Al-Ayen University
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Mechanics

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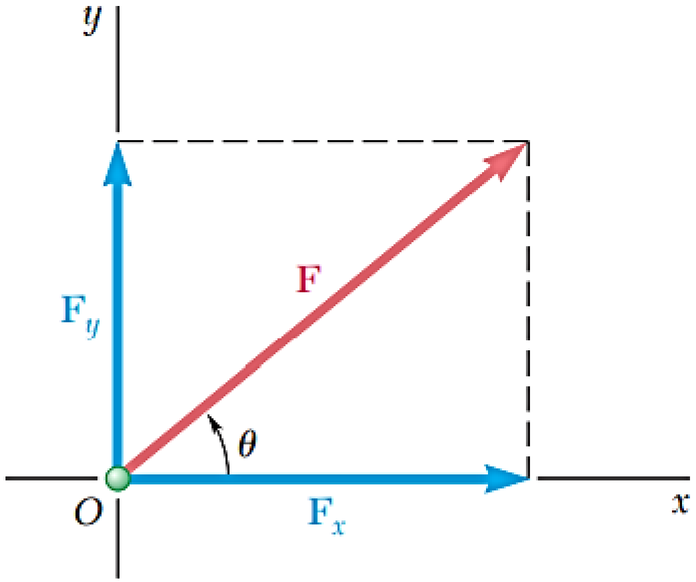
Title: Moment of a force

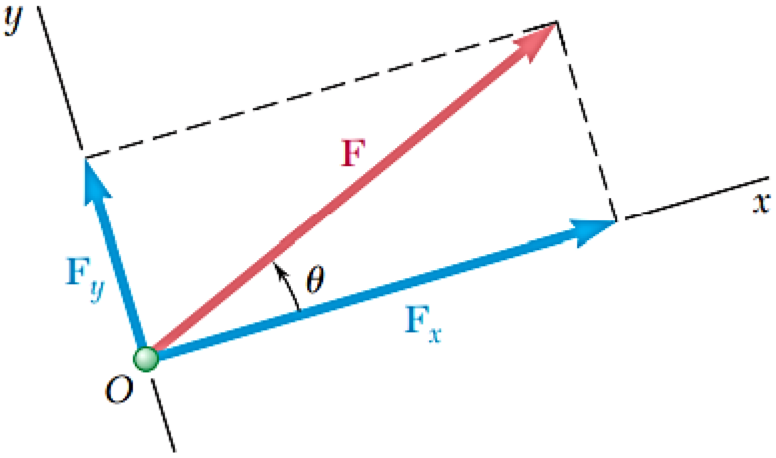
Lec. Number: 6

Rectangular components of a force

In many problems it will be found desirable to resolve a force into two components which are perpendicular to each other. In figure below, the force F has been resolved into a component F_x along the x axis and a component F_y along the y axis. The parallelogram drawn to obtain the two components is a rectangle, and F_x and F_y are called rectangular components.



Normal coordinates

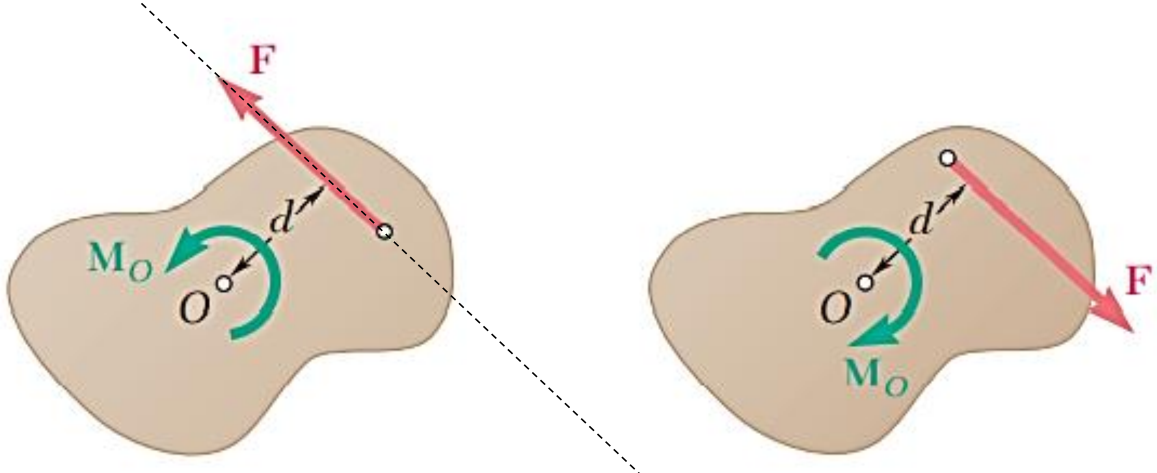


Inclines coordinates

Moment of a force about a point: Two dimensions

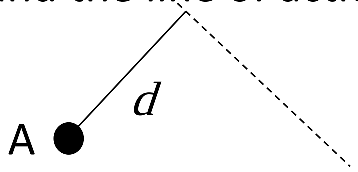
Many applications deal with two-dimensional structures, i.e., structures which have length and breadth but only negligible depth and which are subjected to forces contained in the plane of the structure. Two-dimensional structures and the forces acting on them can be readily represented on a sheet of paper or on a blackboard. Their analysis is therefore considerably simpler than that of three-dimensional structures and forces.

The magnitude of $\mathrm{M_O}$ measures the tendency of the force F to make the rigid body rotate about a fixed axis at point O normal to the plane containing F and d directed according to the right hand-rule.

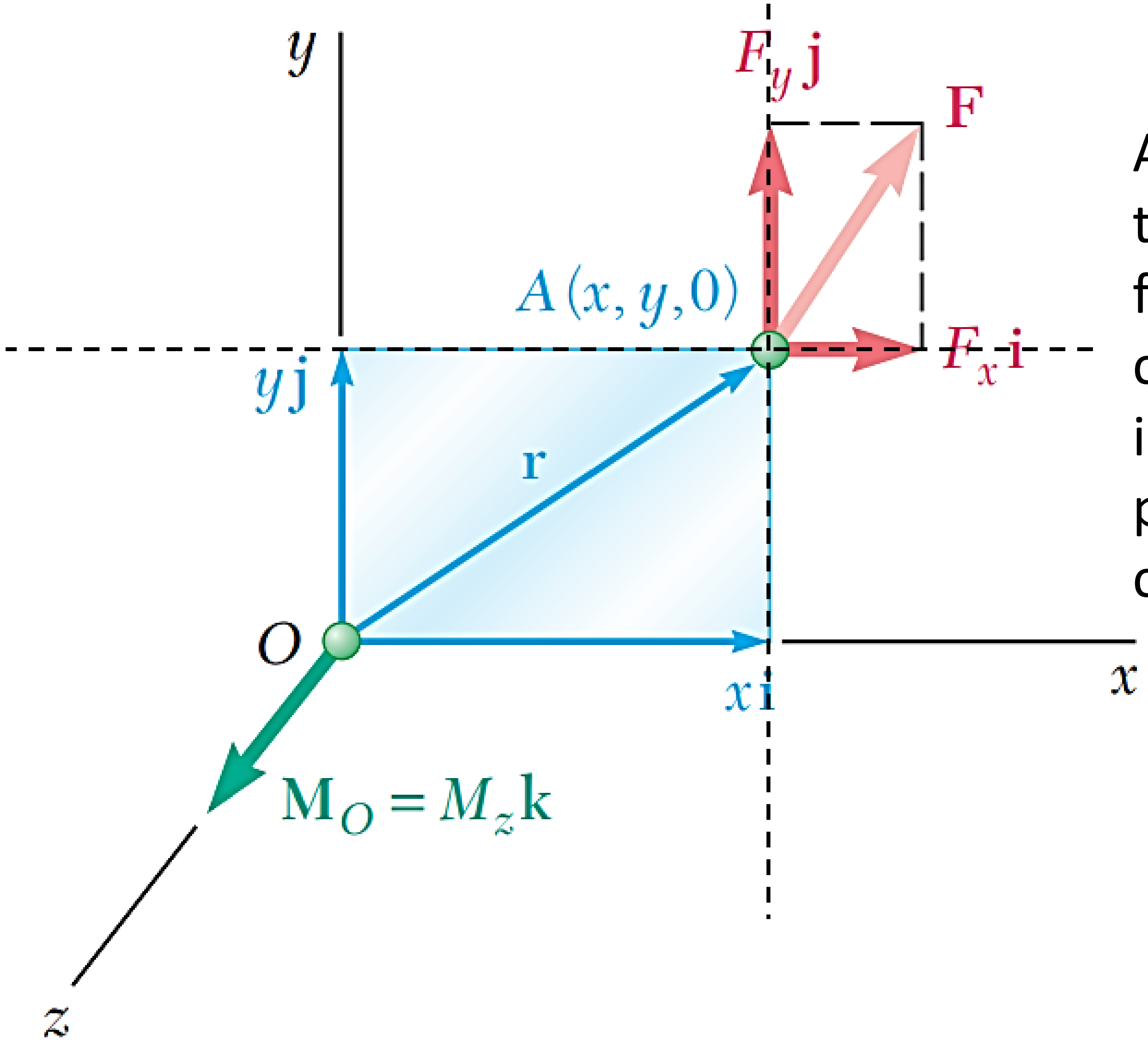


(a) $\boldsymbol{M}_O = +Fd$ (b) $\boldsymbol{M}_O = -Fd$

d = vertical distance between point O and the line of action of force F



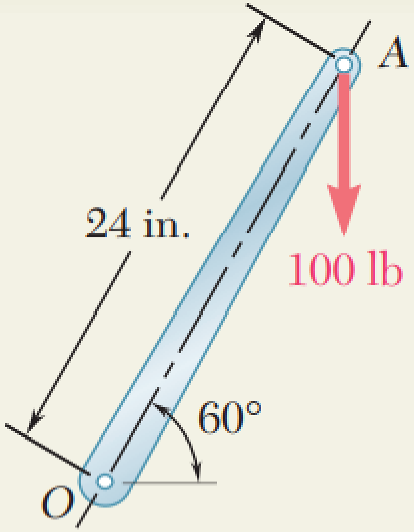
The moment of F about O is perpendicular to the plane of the figure and that it is completely defined by the scalar.



As noted earlier, a positive value for M_O indicates that the vector M_O points out of the paper (the force F tends to rotate the body counter clockwise about O), and a negative value indicates that the vector M_O points into the paper (the force F tends to rotate the body clockwise about O).

$$M_O = M_z = xF_y - yF_x$$

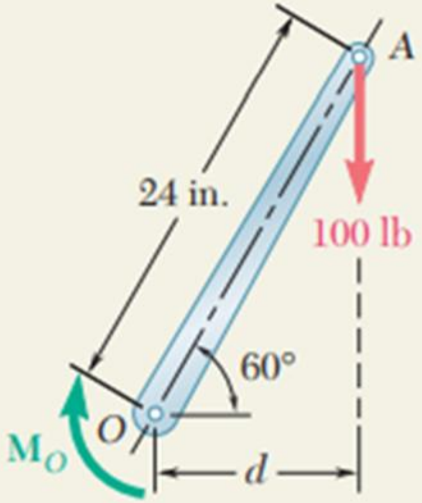
Example 1



A 100-lb vertical force is applied to the end of a lever which is attached to a shaft at O. Determine (*a*) the moment of the 100-lb force about O; (*b*) the horizontal force applied at A which creates the same moment about O; (*c*) the smallest force applied at A which creates the same moment about O; (*d*) how far from the shaft a 240-lb vertical force must act to create the same moment about O; (*e*) whether any one of the forces obtained in parts *b*, *c*, and *d* is equivalent to the original force.

Solution

(a)



a. Moment about O. The perpendicular distance from O to the line of action of the 100-lb force is

$$d = (24 \text{ in.}) \cos 60° = 12 \text{ in.}$$

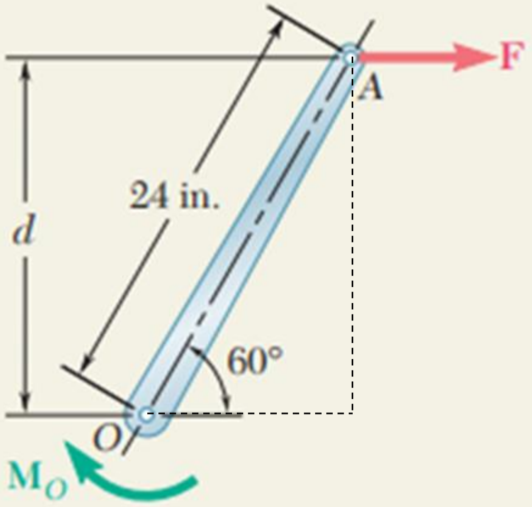
The magnitude of the moment about O of the 100-lb force is

$$M_O = Fd = (100 \text{ lb})(12 \text{ in.}) = 1200 \text{ lb} \cdot \text{in.}$$

Since the force tends to rotate the lever clockwise about O, the moment will be represented by a vector $\mathbf{M}_O$ perpendicular to the plane of the figure and pointing *into* the paper. We express this fact by writing

$$\mathbf{M}_O = 1200 \text{ lb} \cdot \text{in.} \downarrow$$

(b)



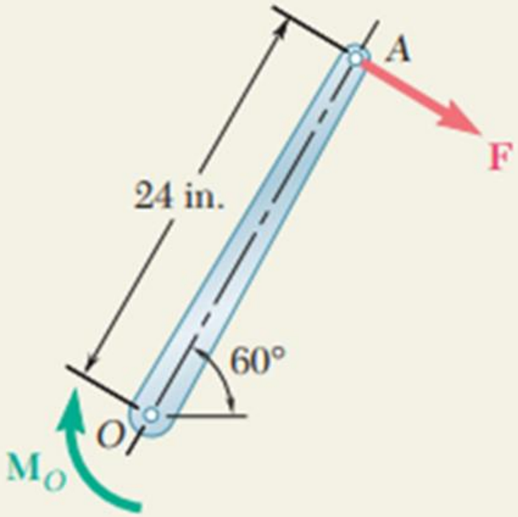
b. Horizontal Force. In this case, we have

$$d = (24 \text{ in.}) \sin 60° = 20.8 \text{ in.}$$

Since the moment about O must be 1200 lb · in., we write

$$M_O = Fd$$
$$1200 \text{ lb} \cdot \text{in.} = F(20.8 \text{ in.})$$
$$F = 57.7 \text{ lb} \qquad \mathbf{F} = 57.7 \text{ lb} \rightarrow$$

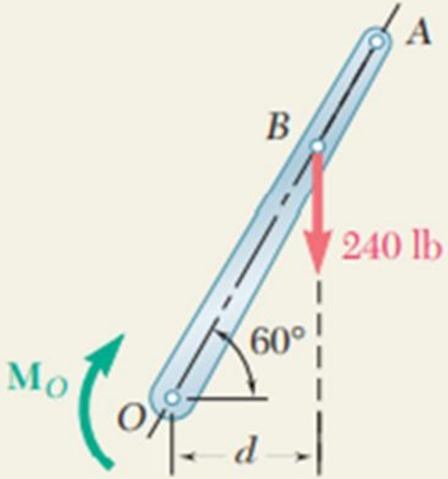
(c)



c. Smallest Force. Since $M_O = Fd$, the smallest value of F occurs when d is maximum. We choose the force perpendicular to OA and note that $d = 24$ in.; thus

$$M_O = Fd$$
$$1200 \text{ lb} \cdot \text{in.} = F(24 \text{ in.})$$
$$F = 50 \text{ lb} \qquad \mathbf{F} = 50 \text{ lb} ⦣ 30°$$

(d)

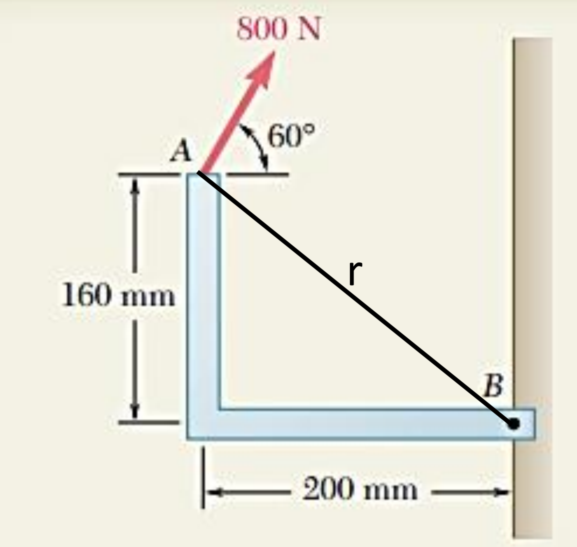


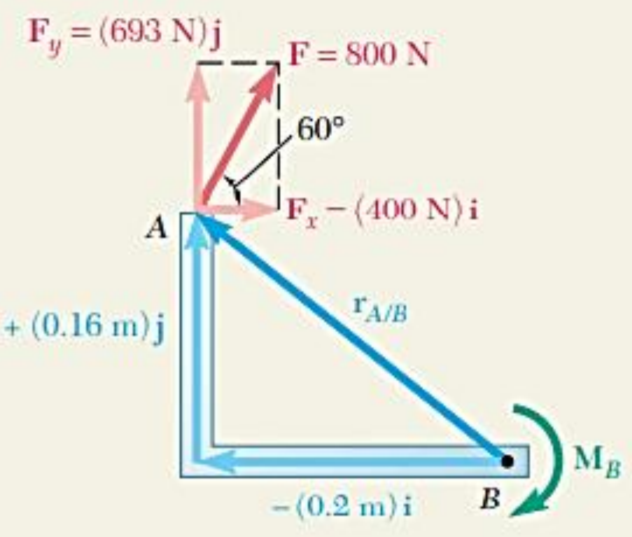
d. 240-lb Vertical Force. In this case $M_O = Fd$ yields

$$1200 \text{ lb} \cdot \text{in.} = (240 \text{ lb})d \qquad d = 5 \text{ in.}$$

but $\quad OB \cos 60° = d \qquad OB = 10$ in.

e. None of the forces considered in parts b, c, and d is equivalent to the original 100-lb force. Although they have the same moment about O, they have different x and y components. In other words, although each force tends to rotate the shaft in the same manner, each causes the lever to pull on the shaft in a different way.





Example 2

A force of 800 N acts on a bracket as shown. Determine the moment of the force about *B*.

Solution

The moment $\mathbf{M}_B$ of the force $\mathbf{F}$ about B is obtained by forming the vector product

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F}$$

where $\mathbf{r}_{A/B}$ is the vector drawn from B to A. Resolving $\mathbf{r}_{A/B}$ and $\mathbf{F}$ into rectangular components, we have

$$\mathbf{r}_{A/B} = -(0.2 \text{ m})\mathbf{i} + (0.16 \text{ m})\mathbf{j}$$

$$\mathbf{F} = (800 \text{ N}) \cos 60°\mathbf{i} + (800 \text{ N}) \sin 60°\mathbf{j}$$

$$= (400 \text{ N})\mathbf{i} + (693 \text{ N})\mathbf{j}$$

Recalling the relations (3.7) for the cross products of unit vectors (Sec. 3.5), we obtain

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F} = [-(0.2 \text{ m})\mathbf{i} + (0.16 \text{ m})\mathbf{j}] \times [(400 \text{ N})\mathbf{i} + (693 \text{ N})\mathbf{j}]$$

$$= -(138.6 \text{ N}\cdot\text{m})\mathbf{k} - (64.0 \text{ N}\cdot\text{m})\mathbf{k}$$

$$= -(202.6 \text{ N}\cdot\text{m})\mathbf{k} \qquad M_B = 203 \text{ N}\cdot\text{m} \circlearrowright$$

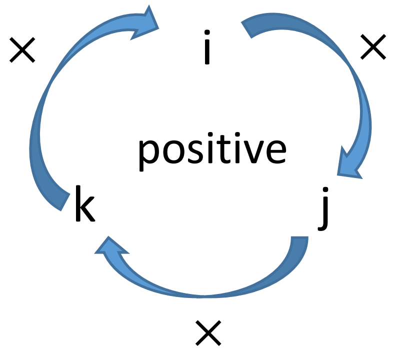
The moment $\mathbf{M}_B$ is a vector perpendicular to the plane of the figure and pointing *into* the paper.

Cross product

$i \times i = 0$

$j \times j = 0$

$k \times k = 0$



$i \times j = k$ $\qquad j \times i = -k$

$j \times k = i$ $\qquad k \times j = -i$

$k \times i = j$ $\qquad i \times k = -j$