is the difference between **normal** and **inverted** well arrangements. Note that the four-spot and inverted seven-spot patterns are identical.

### Crestal and Basal Injection Patterns

In crestal injection, as the name implies, the injection is through wells located at the top of the structure. Gas injection projects typically use a crestal injection pattern. In basal injection, the fluid is injected at the bottom of the structure. Many water-injection projects use basal injection patterns with additional benefits being gained from gravity segregation. A schematic illustration of the two patterns is shown in Figure 14-10.

### OVERALL RECOVERY EFFICIENCY

The overall recovery factor (efficiency) RF of any secondary or tertiary oil recovery method is the product of a combination of three individual efficiency factors as given by the following generalized expression:

$$RF = E_D E_A E_V \quad (14-5)$$

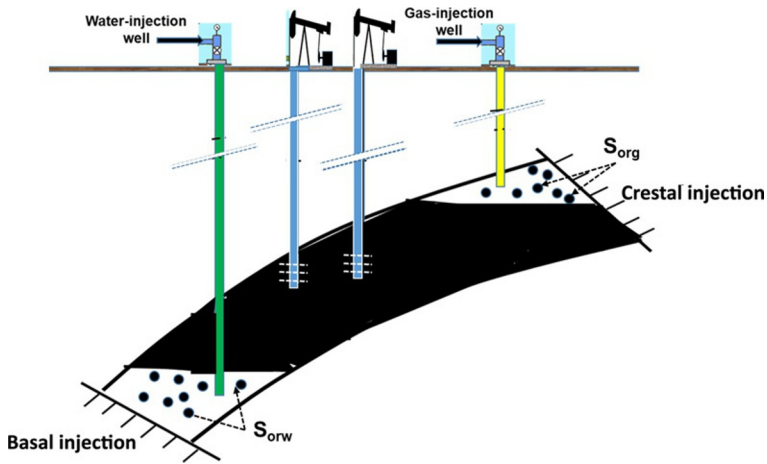


FIGURE 14-10 Well managements for dipping reservoirs.

In terms of cumulative oil production, Equation 14-5 can be written as:

$$N_P = N_S E_D E_A E_V \quad (14-6)$$

Where:

- RF = overall recovery factor
- $N_S$  = initial oil in place at the start of the flood, STB
- $N_P$  = cumulative oil produced, STB
- $E_D$  = displacement efficiency
- $E_A$  = areal sweep efficiency
- $E_V$  = vertical sweep efficiency

The displacement efficiency  $E_D$  is defined as the fraction of movable oil that has been displaced from the swept zone at any given time or pore volume injected. Because an immiscible gas injection or waterflood will always leave behind some residual oil,  $E_D$  will always be less than 1.0.

The areal sweep efficiency  $E_A$  is the fractional area of the pattern that is swept by the displacing fluid. The major factors determining areal sweep are:

- o Fluid mobilities
- o Pattern type
- o Areal heterogeneity
- o Total volume of fluid injected

The vertical sweep efficiency  $E_V$  is the fraction of the vertical section of the pay zone that is contacted by injected fluids. The vertical sweep efficiency is primarily a function of:

- o Vertical heterogeneity
- o Degree of gravity segregation
- o Fluid mobilities
- o Total volume injection

Note that the product of  $E_A E_V$  is called the **volumetric sweep efficiency “ $E_{vol}$ ”** and represents the overall fraction of the flood pattern that is contacted by the injected fluid.

In general, reservoir heterogeneity probably has more influence than any other factor on the performance of a secondary or tertiary injection project.

**Unfavorable reservoir heterogeneity** can negatively impact the performance of the waterflood. For example, the presence of **sealing faults and permeability discontinuities can reduce the effectiveness of water injectors** in providing sufficient pressure support to maintain reservoir pressure. **High permeability streaks are another type of unfavorable reservoir heterogeneity** that can adversely impact the performance of the waterflood. **These high permeability streaks can cause a reduction in the projected oil recovery factor for several reasons, including:**

- 1) **distortion in the waterflood flow pattern**
- 2) **reduction in the waterflood overall sweep efficiency**
- 3) **water channeling and an early water breakthrough**

The most important two types of heterogeneity affecting sweep efficiencies are the reservoir vertical heterogeneity and areal heterogeneity.

- **Vertical heterogeneity** is considered by far the most significant parameter influencing the vertical sweep and in particular its degree of heterogeneity variation in the vertical direction. A reservoir may exhibit **many different layers** in the vertical section that have highly different rock properties. **This stratification can result from many factors** such as **change in depositional environment** or **change in depositional source**. When water injected into a stratified system, the injected water will preferentially enter layers with the highest permeabilities and will move at a higher velocity. Consequently, at the time of the injected water breakthrough in higher-permeability zones, a significant portion of the less-permeable zones will remain unflooded. Although a flood will generally continue beyond breakthrough, the economic limit is often reached at an earlier time.
- **Areal heterogeneity** includes areal variation in formation properties and geometrical factors such as:
  - **Rock characteristics, e.g. h, k,  $\phi$ ,  $S_{wc}$**
  - **Position and type of faults**
  - **boundary conditions due to the presence of an aquifer or gas cap.**

Logging and coring appraisal wells will all permit direct observation of vertical heterogeneity. Therefore, if the data are interpreted correctly, it should be possible to quantify the vertical sweep “ $E_V$ ” quite accurately. In terms of the areal heterogeneity, however, uncertainties exist in defining this heterogeneity since methods used are indirect, e.g.:

- **Locating faults from well testing analysis**
- **The use of Kriging approach**

Consequently, the areal sweep efficiency is traditionally regarded as the unknown in reservoir-development studies.

It should be noted that all three efficiency factors (i.e.,  $E_D$ ,  $E_A$ , and  $E_V$ ) are variables that increase during the flood and reach maximum values at the economic limit of the injection project. Each of the three efficiency factors is discussed individually and methods of estimating these efficiencies are presented.

## I. DISPLACEMENT EFFICIENCY

As defined previously, displacement efficiency is the fraction of movable oil that has been recovered from the swept zone at any given time. Mathematically, the displacement efficiency is expressed as:

$$E_D = \frac{\text{Volume of oil at start of flood} - \text{Remaining oil volume}}{\text{Volume of oil at start of flood}}$$

$$E_D = \frac{(\text{Pore volume}) \left( \frac{S_{oi}}{B_{oi}} \right) - (\text{Pore volume}) \left( \frac{\bar{S}_o}{B_o} \right)}{(\text{Pore volume}) \left( \frac{S_{oi}}{B_{oi}} \right)}$$

or

$$E_D = \frac{\frac{S_{oi}}{B_{oi}} - \frac{\bar{S}_o}{B_o}}{\frac{S_{oi}}{B_{oi}}} \quad (14-7)$$

Where:

$S_{oi}$  = initial oil saturation at start of flood

$B_{oi}$  = oil FVF at start of flood, bbl/STB

$\bar{S}_o$  = average oil saturation in the flood pattern at a particular point during the flood

Assuming a constant oil formation volume factor during the flood life, Equation 14-7 is reduced to:

$$E_D = \frac{S_{oi} - \bar{S}_o}{S_{oi}} \quad (14-8)$$

where the initial oil saturation  $S_{oi}$  is given by:

$$S_{oi} = 1 - S_{wi} - S_{gi}$$

Assuming that trapped gas " $S_{gt}$ " exists in the flooded; the average oil saturation is given by the expressions:

$$\bar{S}_o = 1 - \bar{S}_w - S_{gt}$$

And the displacement efficiency  $E_D$  can be then expressed as:

$$E_D = \frac{S_{oi} - \bar{S}_o}{S_{oi}} = \frac{(1 - S_{wi} - S_{gi}) - (1 - \bar{S}_w - S_{gt})}{1 - S_{wi} - S_{gi}}$$

Simplifying, to give the following generalized for:

$$E_D = \frac{S_{oi} - \bar{S}_o}{S_{oi}} = \frac{(\bar{S}_w - S_{wi}) - (S_{gi} - S_{gt})}{1 - S_{wi} - S_{gi}}$$

The above shows the positive impact of the trapped gas saturation on the improvement of waterflood displacement efficiency.

However, if the trapped gas saturation in the swept area, is considered zero, thus:

$$\bar{S}_o = 1 - \bar{S}_w$$

The displacement efficiency  $E_D$  can be expressed more conveniently in terms of water saturation by substituting the above relationships into Equation 14-8, to give:

$$E_D = \frac{\bar{S}_w - S_{wi} - S_{gi}}{1 - S_{wi} - S_{gi}} \quad (14-9)$$

Where:

$\bar{S}_w$  = average water saturation in the swept area

$S_{gi}$  = initial gas saturation at the start of the flood

$S_{wi}$  = initial water saturation at the start of the flood

If no initial gas is present at the start of the flood, Equation 14-9 is reduced to:

$$E_D = \frac{\bar{S}_w - S_{wi}}{1 - S_{wi}} \quad (14-10)$$

The displacement efficiency  $E_D$  will continually increase at different stages of the flood, i.e., with increasing  $\bar{S}_w$ . Equation 14-8 or 14-10 suggests that  $E_D$  reaches its maximum when the average oil saturation in the area of the flood pattern is reduced to the residual oil saturation  $S_{or}$  or, equivalently, when  $\bar{S}_w = 1 - S_{or}$ .

#### Example 14-4

A saturated oil reservoir is under consideration to be waterflooded immediately after drilling and completion. Core analysis tests indicate that the initial and residual oil saturations are 70 and 35%, respectively. Calculate the displacement efficiency when the oil saturation is reduced to 65, 60, 55, 50, and 35%. Assume that  $B_o$  will remain constant throughout the project life.

#### Solution

Step 1. Calculate initial water saturation:

$$S_{wi} = 1 - 0.7 = 0.3$$

Step 2. Calculate  $E_D$  from Equation 14-10:

$$E_D = \frac{\bar{S}_w - S_{wi}}{1 - S_{wi}}$$

$\bar{S}_o$	$\bar{S}_w = 1 - \bar{S}_o$	$E_D = \frac{\bar{S}_w - S_{wi}}{1 - S_{wi}}$
0.65	0.35	0.071
0.60	0.40	0.142
0.55	0.45	0.214
0.50	0.50	0.286
$S_{or} = 0.35$	0.65	0.500 (maximum)

Example 14-4 shows that  $E_D$  will continually increase with increasing water saturation in the reservoir. The problem, of course, lies with developing an approach for determining the increase in the average water saturation in the swept area as a function of cumulative water injected (or injection time). Buckley and Leverett (1942) developed a well-established theory, called the **frontal displacement theory**, which provides the basis for establishing such a relationship. This classic theory consists of two equations:

- Fractional flow equation
- Frontal advance equation

The frontal displacement theory and its main two components are discussed next.

### A. Fractional Flow Equation

The development of the fractional flow equation is attributed to Buckley and Leverett (1942). For two immiscible fluids, oil and water, the fractional flow of water,  $f_w$  (or any immiscible displacing fluid), is defined as the water flow rate divided by the total flow rate, or:

$$f_w = \frac{q_w}{q_t} = \frac{q_w}{q_w + q_o} \tag{14-11}$$

Where:

- $f_w$  = fraction of water in the flowing stream, i.e., water cut, bbl/bbl
- $q_t$  = total flow rate, bbl/day
- $q_w$  = water flow rate, bbl/day
- $q_o$  = oil flow rate, bbl/day

Consider the steady-state flow of two immiscible fluids (oil and water) through a tilted-linear porous media as shown in Figure 14-11. Assuming a homogeneous system, Darcy's equation can be applied for each of the fluids:

$$q_o = \frac{-k_o A}{\mu_o} \left[ \frac{\partial P_o}{\partial x} + g\rho_o \sin(\alpha) \right] \tag{14-12}$$

$$q_w = \frac{-k_w A}{\mu_w} \left[ \frac{\partial P_w}{\partial x} + g\rho_w \sin(\alpha) \right] \tag{14-13}$$

Where:

- subscripts o, w = oil and water
- $k_o, k_w$  = effective permeability
- $\mu_o, \mu_w$  = viscosity

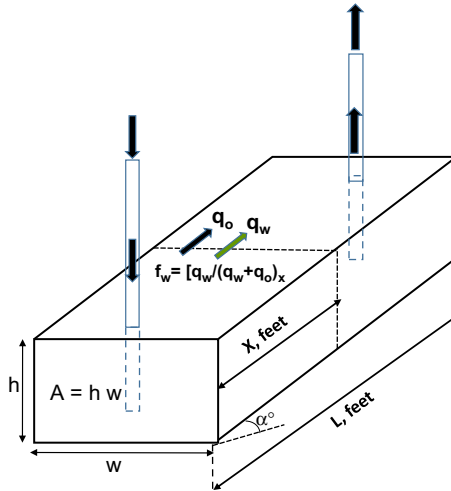


FIGURE 14-11 Linear water flood in a tilted reservoir with a dip-angle of  $\alpha^\circ$ .

$p_o, p_w$  = pressure

$\rho_o, \rho_w$  = density

$A$  = cross-sectional area

$x$  = distance

$\alpha$  = dip angle

$\sin(\alpha)$  = **positive number** if the injection well is located **down**dip

$\sin(\alpha)$  = **negative number** if the injection well is located **up**dip

Rearranging Equations 14-12 and 14-13 gives:

$$\frac{q_o \mu_o}{A k_o} = -\frac{\partial p_o}{\partial x} - g \rho_o \sin(\alpha)$$

$$\frac{q_w \mu_w}{A k_w} = -\frac{\partial p_w}{\partial x} - g \rho_w \sin(\alpha)$$

Subtracting the above two equations yields:

$$\frac{q_w \mu_w}{A k_w} - \frac{q_o \mu_o}{A k_o} = \left( \frac{\partial p_o}{\partial x} - \frac{\partial p_w}{\partial x} \right) - g(\rho_w - \rho_o) \sin \alpha \quad (14-14)$$

From the definition of the capillary pressure  $p_c$ :

$$P_c = p_o - p_w$$

Differentiating the above expression with respect to the distance  $x$  gives:

$$\frac{\partial p_c}{\partial x} = \frac{\partial p_o}{\partial x} - \frac{\partial p_w}{\partial x} \quad (14-15)$$

Combining Equation 14-15 with 14-16 gives:

$$\frac{q_w \mu_w}{A k_w} - \frac{q_o \mu_o}{A k_o} = \frac{\partial p_c}{\partial x} - g \Delta \rho \sin(\alpha) \quad (14-16)$$

where  $\Delta\rho = \rho_w - \rho_o$ . From the water cut equation, i.e., Equation 14-11:

$$q_w = f_w q_t \text{ and } q_o = (1 - f_w) q_t \quad (14-17)$$

Replacing  $q_o$  and  $q_w$  in Equation 14-16 with those of Equation 14-17 gives:

$$f_w = \frac{1 + \left( \frac{k_o A}{\mu_o q_t} \right) \left[ \frac{\partial p_c}{\partial x} - g \Delta\rho \sin(\alpha) \right]}{1 + \frac{k_o \mu_w}{k_w \mu_o}}$$

In field units, the above equation can be expressed as:

$$f_w = \frac{1 + \left( \frac{0.001127 k_o A}{\mu_o q_t} \right) \left[ \frac{\partial p_c}{\partial x} - 0.433 \Delta\rho \sin(\alpha) \right]}{1 + \frac{k_o \mu_w}{k_w \mu_o}} \quad (14-18)$$

Where:

- $f_w$  = fraction of water (water cut), bbl/bbl
- $k_o$  = effective permeability of oil, md
- $k_w$  = effective permeability of water, md
- $\Delta\rho$  = water–oil density differences, g/cm<sup>3</sup>
- $k_w$  = effective permeability of water, md
- $q_t$  = total flow rate, bbl/day
- $\mu_o$  = oil viscosity, cp
- $\mu_w$  = water viscosity, cp
- $A$  = cross-sectional area, ft<sup>2</sup>

Noticing that for two-phase flow:

- o The relative permeability ratios  $k_{ro}/k_{rw} = k_o/k_w$  and
- o the total flow rate  $q_t$  is essentially equal to the water injection rate, i.e.,  $i_w = q_t$ ,

Equation 14-18 can then be expressed more conveniently in terms of  $k_{ro}/k_{rw}$  and  $i_w$  as:

$$f_w = \frac{1 + \left( \frac{0.001127 (k k_{ro}) A}{\mu_o i_w} \right) \left[ \frac{\partial p_c}{\partial x} - 0.433 \Delta\rho \sin(\alpha) \right]}{1 + \frac{k_{ro} \mu_w}{k_{rw} \mu_o}} \quad (14-19)$$

Where:

- $i_w$  = water injection rate, bbl/day
- $f_w$  = water cut, bbl/bbl
- $k_{ro}$  = relative permeability to oil
- $k_{rw}$  = relative permeability to water
- $k$  = absolute permeability, md



The fractional flow equation as expressed by the above relationship suggests that for a given rock–fluid system, all the terms in the equation are defined by the characteristics of the reservoir, excluding (except):

- water injection rate,  $i_w$
- water viscosity,  $\mu_w$
- direction of the flow, i.e., updip or downdip injection

Equation 14-19 can be expressed in a more generalized form to describe the fractional flow of any displacement fluid as:

$$f_D = \frac{1 + \left( \frac{0.001127(kk_{rD})A}{\mu_o i_D} \right) \left[ \frac{\partial p_c}{\partial x} - 0.433 \Delta \rho \sin(\alpha) \right]}{1 + \frac{k_{ro} \mu_D}{k_{rD} \mu_o}} \quad (14-20)$$

where the subscript **D** refers to the displacement fluid and  $\Delta \rho$  is defined as:

$$\Delta \rho = \rho_D - \rho_o$$

For example, when the displacing fluid is immiscible gas, then:

$$f_g = \frac{1 + \left( \frac{0.001127(kk_{rg})A}{\mu_o i_g} \right) \left[ \frac{\partial p_c}{\partial x} - 0.433 (\rho_g - \rho_o) \sin(\alpha) \right]}{1 + \frac{k_{ro} \mu_g}{k_{rg} \mu_o}} \quad (14-21)$$

The effect of capillary pressure is usually neglected because the capillary pressure gradient is generally small and, thus, Equations 14-19 and 14-21 are reduced to:

$$f_w = \frac{1 - \left( \frac{0.001127(kk_{ro})A}{\mu_o i_w} \right) [0.433 (\rho_w - \rho_o) \sin(\alpha)]}{1 + \frac{k_{ro} \mu_w}{k_{rw} \mu_o}} \quad (14-22)$$

and

$$f_g = \frac{1 - \left( \frac{0.001127(kk_{ro})A}{\mu_o i_g} \right) [0.433 (\rho_g - \rho_o) \sin(\alpha)]}{1 + \frac{k_{ro} \mu_g}{k_{rg} \mu_o}}$$

Where:

$i_g$  = gas injection rate, bbl/day

$\mu_g$  = gas viscosity, cp

$\rho_g$  = gas density, g/cm<sup>3</sup>

From the definition of water cut, i.e.,  $f_w = q_w / (q_w + q_o)$ , it indicates that the limits of the water cut are 0 and 100%. At the irreducible (connate) water saturation “ $S_{wc}$ ”, the water flow rate  $q_w$  is zero and, therefore, the water cut is 0%. At the residual oil saturation point “ $S_{orw}$ ”, the oil flow rate is zero and the water cut reaches its upper limit of 100%.

The shape of the water cut versus water saturation curve is characteristically has the S-shaped profile, as shown in Figure 14-12. The limits of the  $f_w$  curve (0 and 1) are defined by the end points of the relative permeability curves. The implications of the above discussion can be also applied to define the relationship that exists between gas water-cut “ $f_g$ ” and gas saturation, as shown in Figure 14-12.

It should be pointed out that, in general, any influences that cause the fractional flow curve to shift upward and to the left (i.e., increase in  $f_w$  or  $f_g$ ) will result in a *less efficient displacement* process. It is essential, therefore, to determine the impact of changing the various component parts of the fractional flow equation. Note that for any two immiscible fluids (e.g., water and oil) the fraction (proportion) of the oil cut “ $f_o$ ” flowing at any point in the reservoir is given by:

$$f_o + f_w = 1$$

Or:

$$f_o = 1 - f_w$$

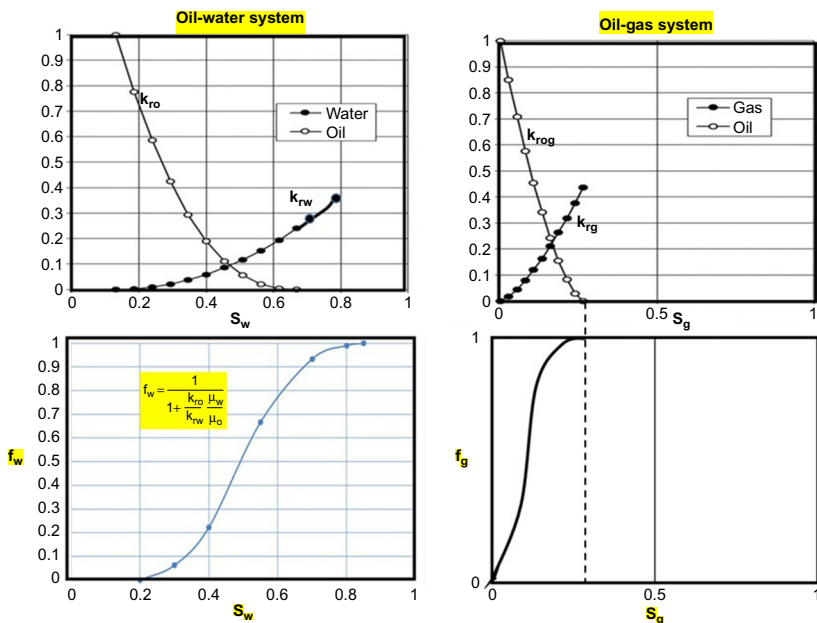


FIGURE 14-12 Fractional flow curves as a function of saturation.

The above expression indicates that during the displacement of oil by water-flood, an **increase in  $f_w$**  at any point in the reservoir will cause a proportional decrease in:

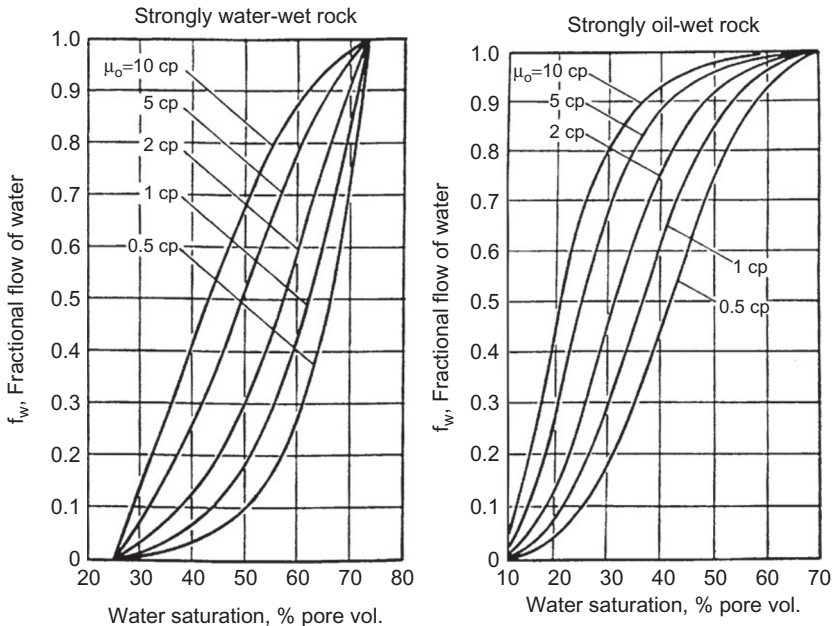
- **Oil-cut " $f_o$ "** and
- **Oil mobility.**

**Therefore, the objective is to select the proper injection scheme that could possibly reduce the water fractional flow.** This can be achieved by investigating the effect of the injected **water viscosity**, **formation dip angle**, and **water-injection rate** on the water cut. The overall effect of these parameters on the water fractional flow curve are discussed next that includes the impact of:

- **Oil and water viscosities**
- **Water injection rate as related to the formation dip angle**

## Effect of Water and Oil Viscosities

Figure 14-13 shows the general effect of oil viscosity on the fractional flow curve for both water-wet and oil-wet rock systems. This illustration reveals that regardless of the system wettability, a higher oil viscosity results in an upward shift (an increase) in the fractional flow curve. The apparent effect of the water viscosity on the water fractional flow is clearly indicated by examining



**FIGURE 14-13** Effect of oil viscosity on  $f_w$ .