Al-Ayen University
College of Petroleum Engineering

Reservoir Engineering II

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Lecture 14: Principle of Superposition

Refs.: Reservoir Engineering Handbook by Tarek Ahmed

Outline

- Principle of Superposition
  - Effects of Multiple Wells
    - Example
  - Effects of Variable Flow Rates
    - Example
- Summary

Principle of Superposition

- Mathematically the *superposition theorem* states that *any sum of individual solutions to the diffusivity equation is also a solution to that equation*.
- This concept can be applied to account for the following effects on the transient flow solution:
  - *Effects of multiple wells*
  - *Effects of rate change*
  - *Effects of the boundary*
  - *Effects of pressure change*

Effects of Multiple Wells

- The superposition concept states that the total pressure drop at any point in the reservoir is the sum of the pressure changes at that point caused by flow in each of the wells in the reservoir.

- Consider the Figure 1, which shows three wells that are producing at *different flow rates* from *an infinite acting reservoir*, i.e., *unsteady-state flow reservoir*.

- The principle of superposition indicates that the *total pressure drop observed at any well, e.g., Well 1*:

$$(\Delta p)_{\text{total drop at well 1}} = (\Delta p)_{\text{drop due to well 1}} + (\Delta p)_{\text{drop due to well 2}} + (\Delta p)_{\text{drop due to well 3}}$$

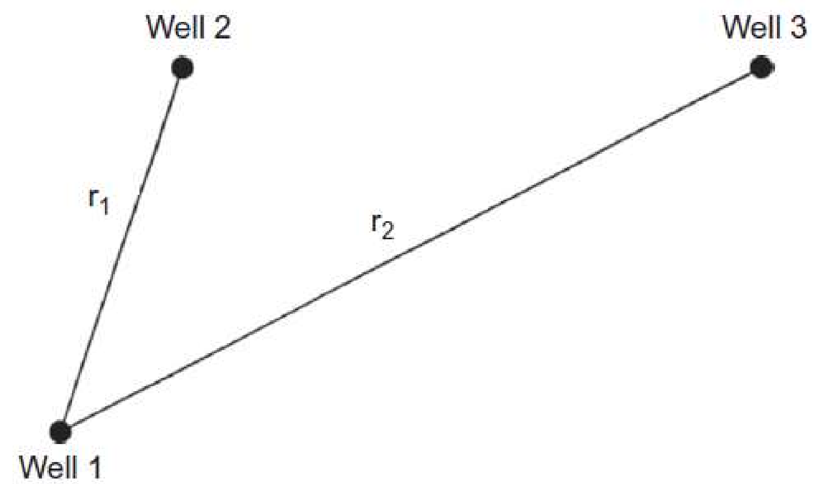

Figure 1: Three wells are producing at different flow rates from an infinite acting reservoir.

Example

Assume that the three wells as shown in Figure 2 are producing under a transient flow condition for 15 hours. The following additional data are available:

$$Q_{o1} = 100 \text{ STB/day}$$
$$Q_{o2} = 160 \text{ STB/day}$$
$$Q_{o3} = 200 \text{ STB/day}$$
$$p_i = 4500 \text{ psi}$$
$$B_o = 1.20 \text{ bbl/STB}$$
$$c_t = 20 \times 10^{-6} \text{ psi}^{-1}$$
$$(s)_{\text{well 1}} = -0.5$$

$$h = 20'$$
$$\phi = 15\%$$
$$k = 40 \text{ md}$$
$$r_w = 0.25'$$
$$\mu_o = 2.0 \text{ cp}$$
$$r_1 = 400'$$
$$r_2 = 700'$$

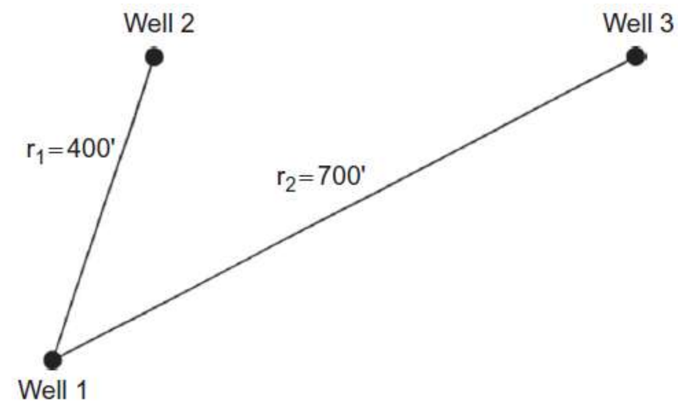

Figure 2: Well layout of the Example

If the three wells are producing at a constant flow rate, calculate the sand face flowing pressure at Well 1.

$$(p_i - p_{wf}) = (\Delta p)_{\text{well1}} = \frac{162.6 Q_{o1} B_o \mu_o}{kh}\left[\log\left(\frac{kt}{\phi \mu c_t r_w^2}\right) - 3.23 + 0.87s\right]$$

Solution

Pwf = Pi – (ΔP) total drop at well 1

$$(\Delta p)_{\text{total drop at well 1}} = (\Delta p)_{\text{drop due to well 1}} + (\Delta p)_{\text{drop due to well 2}} + (\Delta p)_{\text{drop due to well 3}}$$

For a slightly compressible fluid at the unsteady-state flow, the constant-terminal- rate solution of the diffusivity equation is:

$$p(r,t) = p_i + \left[\frac{70.6\, Q_o\, \mu_o\, Bo}{kh}\right] E_i\left[\frac{-948\phi\, \mu_o\, c_t r^2}{kt}\right]$$

The pressure drop at *Well 1* due to its own production and is given by the log-approximation to the Ei-function solution presented by Equation:

$$(p_i - p_{wf}) = (\Delta p)_{\text{well1}} = \frac{162.6 Q_{o1} B_o \mu_o}{kh}\left[\log\left(\frac{kt}{\phi\, \mu\, c_t\, r_w^2}\right) - 3.23 + 0.87s\right] = 270.2 \text{ psi}$$

$$p(r,t) = p_i + \left[\frac{70.6\, Q_o\, \mu_o Bo}{kh}\right] E_i \left[\frac{-948\phi\, \mu_o\, c_t r^2}{kt}\right]$$

$$(\Delta p)_{\text{due to well2}} = -\left(\frac{70.6 Q_{o2} B_o \mu_o}{kh}\right) \times E_i\left[-\frac{948\,\phi\,\mu\, c_t r_1^2}{kt}\right] = 4.41 \text{ psi}$$

$$(\Delta p)_{\text{due to well 3}} = -\left(\frac{70.6 Q_{o3} B_o \mu_o}{kh}\right) \times E_i\left[-\frac{948\,\phi\,\mu\, c_t r_2^2}{kt}\right] = 0.08 \text{ psi}$$

$$(\Delta p)_{\text{total at well 1}} = 270.2 + 4.41 + 0.08 = 274.69 \text{ psi}$$

$$p_{wf} = 4500 - 274.69 = 4225.31 \text{ psi}$$

- The above computational approach can be used to calculate the pressure at Wells 2 and 3. Further, it can be extended to include any number of wells flowing under the unsteady-state flow condition.
- It should also be noted that if the point of interest is an operating well, the skin factor s must be included for that well only.

Effects of Variable Flow Rates

- The concept of superposition states, *"Every flow rate change in a well will result in a pressure response which is independent of the pressure responses caused by other previous rate changes."*
- Accordingly, the total pressure drop that has occurred at any time is the summation of pressure changes caused separately by each net flow rate change.
- Consider the case of a shut-in well, i.e., Q = 0, that was then allowed to produce at a series of constant rates for the different time periods shown in Figure 3.
- The total pressure drop at the sand face at time t_4 :

$$(\Delta p)_{total} = (\Delta p)_{due\ to\ (Q_{o1}-0)} + (\Delta p)_{due\ to\ (Q_{o2}-Q_{o1})} + (\Delta p)_{due\ to\ (Q_{o3}-Q_{o2})} + (\Delta p)_{due\ to\ (Q_{o4}-Q_{o3})}$$

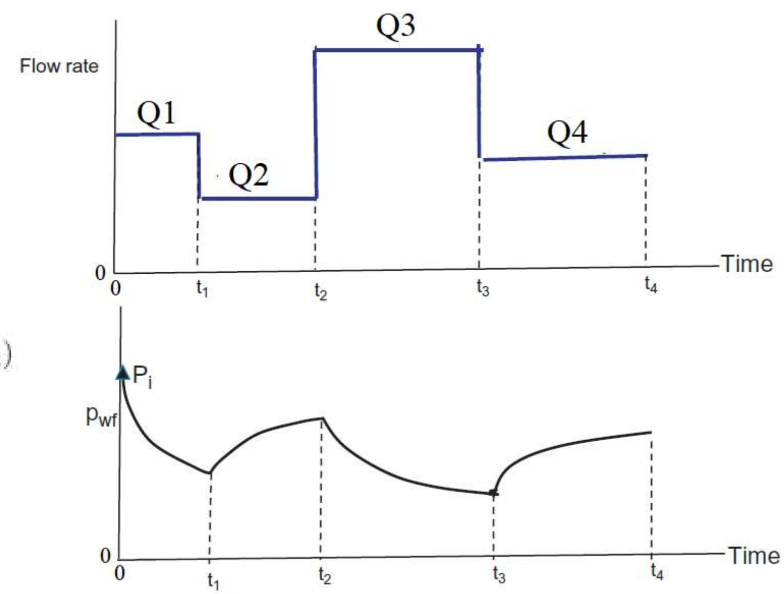

Figure 3: Production and pressure of an oil well at variable flow rates

Example

Figure 4 shows the rate history of a well that is producing under transient flow condition for 15 hours. Given the following data:

$$p_i = 5000 \text{ psi} \qquad h = 20'$$
$$B_o = 1.1 \text{ bbl/STB} \qquad \phi = 15\%$$
$$\mu_o = 2.5 \text{ cp} \qquad r_w = 0.3'$$
$$c_t = 20 \times 10^{-6} \text{ psi}^{-1} \qquad s = 0$$
$$k = 40 \text{ md}$$

Calculate the sand face pressure after 15 hours.

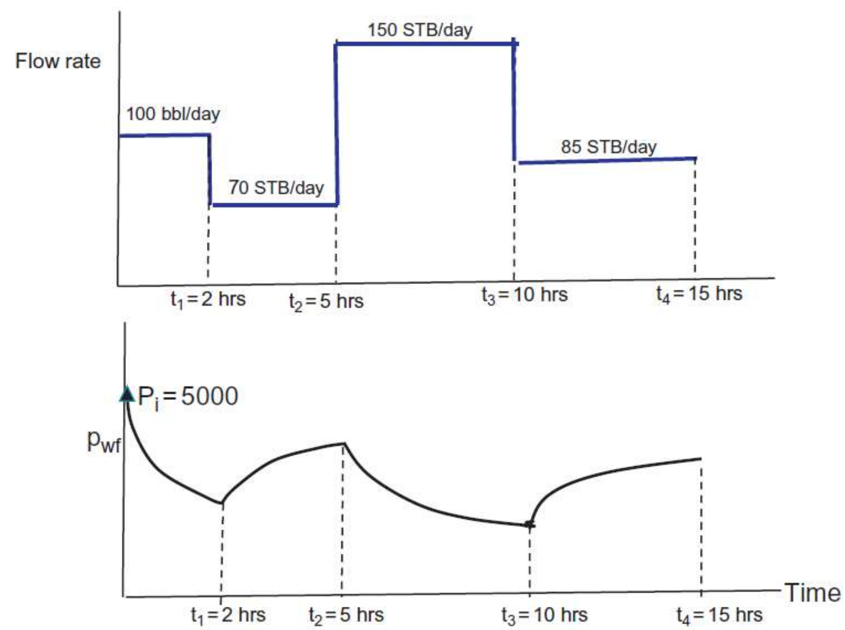

Figure 4: Production and pressure of an oil well of the Example.

Solution

Pwf = Pi – (ΔP) total drop at the well

$$(\Delta p)_{total} = (\Delta p)_{\text{due to }(Q_{o1}-0)} + (\Delta p)_{\text{due to }(Q_{o2}-Q_{o1})} + (\Delta p)_{\text{due to }(Q_{o3}-Q_{o2})} + (\Delta p)_{\text{due to }(Q_{o4}-Q_{o3})}$$

$$(\Delta p)_{Q1-o} = \left[\frac{162.6(Q_1-0)B\mu}{kh}\right]\left[\log\left(\frac{kt_4}{\phi\mu c_t r_w^2}\right) - 3.23 + 0.87s\right] = 319.6\text{ psi}$$

$$(\Delta p)_{Q_2-Q_1} = \left[\frac{162.6(Q_2-Q_1)B\mu}{kh}\right]\left[\log\left(\frac{k(t_4-t_1)}{\phi\mu c_t r_w^2}\right) - 3.23 + 0.87s\right] = -94.85\text{ psi}$$

$$(\Delta p)_{Q_3-Q_2} = \left[\frac{162.6(Q_3-Q_2)B\mu}{kh}\right]\left[\log\left(\frac{k(t_4-t_2)}{\phi\mu c_t r_w^2}\right) - 3.23 + 0.87s\right] = 249.18\text{ psi}$$

$$(\Delta p)_{Q4-Q_3} = \left[\frac{162.6(Q_4-Q_3)B\mu}{kh}\right]\left[\log\left(\frac{k(t_4-t_3)}{\phi\mu c_t r_w^2}\right) - 3.23 + 0.87s\right] = -190.44\text{ psi}$$

$$(\Delta p)_{total} = 319.6 + (-94.85) + 249.18 + (-190.44) = 283.49\text{ psi}$$

$$p_{wf} = 5000 - 283.49 = 4716.51\text{ psi}$$

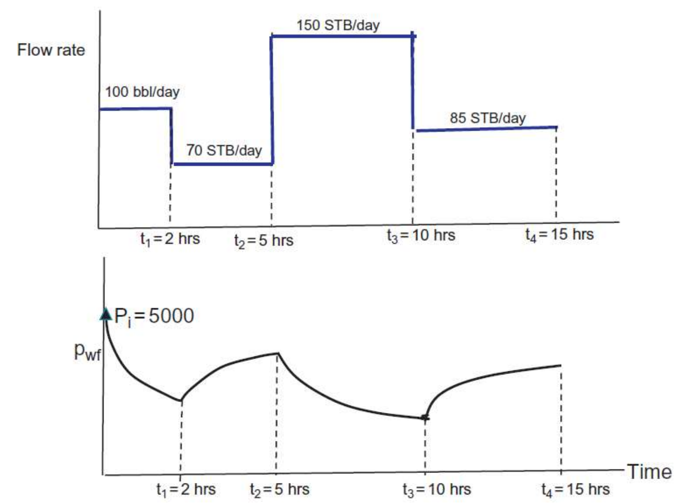

Figure 4: Production and pressure of an oil well of the Example.

Note: This approach is valid only if the well is flowing under the unsteady-state flow condition for the total time elapsed since the well began to flow at its initial rate.

Summary

- The superposition theorem states that any sum of individual solutions to the diffusivity equation is also a solution to that equation.

- This concept can be applied to account for the following effects on the transient flow solution: effects of multiple wells, effects of rate change, effects of the boundary and effects of pressure change.

- The superposition concept states that the total pressure drop at any point in the reservoir is the sum of the pressure changes at that point caused by flow in each of the wells in the reservoir.

- The concept of superposition states that every flow rate change in a well will result in a pressure response which is independent of the pressure responses caused by other previous rate changes.

THANK YOU