Al-Ayen University
Petroleum Engineering College

Mechanics

Dynamics

Title: Curvilinear motion

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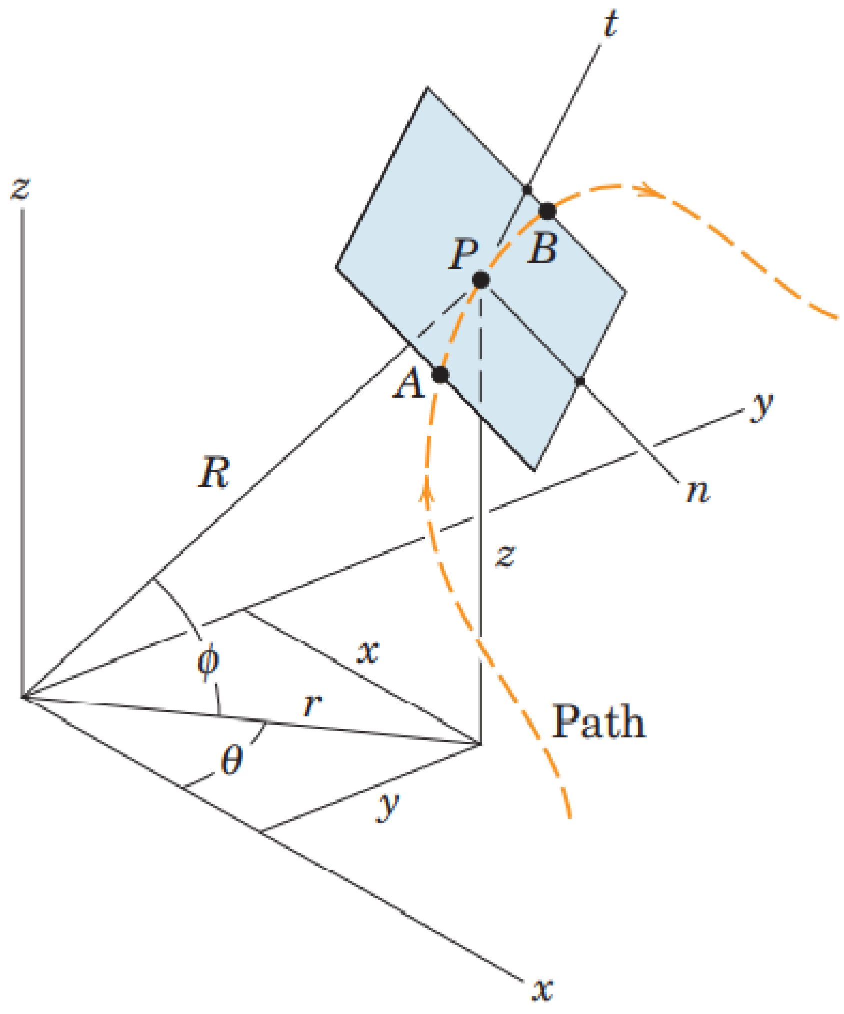
Lecture No. 8

Curvilinear motion

The figure shows a particle P moving along a curvilinear path in space.

The position of particle P at any time t can be described by specifying its rectangular coordinates x, y, z, its cylindrical coordinates r, θ, z, or its spherical coordinates R, θ, Ø. The motion of P can also be described by measurements along the tangent t and normal n to the curve. The direction of n lies in the local plane of the curve.

All movement occurs in or can be represented as occurring in a single plane. A large proportion of the motions of machines and structures in engineering can be represented as plane motion.



Rectangular coordinates (x-y)

This system of coordinates is particularly useful for describing motions where the x- and y-components of acceleration are independently generated or determined. The resulting curvilinear motion is then obtained by a vector combination of the x- and y-components of the position vector, the velocity, and the acceleration.

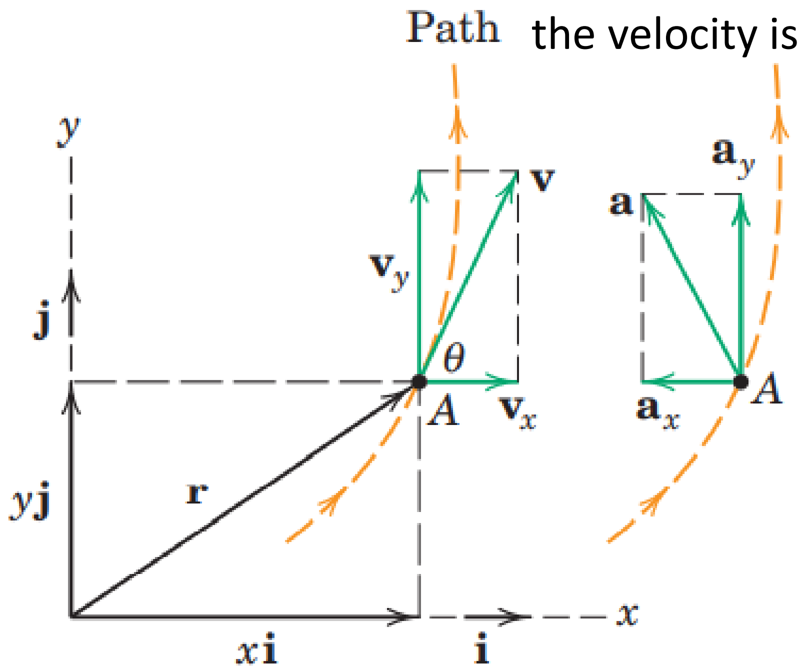
Vector Representation

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}}$$

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$
$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$
$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

As observed previously, the direction of the velocity is always tangent to the path



Scalar Representation

$$v^2 = {v_x}^2 + {v_y}^2 \qquad v = \sqrt{{v_x}^2 + {v_y}^2} \qquad \tan\theta = \frac{v_y}{v_x}$$

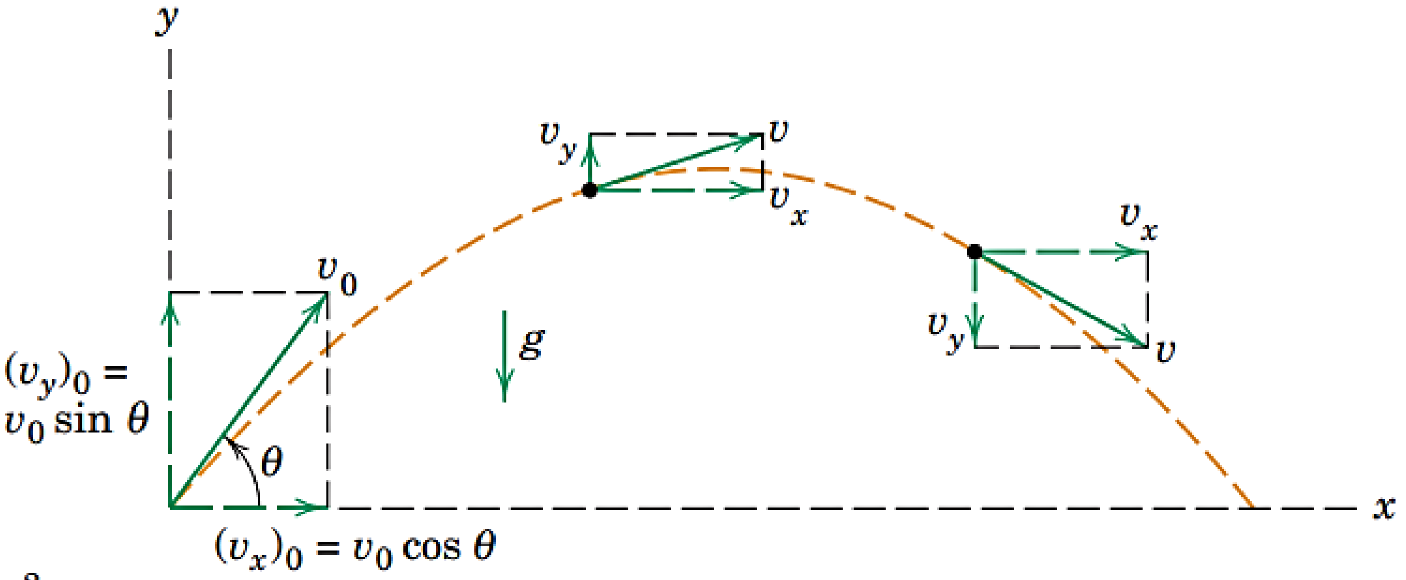
$$a^2 = {a_x}^2 + {a_y}^2 \qquad a = \sqrt{{a_x}^2 + {a_y}^2}$$

Projectile Motion

An important application of two-dimensional kinematic theory is the problem of projectile motion. For a first treatment of the subject, we neglect aerodynamic drag and the curvature and rotation of the earth, and we assume that the altitude change is small enough so that the acceleration due to gravity can be considered constant. With these assumptions, rectangular coordinates are useful for the trajectory analysis.

For the axes shown in the Figure, the acceleration components are

$$a_x = 0 \qquad a_y = -g$$



Integration of these accelerations follows the results obtained previously for constant acceleration and yields

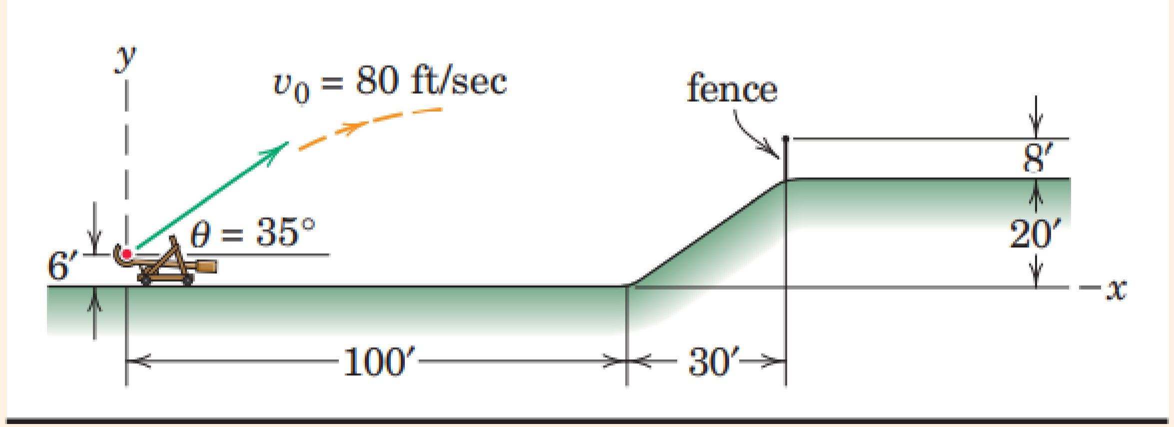
$$v_x = (v_x)_0 \qquad v_y = (v_y)_0 - gt$$

$$x = x_0 + (v_x)_0 t \qquad y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

$$v_y^{\,2} = (v_y)_0^{\,2} - 2g(y - y_0)$$

Example:

A team of engineering students designs a medium-size catapult which launches 8-lb steel spheres. The launch speed is $v_0 = 80$ ft/sec, the launch angle is $\theta = 35°$ above the horizontal, and the launch position is 6 ft above ground level. The students use an athletic field with an adjoining slope topped by an 8-ft fence as shown. Determine:



(a) the time duration t_f of the flight

(b) the x-y coordinates of the point of first impact

(c) the maximum height h above the horizontal field attained by the ball

(d) the velocity (expressed as a vector) with which the projectile strikes the ground (or the fence)

Repeat part (b) for a launch speed of $v_0 = 75$ ft/sec. **(Homework)**

Solution. We make the assumptions of constant gravitational acceleration and no aerodynamic drag. With the latter assumption, the 8-lb weight of the projectile is irrelevant. Using the given x-y coordinate system, we begin by checking the y-displacement at the horizontal position of the fence.

$$[x = x_0 + (v_x)_0 t] \qquad 100 + 30 = 0 + (80 \cos 35°)t \qquad t = 1.984 \text{ sec}$$

$$[y = y_0 + (v_y)_0 t - \tfrac{1}{2}gt^2] \quad y = 6 + 80 \sin 35°(1.984) - \tfrac{1}{2}(32.2)(1.984)^2 = 33.7 \text{ ft}$$

(a) Because the y-coordinate of the top of the fence is $20 + 8 = 28$ feet, the projectile clears the fence. We now find the flight time by setting $y = 20$ ft:

$$[y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2] \quad 20 = 6 + 80 \sin 35°(t_f) - \frac{1}{2}(32.2)t_f^2 \quad t_f = 2.50 \text{ sec} \quad \textit{Ans.}$$

$$[x = x_0 + (v_x)_0 t] \quad x = 0 + 80 \cos 35°(2.50) = 164.0 \text{ ft}$$

(b) Thus the point of first impact is $(x, y) = (164.0, 20)$ ft. *Ans.*

(c) For the maximum height:

$[v_y{}^2 = (v_y)_0{}^2 - 2g(y - y_0)]$ $0^2 = (80 \sin 35°)^2 - 2(32.2)(h - 6)$ $h = 38.7$ ft *Ans.*

(d) For the impact velocity:

$[v_x = (v_x)_0]$ $v_x = 80 \cos 35° = 65.5$ ft/sec

$[v_y = (v_y)_0 - gt]$ $v_y = 80 \sin 35° - 32.2(2.50) = -34.7$ ft/sec

So the impact velocity is $\mathbf{v} = 65.5\mathbf{i} - 34.7\mathbf{j}$ ft/sec. *Ans.*

If $v_0 = 75$ ft/sec, the time from launch to the fence is found by

$[x = x_0 + (v_x)_0 t]$ $100 + 30 = (75 \cos 35°)t$ $t = 2.12$ sec

and the corresponding value of y is

$[y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2]$ $y = 6 + 80 \sin 35°(2.12) - \frac{1}{2}(32.2)(2.12)^2 = 24.9$ ft

For this launch speed, we see that the projectile hits the fence, and the point of impact is

$$(x, y) = (130, 24.9) \text{ ft}$$ *Ans.*

For lower launch speeds, the projectile could land on the slope or even on the level portion of the athletic field.