Lecture Four

4.1 Natural Gas Production

In traveling from its original location in the reservoir to the final point of consumption, the gas must first travel through the reservoir rock or porous medium. A certain amount of energy is required to overcome the resistance to flow through the rock, which is manifested in a pressure decrease in the direction of flow, toward the well. This pressure drop or decrease depends on the gas flow rate, properties of the reservoir fluids, and properties of the rock. The fluid properties were discussed in Lecture 3, and a brief discussion of the rock properties is given in this lecture.

The engineer involved in gas production operations must be able to predict not only the rate at which a well or field will produce, but also how much gas is originally in the reservoir and how much of it can be recovered economically. This requires the ability to relate volumes of gas existing in the reservoir to reservoir pressure. Because the flow capacity of a well depends on the reservoir pressure, both reservoir gas flow and reserve estimates are discussed in this lecture.

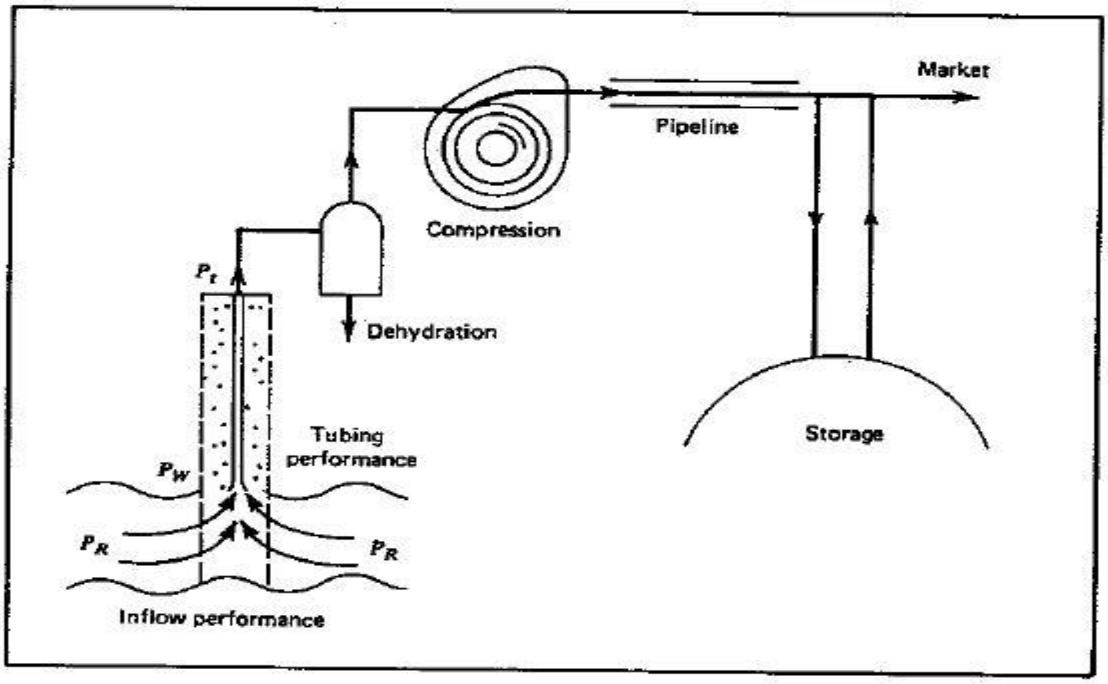


Figure 4.1: Gas Production schematic

4.1.1 Flow of Natural Gas in Porous Media

Determination of the inflow performance or reservoir flow capacity for a gas well requires a relationship between flow rate coming into a well and the sand-face pressure or flowing bottom-hole pressure. This relationship may be established by the proper solution of Darcy's Law, which is the accepted expression relating pressure drop and fluid velocity in a porous medium, provided that the flow is laminar. Solution of Darcy's Law depends on the conditions of flow existing in the reservoir or the flow regime. The flow type or regime may be independent of time or *steady-state*, or if conditions at a particular location change with time, the flow regime is transient or *unsteady-state*. Under certain conditions of transient flow, conditions change at a constant rate at all locations in the reservoir. This condition is called *pseudo-steady-state* and may be analyzed more simply than the transient condition.

The flow regimes will be discussed qualitatively first, the equations for each regime will be presented, and then the application of the equations for determining inflow performance or well flow capacity will be presented.

4.1.1.1 Flow Regime Characteristics

When a well is opened to production from a shut-in condition, the pressure disturbance created at the well travels outward through the rock at a velocity governed by the rock and fluid properties. The various flow regimes are discussed with respect to the behavior of this pressure disturbance.

- ***Steady-State Flow***

Figure 4-2 illustrates the pressure and flow rate distribution occurring during radial, steady-state flow into a well. This pressure distribution will remain constant as long as the radius being drained by the well remains constant.

For such a situation to be strictly true it is necessary that the flow across the external drainage radius r_e be equal to the flow across the well radius at r_w. This is never strictly met in a reservoir other than a strong water drive, whereby the water influx rate equals the producing rate. Pressure maintenance by water injection down-dip or by gas injection up dip would also approximate steady-state conditions as would most pattern water-floods after the initial stages of injection have passed.

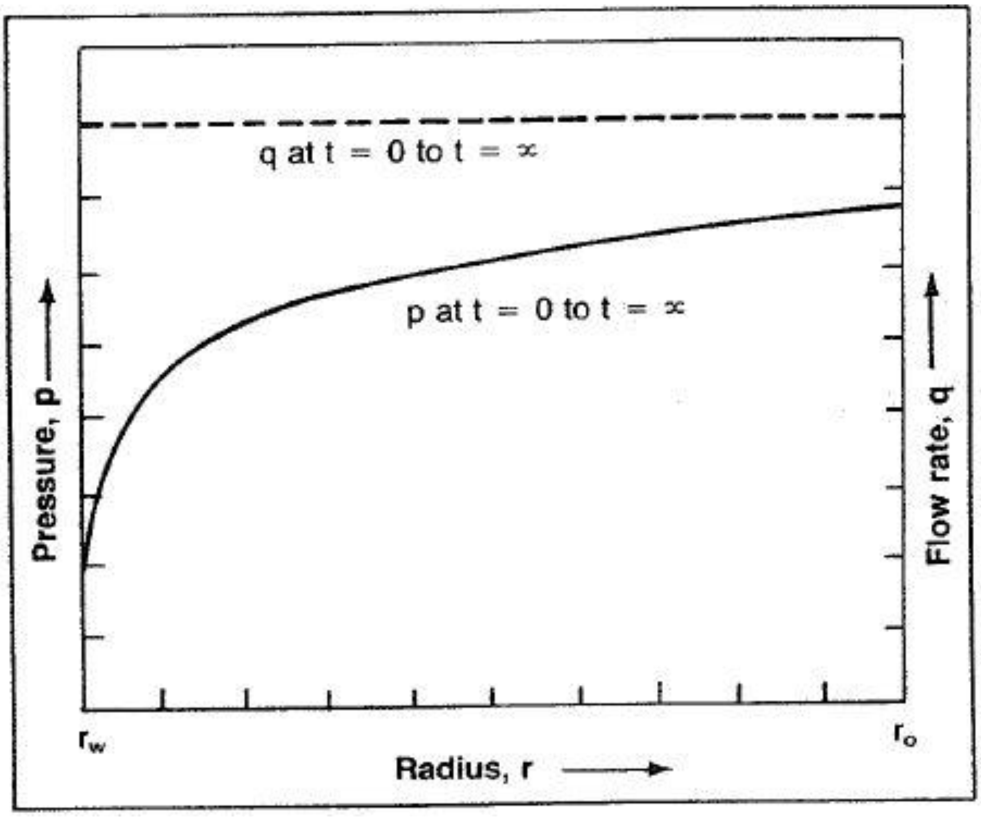


Figure 4-2: Radial steady-state flow.

Steady-state equations are also useful in analyzing the conditions near the wellbore because even in an unsteady-state system the flow rate near the wellbore is al most constant so that the conditions around the wellbore are almost constant. Thus, steady-state flow equations can be applied to this portion of the reservoir without any significant error.

- ***Unsteady-State Flow***

Figure 4.3 shows the pressure and rate distributions for a radial system at various times for a closed reservoir (no flow across r_e). In this case all of the production is due to the expansion of the fluid in the reservoir. This causes the rate at r_e to be zero and the rate increases to a maximum at the well radius r_w. In the steady-state case the flow across the outer boundary, r_e was equal to the flow across r_w the well radius. With flow across r_e zero, the only energy causing the flow of fluid is the expansion of the fluids themselves. Initially the pressure is uniform throughout the reservoir at P_i. This represents the zero producing time.

The production rate is controlled so that the pressure at the well is constant. A pressure distribution shown as p at t_1 is obtained after a short period of time of producing the well at such a rate that the well pressure remains constant. At this time only a small portion of the reservoir has been affected or has had a significant pressure drop.

For a closed reservoir, flow occurs due to expansion of the fluid. Consequently, if no pressure drop exists in the reservoir at a particular point, or outside of that point, no flow could be taking place at that particular radius, the fluid could not expand without a drop in pressure.

Thus, as shown in the plot of q at t_1, the rate at r_e, is zero and increases with a reduction in radius until the maximum rate in the reservoir is obtained at r_w. The pressure and rate distributions at time t_1 represent an instant in time, and the pressure and rate distributions move on through these positions immediately as the production continues to affect more and more of the reservoir. That is, more and more of the reservoir continues to experience a significant pressure drop and is subjected to flow until the entire reservoir is affected as shown by the pressure at t_2. The rate, q, at t_1 indicates that the flow rate at this time extends throughout the reservoir since all of the reservoir has been affected and has had a significant pressure drop.

Notice that the rate at the well has declined somewhat from time t_1 to t_2 since the same pressure drop (P_i - P_w), is effective over a much larger volume of the reservoir. Once the pressure in the entire reservoir has been affected the pressure will drop throughout the reservoir as production continues so that the pressure distribution might be as shown for p at t_3. The rate will have declined somewhat during time t_1, to t_2 due to the increase in the radius over which flow is taking place, and it will continue to decline from t_2 to t_3 due to the decline of the total pressure drop from r_e to r_w, (P_e - P_w).

Note that from time t = 0 to time t_2, when a pressure drop is finally affected throughout the entire reservoir, the pressure and rate distributions would not be affected by the size of the reservoir or the position of the external drainage radius r_e. During this time the reservoir is said to be infinite-acting because during this period the outer drainage radius, r_e, could mathematically be infinite. Even in reservoir systems that are dominated by steady-state flow, the effect of changes in well rates or well pressures at the well will be governed by unsteady-state flow equations until the changes have been in effect for a sufficient length of time to affect the entire reservoir and have the reservoir again reach a steady-state condition.

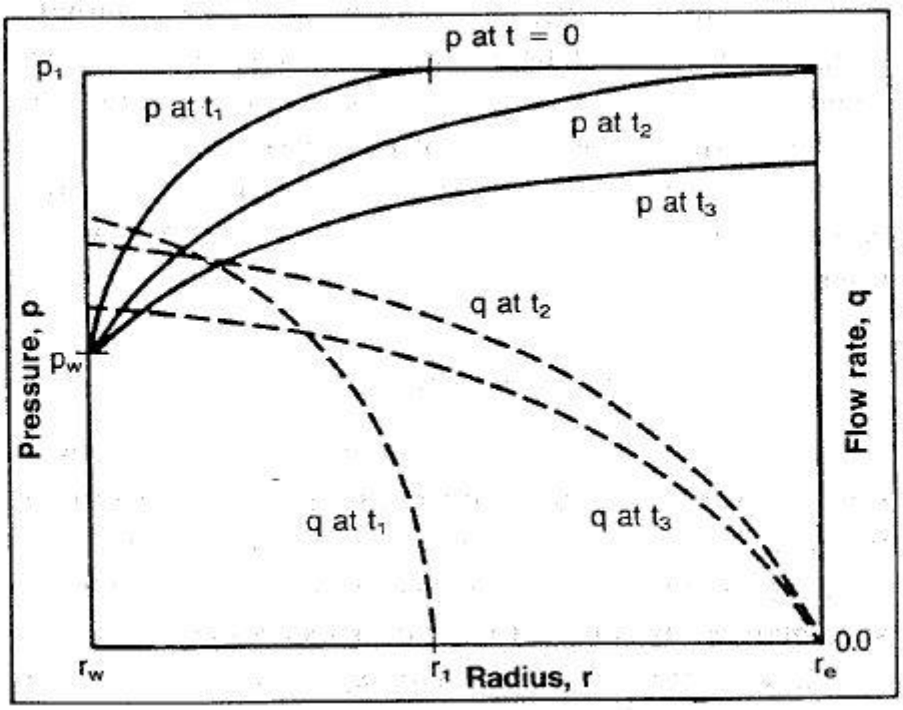


Fig. 4.3: Unsteady-state radial flow with constant well pressure.

- ***Pseudo-Steady-State Flow***

Figure 4.4 illustrates the pressure and rate distribution for the same unsteady-state system except that in this case the rate at the well, q_w, is held constant. This might be comparable to a prorated well or one that is pumping at a constant rate. Again at time = 0 the pressure throughout the reservoir is uniform at P_i. Then after some short producing time t_1 at a constant rate, only a small portion of the reservoir will have experienced a significant pressure drop, and consequently the reservoir will be flowing only out to a radius r_1. As production continues at the constant rate, the entire reservoir will eventually experience a significant pressure drop as shown at t_2. Shortly after the entire reservoir pressure has been affected, the change in the pressure with time at all radius in the reservoir becomes uniform so that the pressure distributions at subsequent times are parallel as illustrated by the pressure distributions at times t_3,t_4 , and t_5. This situation will continue with constant changes in pressure with time at all radiuses and with subsequent parallel pressure distributions until the reservoir is no longer able to sustain a constant flow rate at the wellbore. This will occur when the pressure at the well, p_w, has reached its physical lower limit.

Pseudo-steady-state flow occurs in the reservoir after it has been produced at a constant rate for a long enough period of time to cause a constant change in pressure at all radius, resulting in parallel pressure distributions and corresponding constant rate distributions. Pseudo-steady- state flow is a specialized case of unsteady-state flow, and is sometimes referred to as stabilized flow. Most of the life of a reservoir will exist in pseudo-steady-state flow.

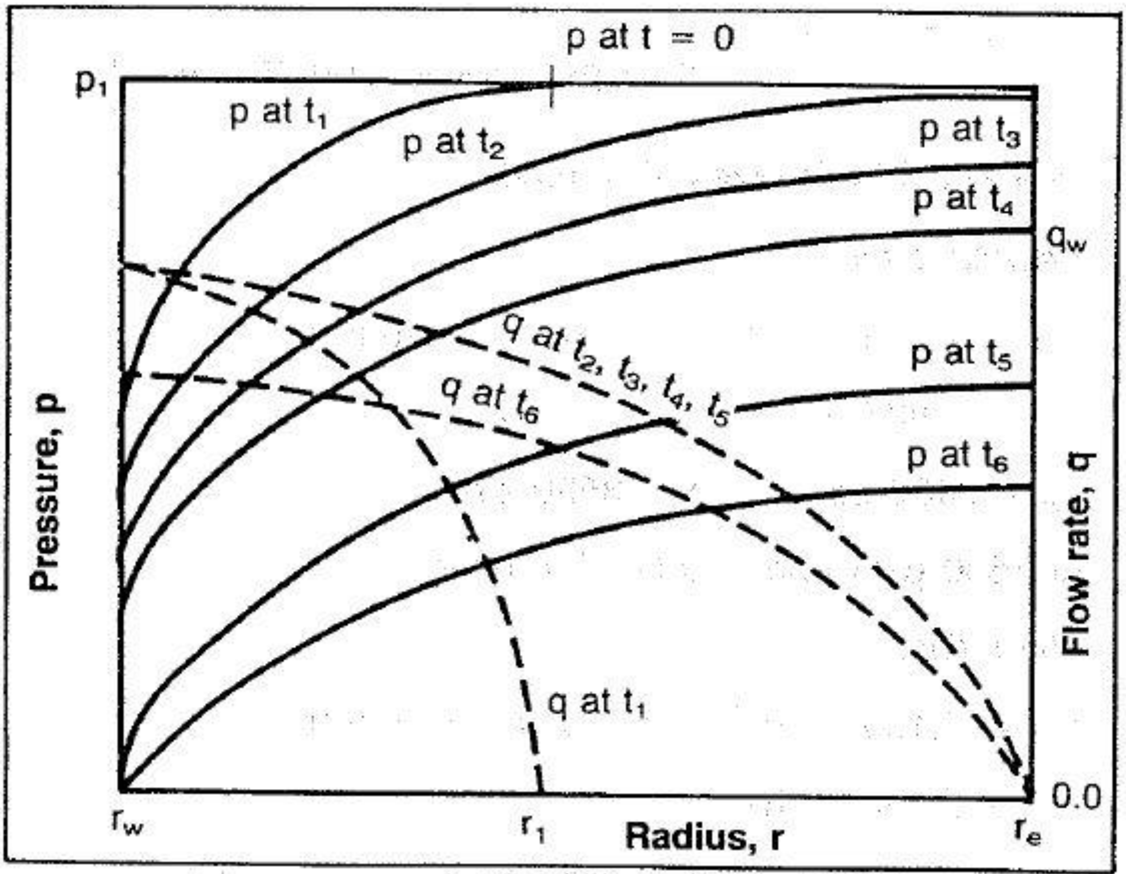


Figure 4-4: Unsteady-state radial flow with constant producing rate pseudo-steady-state t_2 to t_5.

4.1.1.2 Flow Equations

From the previous description of the various flow regimes it is obvious that a particular well will be acting in each of these regimes at some time in the life of the well. The applicable equations for each flow regime will be derived or represent this section.

Steady-State Flow

Darcy's Law for flow in a porous medium is

$$v = -\frac{k}{\mu}\frac{dp}{dx} \quad \text{or,}$$

$$q = vA = -\frac{kA}{\mu}\frac{dp}{dx} \qquad \text{.................. (4.1)}$$

Where:

v = fluid velocity,

q = volumetric flow rate,

k = effective permeability,

μ = fluid viscosity, and

dp/dx = pressure gradient in the direction of flow.

For radial flow in which the distance is defined as positive moving away from the well, the equation becomes

$$q = \frac{k(2\pi rh)}{\mu}\frac{dp}{dr} \qquad \text{..................... (4.2)}$$

Where:

r = radial distance, and

h = reservoir thickness.

Darcy's Law describes the pressure loss due to viscous shear occurring in the flowing fluid. If the formation is not horizontal, the hydrostatic or potential energy term must be included. This is usually negligible for gas flow in reservoirs. Equation 4-2 is a differential equation and must be integrated for application. Before integration the flow equation must be combined with an equation of state and the continuity equation. The continuity equation is

$$\rho_1 q_1 = \rho_2 q_2 = \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.3)$$

From lecture 3, the equation of state for a real gas is

$$\rho = \frac{pM}{ZRT}. \quad \ldots\ldots\ldots\ldots\ldots (4.4)$$

The flow rate for a gas is usually desired at some standard conditions of pressure and temperature, P_{sc} and T_{sc}. Using these conditions in Equation 4-3 and combining Equations 4-3 and 4-4:

$$\rho q = \rho_{sc}\, q_{sc},$$

or

$$q\,\frac{pM}{ZRT} = q_{sc}\,\frac{p_{sc}\,M}{Z_{sc}\,R\,T_{sc}}.$$

Solving for q_{sc} and expressing q with Equation 4-2 gives

$$q_{sc} = \frac{p\,T_{sc}}{p_{sc}\,Z\,T}\,\frac{2\pi rhk}{\mu}\,\frac{dp}{dr}$$

The variables in this equation are p and r. Separating the variables and integrating:

$$\int_{p_w}^{p_e} p\,dp = \frac{q_{sc}\,p_{sc}\,T\,\bar{\mu}\,\bar{Z}}{T_{sc}\,2\pi kh}\int_{r_w}^{r_e}\frac{dr}{r}$$

$$\frac{p_e^2 - p_w^2}{2} = \frac{q_{sc}\,p_{sc}\,T\,\bar{\mu}\,\bar{Z}}{T_{sc}\,2\pi kh}\ln\,(r_e/r_w)$$

or

$$q_{sc} = \frac{\pi kh\,T_{sc}\,(p_e^2 - p_w^2)}{p_{sc}\,T\,\bar{\mu}\,\bar{Z}\ln\,(r_e/r_w)}. \quad \ldots\ldots\ldots (4.5)$$

In this derivation it was assumed that µ and Z were independent of pressure. They may be evaluated at reservoir temperature and average pressure in the drainage area

$$\bar{p} = \frac{p_e + p_w}{2}.$$

Equation 4-5 is applicable for any consistent set of units. In so-called conventional oil field units the equation becomes

$$q_{sc} = \frac{703 \times 10^{-6} kh\,(p_e^2 - p_w^2)}{\bar{T}\,\bar{\mu}\,\bar{Z}\ln(r_e/r_w)} \quad \text{……………….. (4.6)}$$

Where:

q_{sc} = Mscf/day,

k = permeability in millidarcies,

h = formation thickness in feet,

P_e = pressure at r_e, psia,

P_w = wellbore pressure at r_w, psia, and

μ = gas viscosity, cp.

This equation incorporates the following values for standard pressure and temperature:

$$P_{sc} = 14.7 \text{ psia}$$

$$T_{sc} = 60°F = 520°R.$$

These units will be used in all equations in the text un- less otherwise stated.

Example 4-1:

Given the following data, determine the wellbore pres- sure required for an inflow rate of 3900 Mscfd assuming steady-state flow.

$k = 1.5$ md	$h = 30$ ft
$p_e = 4625$ psia	$\bar{\mu} = 0.027$ cp
$T = 712°R = 252°F$	$\gamma_g = 0.76$
$r_e = 550$ ft	$r_w = 0.333$ ft

Solution:

The solution is iterative since

$$\bar{Z} = f(\bar{p}),\ \text{where } \bar{p} = \frac{p_e + p_w}{2}, \text{ and}$$

P_w is the unknown. As a first estimate, assume $\bar{Z} = 1$

First Trial

$$p_w^2 = p_e^2 - \frac{\bar{\mu}\, T \ln(r_e/r_w)\, q_{sc}\, \bar{Z}}{703 \times 10^{-6}\, kh}$$

$$p_w^2 = 2.139 \times 10^7 - \frac{(0.027)(712)(7.41)(3900)\, \bar{Z}}{703 \times 10^{-6}\,(1.5)(30)}$$

$$p_w^2 = 2.139 \times 10^7 - 1.756 \times 10^7\, \bar{Z}$$

$$= 2.139 \times 10^7 - 1.756 \times 10^7\,(1)$$

$$p_w^2 = 3.83 \times 10^6 \qquad p_w = 1957 \text{ psia}$$

$\bar{p} = \dfrac{4625 + 1957}{2} = 3291$ psia. Evaluation of $\bar{Z}$ at 3291 psia and 712°R gives $\bar{Z} = 0.88$

Second Trial

$$p_w^2 = 2.139 \times 10^7 - 1.756 \times 10^7\,(0.88)$$

$$= 5.937 \times 10^6$$

$$p_w = 2436 \text{ psia} \qquad \bar{p} = 3530 \text{ psia}$$

At 3530 psia and 712°R, $\bar{Z} = 0.89$

Third Trial

$$p_w^2 = 2.139 \times 10^7 - 1.756 \times 10^7\,(0.89)$$

$$= 5.762 \times 10^6$$

$$p_w = 2400 \text{ psia} \qquad \bar{p} = 3512 \text{ psia} \qquad \bar{Z} = 0.89$$

Since the value for Z is the same as for Trial 2, the solution has converged and the required well pressure is 2400 psia. The solution would have been more complicated if a constant value for μ had not been assumed.

The above treatment of steady-state flow assumes no turbulent flow in the formation and no formation or skin damage around the wellbore. The effects of turbulence and skin will be examined in a following section.

Although steady-state flow in a gas reservoir is seldom reached, the conditions around the wellbore can approach steady-state. The steady-state equation including turbulence is

$$p_e^2 - p_w^2 = \frac{1422\, T\, \bar{\mu}\, \bar{Z}\, q_{sc} \ln (r_e/r_w)}{kh} \ldots$$

$$\ldots + \frac{3.161 \times 10^{-12}\, \beta\, \gamma_g\, \bar{Z}\, q_{sc}^2\, T}{h^2} \left(\frac{1}{r_w} - \frac{1}{r_e}\right). \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4.7)$$

The first term on the right hand side is the pressure drop from laminar or Darcy flow, while the second term gives the additional pressure drop due to turbulence. If the fluid properties are known and the permeability is known from some source such as a drawdown test, the turbulent effects can be calculated using the results of a test. This will be used later to distinguish between actual formation damage and turbulence. Values of the velocity coefficient B for various perm abilities and porosities can be obtained from Figure 4.5 or calculated from Equation 4-8.

$$\beta = \frac{2.33 \times 10^{10}}{k^{1.2}} \qquad \ldots\ldots\ldots\ldots (4.8)$$

Where: k is in millidarcies.

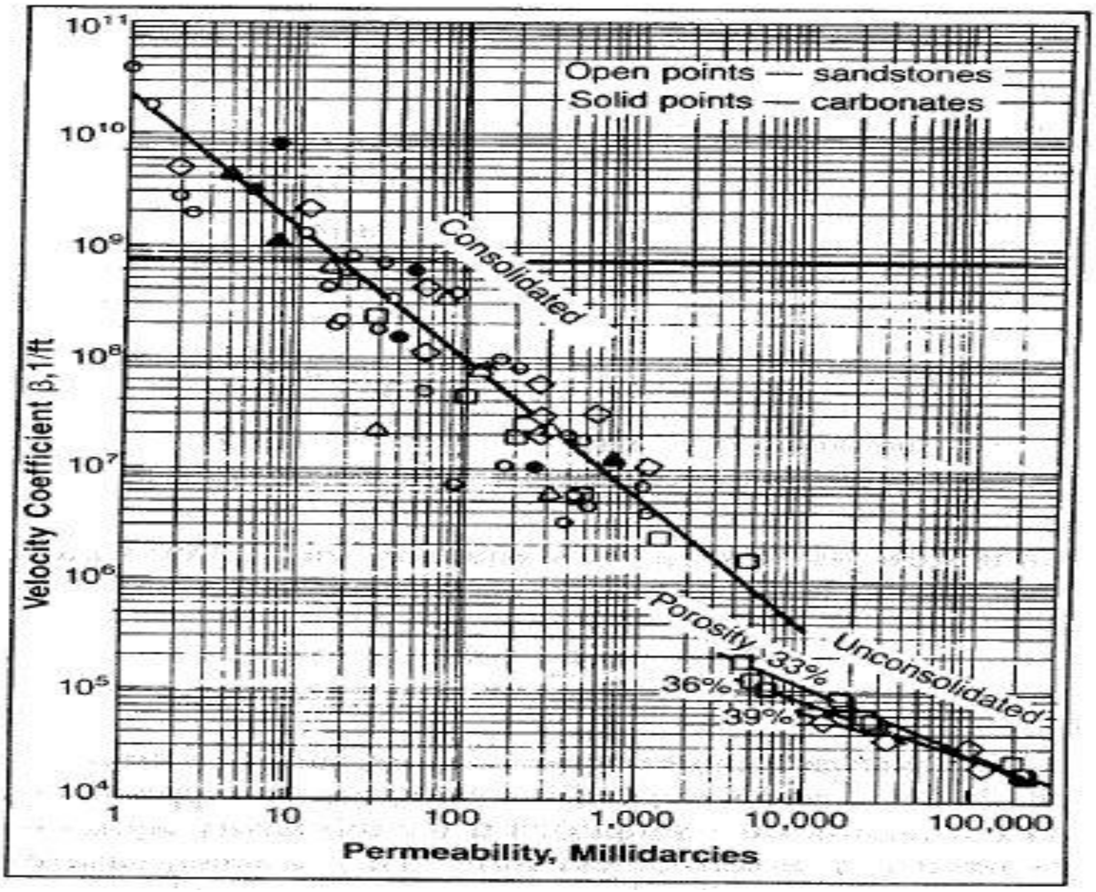


Figure 4-5: Gas velocity coefficient.

Pseudo-Steady-State Flow

An equation for pseudo-steady-state flow can be de- rived that will show that

$$q_{sc} = \frac{703 \times 10^{-6}\, kh(\bar{p}_R^{\,2} - p_w^{\,2})}{T\,\bar{\mu}\,\bar{Z}\ln(0.472 r_e/r_w)}. \quad \ldots\ldots\ldots\ldots\ldots\ (4.9)$$

Although time does not appear explicitly in Equation 4-9, it should be remembered that both $\overline{p_r}$ and p_w. will be declining at the same rate for a constant q once the pressure disturbance has reached the reservoir boundary.

The effects of skin damage and turbulence are sometimes included in Equation 4-9 as follows:

$$q_{sc} = \frac{703 \times 10^{-6}\, kh(\bar{p}_R^{\,2} - p_w^{\,2})}{T\,\bar{\mu}\,\bar{Z}[\ln(.472 r_e/r_w) + S + D\,q_{sc}]} \quad \ldots\ldots\ldots\ldots\ (4.10)$$

Where

S= dimensionless skin factor, and

D = turbulence coefficient,

It is frequently necessary to solve Equation 4-10 for pressure or pressure drop for a known q_{sc}.

$$\bar{p}_R^{\,2} - p_w^{\,2} = \frac{1422\,T\,\bar{\mu}\,\bar{Z}\,q_{sc}}{kh} \cdot [\ln(.472\, r_e/r_w) + S + D\,q_{sc}] \quad \ldots\ldots\ldots\ldots..\ (4.11)$$

Unsteady-State Flow

It was stated earlier that any well flows in the un- steady-state or transient regime until the pressure disturbance reaches a reservoir boundary or until interference from other wells takes effect. Although the flow capacity of a well is desired for pseudo-steady-state or stabilized conditions, much useful information can be obtained from transient tests. This information includes permeability, skin factor, turbulence coefficient, and average reservoir pressure.

The procedures are developed in the section on transient testing. The relationships among flow rate, pressure, and time will be presented in this section for various conditions of well performance and reservoir types. It will be seen that the steady-state and pseudo-steady-state equations can be obtained from solution of the diffusivity equation as special cases.

The diffusivity equation can be derived by combining an unsteady-state continuity equation with Darcy's Law and the gas equation of state. The equation is

$$\frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r}\frac{\partial p^2}{\partial r} = \frac{\phi \mu C}{k}\frac{\partial p^2}{\partial t} \quad \text{…………….} (4.12)$$

This equation can be solved for pressure as a function of flow rate and time, but the solutions and application of the solutions are simplified if the diffusivity equation is written in dimensionless form. This is accomplished by defining the following dimensionless variables:

Table 4.1: Dimensionless variables

Dimensionless Variable	Symbol	Definition
Time	t_D	$\frac{2.64 \times 10^{-4} kt}{\phi \bar{\mu} \bar{C} r^2}$ (4-13)
Radius	r_D	$\frac{r}{r_w}$ (4-14)
Flow rate	q_D	$\frac{1422\, T q_{sc} \bar{Z} \bar{\mu}}{kh\, p_i^2}$ (4-15)
Pressure	p_D	$\frac{p^2}{p_i^2 q_D}$ (4-16)
Pressure drop	Δp_D	$\frac{p_i^2 - p^2}{p_i^2 q_D}$ (4-17)

The following units are to be used in calculating values for the dimensionless numbers in Table 4-1:

k = millidarcies $\quad C = psi^{-1}$
t = hours $\quad r$ = ft
μ = cp $\quad T$ = °R
q_{sc} = Mscfd $\quad h$ = ft
p = psia

The diffusivity equation in dimensionless variables becomes

$$\frac{\partial^2 \Delta p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial \Delta p_D}{\partial r_D} = \frac{\partial \Delta p_D}{\partial t_D}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (4\text{-}18)$$

Solutions to Equation 4-18 depend on the reservoir type and boundary conditions. The following solutions will be presented:

1. Constant rate at well, infinite-acting reservoir (transient)

2. Constant rate at well, finite-acting (closed) reservoir (pseudo-steady-state)

3. Constant rate at well, constant pressure at outer boundary (steady-state)

4. Constant well pressure.

Case 1:

The most useful solution for transient flow is the so- called line source solution. The solution is

$$\Delta p_D = -0.5\, E_i\left(-\frac{1}{4t_D}\right). \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.19)$$

Values for the E_i or exponential integral term as a function of t, can be found in various mathematics hand- books, but for all practical purposes the function may be represented by a logarithmic approximation. That is,

$$\Delta p_D = 0.5\,(\ln t_D + 0.809) \quad \ldots\ldots\ldots\ldots (4.20)$$

Once a value of the dimensionless pressure drop App is obtained, the actual pressures may be calculated by using the definition of Δp_D, from Table 4-1.

Example 4-2:

Using the following data and assuming the well is still in the transient regime, calculate the pressure at the well after a flowing time of 1.5 days.

$h = 36$ ft $\quad T = 580°R$
$\phi = 0.15 \quad q_{sc} = 7000$ Mscfd
$k = 20$ md $\quad \bar{Z} = 0.85$
$p_i = 2000$ psia $\quad \bar{\mu} = 0.0152$ cp
$r_w = 0.4$ ft $\quad \bar{C} = 0.00061$ psi^{-1}
$r_e = 2000$ ft

Solution:

Calculate t_D for $r = r_w$:

$$t_{Dw} = \frac{2.64 \times 10^{-4}\, kt}{\phi\, \bar{\mu}\, \bar{C}\, r_w^2}$$

$$= \frac{2.64 \times 10^{-4}\,(20)(36)}{0.15\,(0.0152)(0.00061)(0.4)^2}$$

$$t_{Dw} = 8.54 \times 10^5$$

$$\Delta p_D = 0.5\,(\ln(8.54 \times 10^5) + 0.809) = 7.23$$

$$q_D = \frac{1422\, T\, q_{sc}\, \bar{Z}\, \bar{\mu}}{kh\, p_i^2}$$

$$= \frac{1422(580)(7000)(0.85)(0.0152)}{20\,(36)(2000)^2}$$

$$q_D = 0.0259$$

$$p_w^2 = p_i^2 - p_i^2\, q_D\, \Delta p_D$$

$$= (2000)^2 - (2000)^2\,(0.0259)(7.23)$$

$$p_w^2 = 3250972 \qquad p_w = 1803 \text{ psia}$$

Equation 4-20 applies for values of dimensionless time based on the well's drainage radius, t_{De}, less than 0.25. That is, the well will still be infinite-acting if

$$t_{De} = \frac{2.64 \times 10^{-4}\, kt}{\phi\, \bar{\mu}\, \bar{C}\, r_e^2} < 0.25 \qquad \text{…………… (4.21)}$$

Another restriction on the validity of Equation 4-20 is that t_D should be greater than 100. If t_D is less than 100, the E_i, solution (Equation 4-19) must be used. For most practical cases to will be greater than 100.

Equation 4-20 may also be used to calculate pressure at a location other than the well. That is, r need not always be r_w. For the solution to be valid, the dimensionless time based on the radius of interest must be greater than 100. That is,

$$t_D = \frac{2.64 \times 10^{-4}\, kt}{\phi\, \bar{\mu}\, \bar{C}\, r^2} > 100. \qquad \ldots\ldots\ldots\ldots\ldots.. (4.22)$$

Case 2:

The solution for wells that have reached pseudo-steady- state was presented by Van Everdingen and Hurst in 1949. The solution can be applied to calculate the pres- sure at any radius where the flow rate is known, which effectively limits its application to calculating well pressures. The solution is presented both in graphical form and equation form. Values of Δp_D versus t_{Dw} are presented in Figure 4-6 for various reservoir sizes, that is for various values of r_{De}. The equation form of the solution is

$$\Delta p_{Dw} = \frac{2t_{Dw}}{r_{De}^{\ 2}} + \ln(0.472\, r_{De}) \qquad \ldots\ldots\ldots\ldots\ldots (4.23)$$

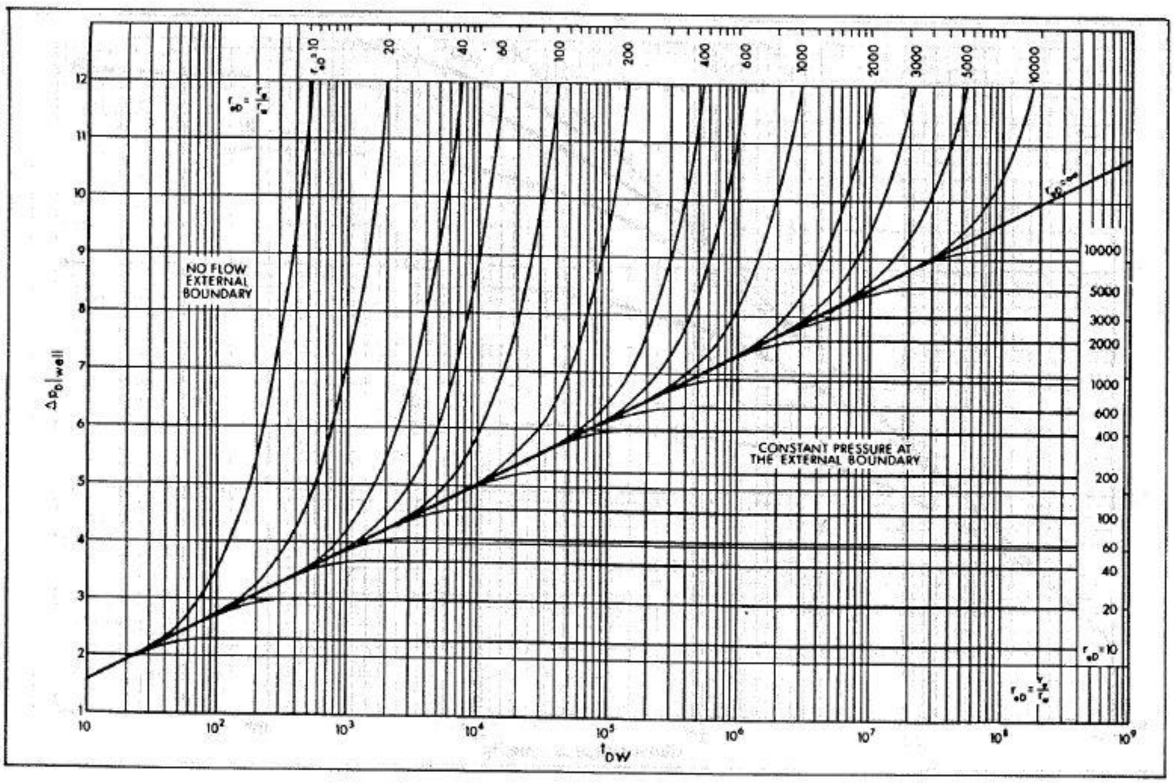


Figure 4-6: Values of Δp_D/well for infinite reservoirs, for finite circular reservoirs with no flow at the external boundary, and for finite circular reservoirs with constant pressure at the external boundary.

Example 4-3:

Using the data given in Example 4-2, find the pressure at the well after the well has been flowing for 1800 hours.

Solution:

Check for finite or pseudo-steady-state validity:

$$t_{De} = \frac{2.64 \times 10^{-4} kt}{\phi \bar{\mu} \bar{C} r_e^2}$$

$$= \frac{2.64 \times 10^{-4}(20)(1800)}{0.15(0.0152)(0.00061)(2000)^2} = 1.71$$

Since $t_{De} > 0.25$, the well is finite acting.

$$t_{Dw} = \frac{t_{De} r_e^2}{r_w^2} = 4.271 \times 10^7 \ (>100)$$

$$r_{De} = \frac{r_e}{r_w} = \frac{2000}{0.4} = 5000$$

$$\Delta p_{Dw} = \frac{2t_{Dw}}{r_{De}^2} + \ln(0.472\, r_{De})$$

$$= \frac{2(4.271 \times 10^7)}{(5000)^2} + \ln(0.472(5000))$$

$$= 3.417 + 7.766$$

$$\Delta p_{Dw} = 11.183$$

From Example 3-2, $q_D = 0.0259$

$$p_w^2 = p_i^2(1 - q_D \Delta p_{Dw})$$

$$= (2000)^2(1 - 0.0259(11.183))$$

$$p_w^2 = 2.841 \times 10^6 \qquad p_w = 1686 \text{ psia}$$

Case 3:

If a well is producing from a reservoir with constant pressure at the outer boundary, that is, in the steady-state condition, the solution to the diffusivity equation is

$$\Delta p_D = \ln r_{De}. \quad \ldots\ldots\ldots\ldots (4.24)$$

In this case the fixed pressure used in the definition of Δp_D is p_e. rather than p_i, where p_e, is the constant pres- sure at the outer boundary. Substituting the definitions of the dimensionless variables will result in Equation 4- 6 presented earlier for steady-state flow.

Case 4:

The constant pressure solution to the diffusivity equation can be expressed as a function of a dimensionless cumulative production, Q_{tD}. Q_{tD} is defined as

$$Q_{tD} = \frac{G_p \bar{Z} T}{0.111\, \phi\, h\, r_w^2\, C\, (p_i^2 - p_w^2)}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.25)$$

Where G_p, is the cumulative gas produced in Mscf. Values of Q_{tD} as a function of dimensionless time and radius have been presented both graphically and in table form. Figures 4-7 and 4-8 show values of Q_{tD} versus t_D.

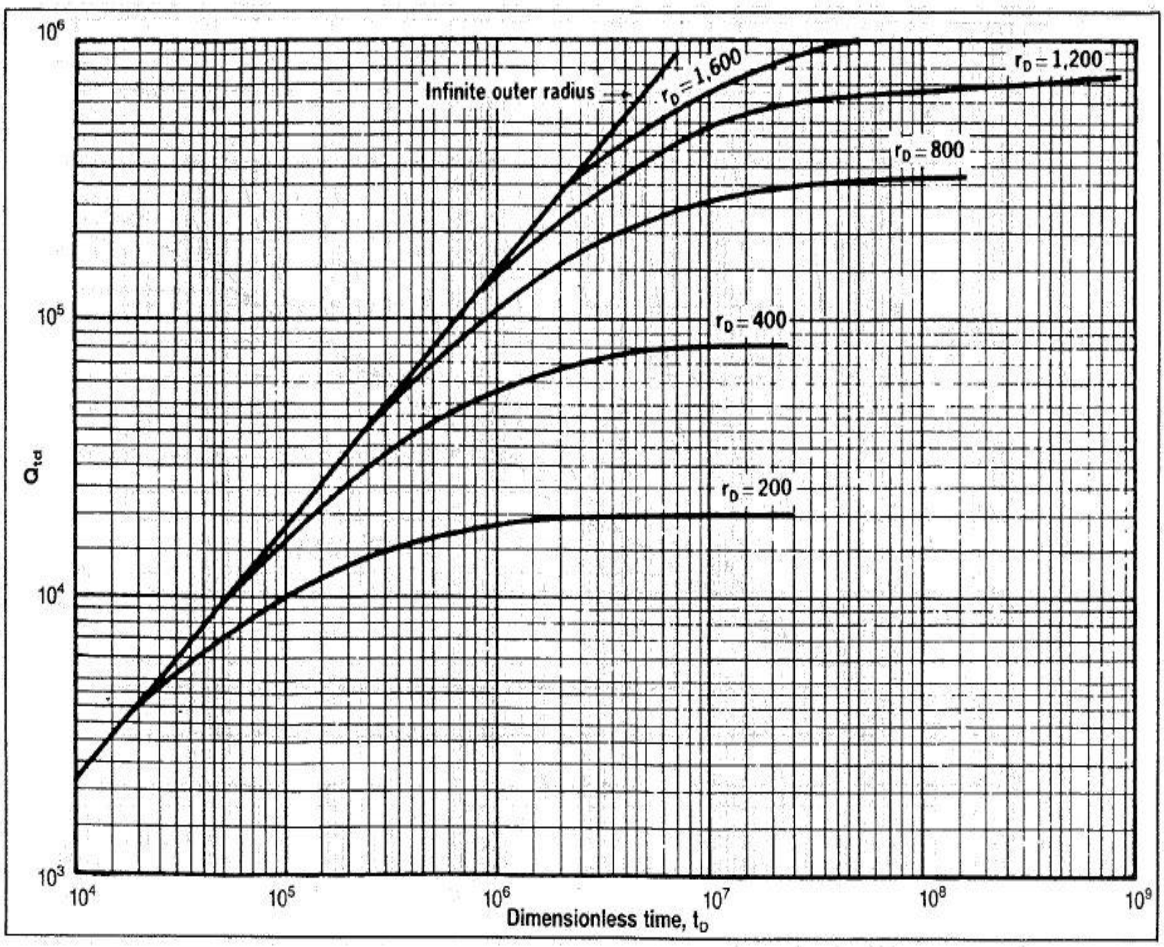


Figure 4-7: Constant pressure functions.

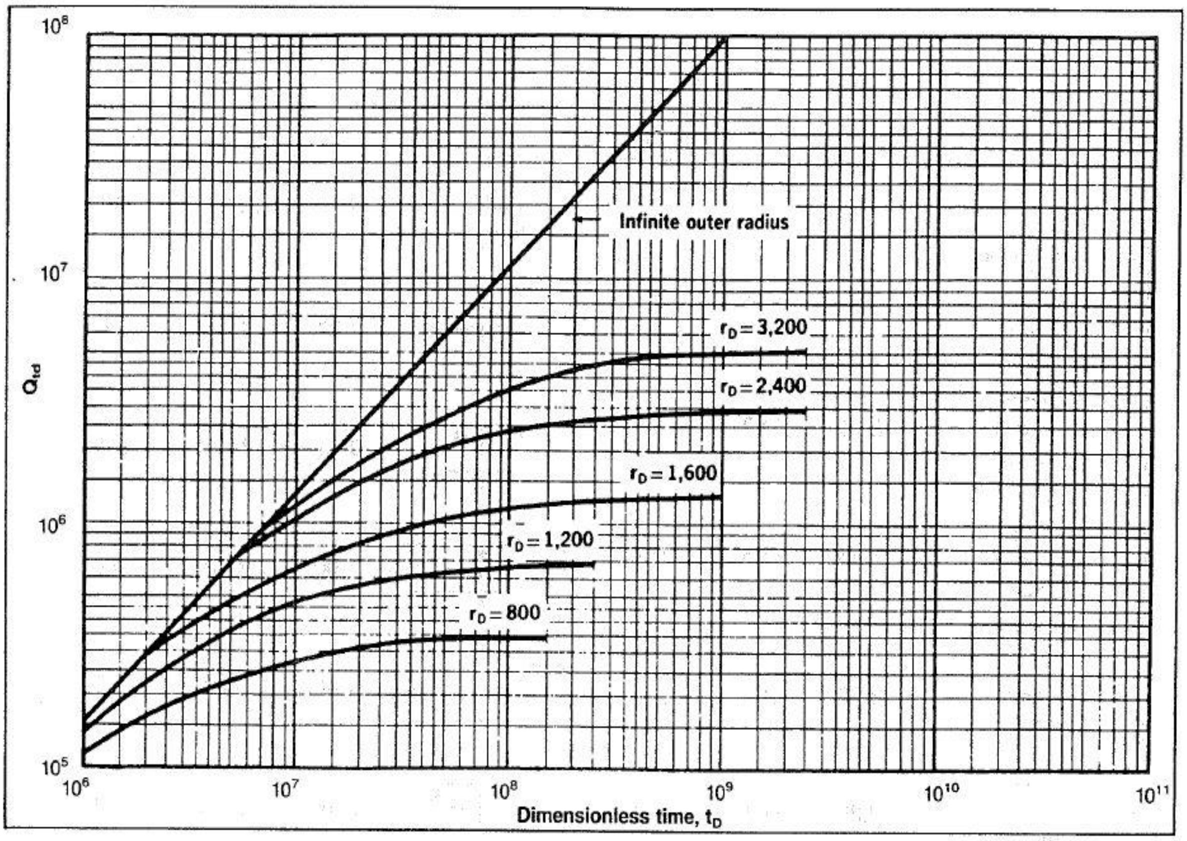


Figure 4-8: Constant pressure functions.

Noncircular Reservoirs

All of the previous equations were based on a single well draining a circular reservoir, which is rarely ever the actual case. The pressure behavior depends on the shape active to the boundaries. The time to reach stabilized or pseudo-steady-state flow also depends on these factors.

The previous equations have been modified by Dietz as follows. If the well is still infinite-acting,

$$\Delta p_D = 0.5\left[\ln\left(\frac{A\,t_{DA}}{r_w^{\,2}}\right) + .809\right], \qquad \ldots\ldots\ldots\ldots\ldots (4.26)$$

Where A is the drainage area, for pseudo-steady-state,

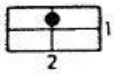
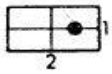
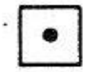
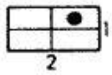
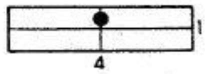
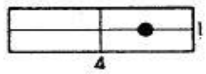
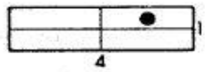
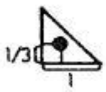
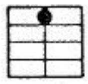
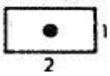
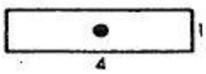
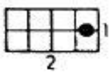
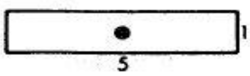
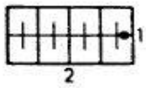
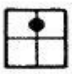
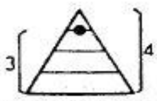
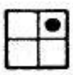
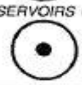
$$\Delta p_D = 0.5\left[\ln\left(\frac{A}{r_w^{\,2}\,C_A}\right)\right] + .809\Big] + 2\pi\,t_{DA}. \qquad \ldots\ldots\ldots (4.27)$$

Calculation of t_{DA} is based on the drainage area and is defined as

$$t_{DA} = \frac{2.64 \times 10^{-4}\, kt}{\phi\, \bar{\mu}\, \bar{C}\, A} \quad \ldots\ldots\ldots\ldots\ldots \text{(4-28)}$$

The shape factor C as well as the value of t_{DA} required to reach pseudo-steady state flow can be obtained from Table 4-2.

TABLE 4-2: PSEUDO-STEADY STATE SHAPE FACTORS FOR VARIOUS RESERVOIRS

	$\ln C_A$	C_A	STABILIZED CONDITIONS FOR $t_{DA}>$		$\ln C_A$	C_A	STABILIZED CONDITIONS FOR $t_{DA}>$
IN BOUNDED RESERVOIRS					2.38	10.8	0.3
	3.45	31.6	0.1		1.58	4.86	1.0
	3.43	30.9	0.1		0.73	2.07	0.8
	3.45	31.6	0.1		1.00	2.72	0.8
	3.32	27.6	0.2		−1.46	0.232	2.5
	3.30	27.1	0.2		−2.16	0.115	3.0
	3.09	21.9	0.4		1.22	3.39	0.6
	3.12	22.6	0.2		1.14	3.13	0.3
	1.68	5.38	0.7		−0.50	0.607	1.0
	0.86	2.36	0.7		−2.20	0.111	1.2
	2.56	12.9	0.6		−2.32	0.098	0.9
	1.52	4.57	0.5	IN WATER-DRIVE RESERVOIRS 	2.95	19.1	0.1
				IN RESERVOIRS OF UNKNOWN PRODUCTION CHARACTER 	3.22	25	0.1

Example 4-3a:

Rework Example 4-3 if the well is located in the center of a rectangular shaped (1 × 4) drainage area containing 220 acres. (9.58 * 10^6 ft^2).

Solution:

Check for pseudo-steady-state:

$$t_{DA} = \frac{2.64 \times 10^{-4} (20)(1800)}{0.15 (0.0152)(0.00061)(9.58 \times 10^6)} = 0.71$$

Since $t_{DA} > 0.7$, the well is in pseudo-steady-state flow.

From Table 4-2 $C_A = 5.38$ and

$$\Delta p_D = 0.5 \left(\ln \frac{9.58 \times 10^6}{(0.4)^2 (5.38)} \right) + .809 + 2\pi(0.71)$$

$$\Delta p_D = 8.517 + 4.461 = 12.978$$

$$p_w^2 = p_i^2 (1 - q_D \Delta p_D) = (2000)^2 (1 - 0.0259(12.978))$$

$$p_w^2 = 2.655 \times 10^6 \qquad p_w = 1630 \text{ psia}$$