Al-Ayen University
College of Petroleum Engineering

Numerical Methods and Reservoir Simulation

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L18: Application of Finite Difference Approximation for 2D Systems (Part 2)

Outline

- 2D Flow Problem/Block-Centered Grid
  - Example

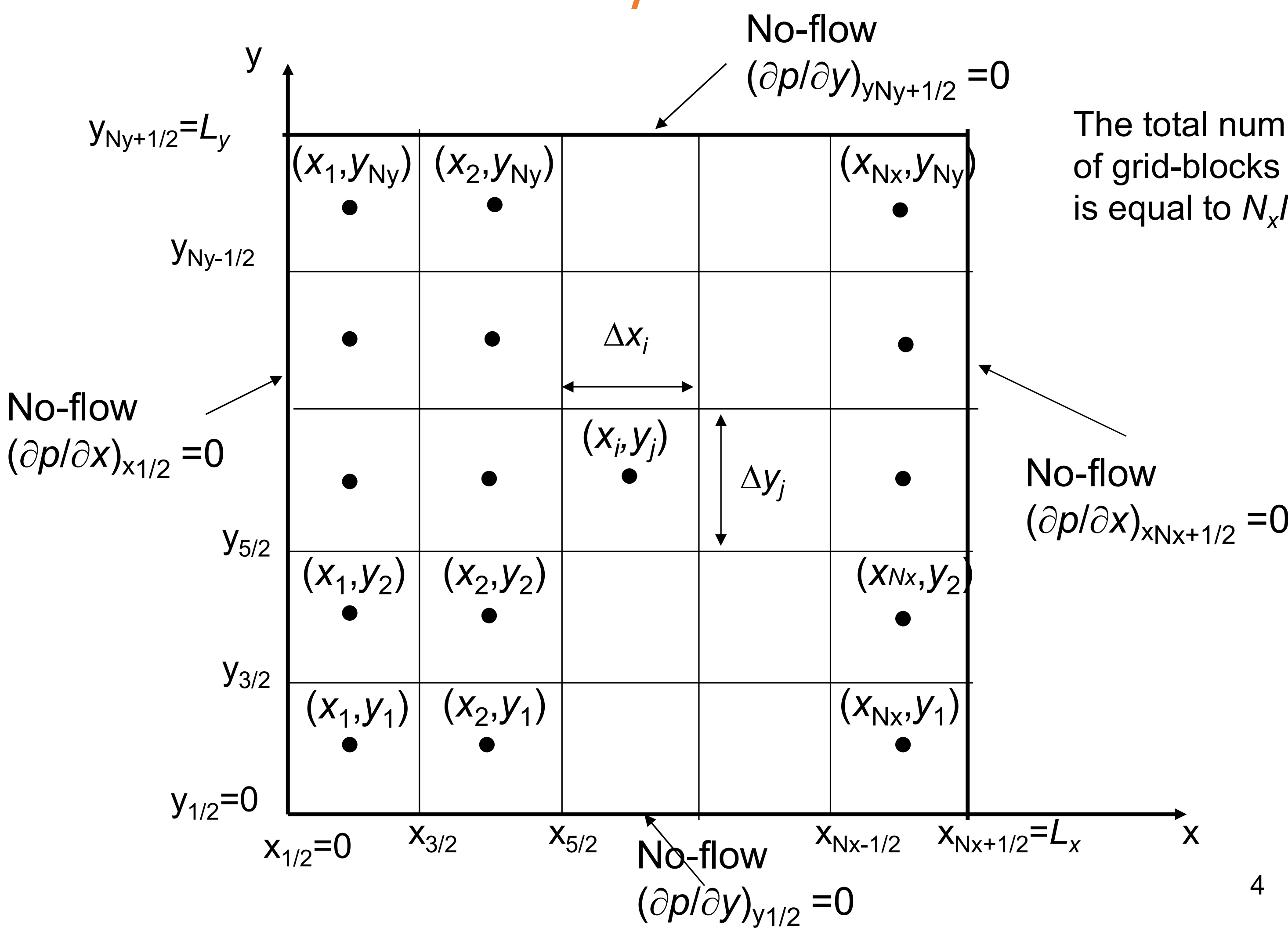
2D Flow Problem/Block-Centered Grid

- Let's consider a block-centered grid system with no-flow boundary conditions. Let N_x be the number of grid-blocks in the *x*-direction and N_y be the number of grid-blocks in the *y*-direction.
- Note that as we consider 2-D flow, we will have four (4) no-flow boundary conditions as given below:

$$\left(\frac{\partial p}{\partial x}\right)_{x=0,y} = 0 \quad \text{and} \quad \left(\frac{\partial p}{\partial x}\right)_{x=L_x,y} = 0$$

$$\left(\frac{\partial p}{\partial y}\right)_{x,y=0} = 0 \quad \text{and} \quad \left(\frac{\partial p}{\partial y}\right)_{x,y=L_y} = 0$$

2D Flow Problem/Block-Centered Grid



The total number of grid-blocks is equal to $N_x N_y$

2D Flow Problem/Block-Centered Grid

- Incorporation of no-flow boundary conditions for 2D problem is similar to that of no-flow conditions for 1-D flow.

- We simply set the transmissibility at the left-, right-, bottom- and top-boundaries:

$$-T_{x,i-1/2,j}\,p_{i-1,j}^{n+1} + \left(T_{x,i-1/2,j} + T_{x,i+1/2,j} + T_{y,i,j-1/2} + T_{y,i,j+1/2} + \widetilde{V}_{i,j}\right)p_{i,j}^{n+1} - T_{x,i+1/2,j}\,p_{i+1,j}^{n+1}$$
$$-T_{y,i,j-1/2}\,p_{i,j-1}^{n+1} - T_{y,i,j+1/2}\,p_{i,j+1}^{n+1} = \widetilde{V}_{i,j}\,p_{i,j}^{n} - q_{sc,i,j}^{n+1}B$$

$$T_{x,1/2,j} = 0 \quad for \quad j = 1,2,\ldots,N_y$$
$$T_{x,N_x+1/2,j} = 0 \quad for \quad j = 1,2,\ldots,N_y$$
$$T_{y,i,1/2} = 0 \quad for \quad i = 1,2,\ldots,N_x$$
$$T_{y,i,N_y+1/2} = 0 \quad for \quad i = 1,2,\ldots,N_x$$

2D Flow Problem/Block-Centered Grid

- The ordering of the unknown pressures determines the structure of the coefficient matrix arising from implicit formulation.

- There are various ordering schemes. The idea is to find the ordering that will minimize the band width. Here, we will not consider these ordering schemes and we only consider what so-called the *natural ordering*:

  - order first in the x- (i-) direction (left to right)

  - then in the y- (j-) direction (bottom to top)

Block ordering schemes used in reservoir simulation

17	18	19	20
13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

Natural ordering

14	17	19	20
10	13	16	18
6	9	12	15
3	5	8	11
1	2	4	7

Diagonal (D2) ordering

8	19	10	20
16	7	18	9
4	15	6	17
12	3	14	5
1	11	2	13

Alternating diagonal (D4) ordering

11	10	9	8
12	19	18	7
13	20	17	6
14	15	16	5
1	2	3	4

Cyclic ordering

9	10	11	12
17	18	19	20
5	6	7	8
13	14	15	16
1	2	3	4

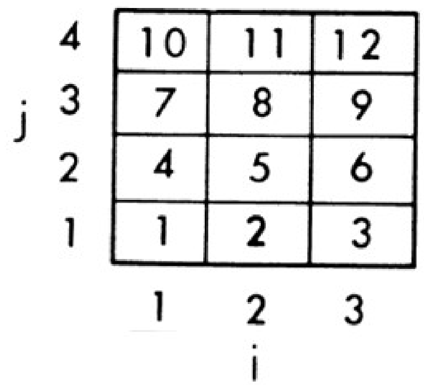
Zebra ordering

9	19	10	20
17	7	18	8
5	15	6	16
13	3	14	4
1	11	2	12

Cyclic-2 ordering

2D Flow Problem/Block-Centered Grid

• Let's consider a block-centered grid with N_x = 3 and N_y = 4.



• It may be simpler to work with a block index (say *m*) instead of coordinate indices *i* and *j* when using this ordering. The block index *m* is related to the coordinate indices *i* and *j* by

$$m = i + (j-1)N_x \quad \text{for} \quad i = 1,2,\ldots,N_x \;\; \text{and} \;\; j = 1,2,\ldots,N_y$$

2D Flow Problem/Block-Centered Grid

- Then, define our unknown vector of pressures:

$$\vec{p}^{\,n+1} = \begin{bmatrix} p_{1,1}^{n+1} \\ p_{2,1}^{n+1} \\ p_{3,1}^{n+1} \\ p_{1,2}^{n+1} \\ p_{2,2}^{n+1} \\ p_{3,2}^{n+1} \\ p_{1,3}^{n+1} \\ p_{2,3}^{n+1} \\ p_{3,3}^{n+1} \\ p_{1,4}^{n+1} \\ p_{2,4}^{n+1} \\ p_{3,4}^{n+1} \end{bmatrix} = \begin{bmatrix} p_{1}^{n+1} \\ p_{2}^{n+1} \\ p_{3}^{n+1} \\ p_{4}^{n+1} \\ p_{5}^{n+1} \\ p_{6}^{n+1} \\ p_{7}^{n+1} \\ p_{8}^{n+1} \\ p_{9}^{n+1} \\ p_{10}^{n+1} \\ p_{11}^{n+1} \\ p_{12}^{n+1} \end{bmatrix}$$

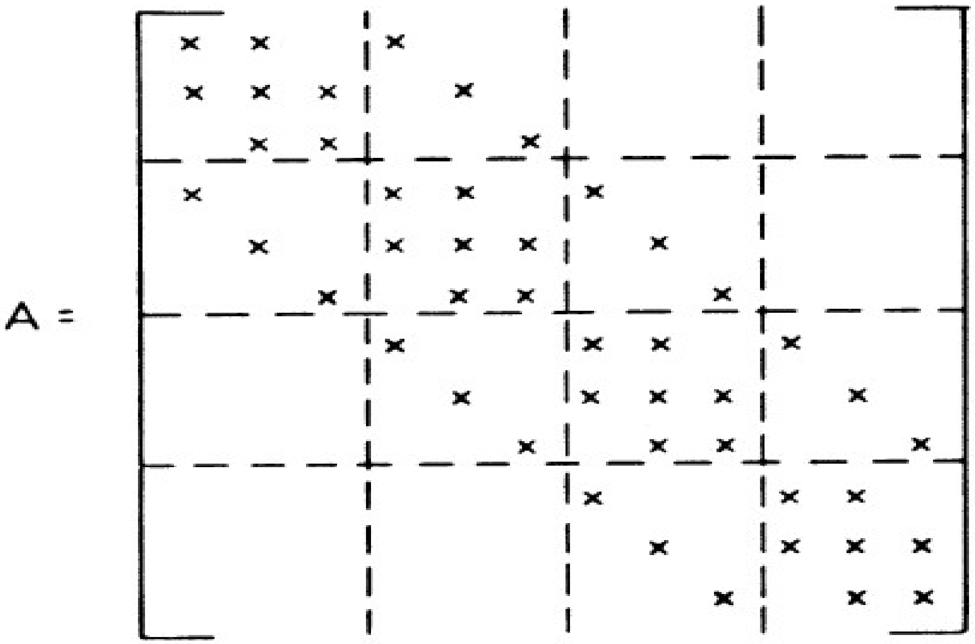
$$\vec{d}^{\,n} = \begin{bmatrix} \widetilde{V}_{1,1} p_{1,1}^{n} - q_{sc,1,1}^{n+1} B \\ \widetilde{V}_{1,2} p_{2,1}^{n} - q_{sc,2,1}^{n+1} B \\ \vdots \\ \widetilde{V}_{i,j} p_{i,j}^{n} - q_{sc,i,j}^{n+1} B \\ \vdots \\ \widetilde{V}_{Nx,Ny} p_{Nx,Ny}^{n} - q_{sc,Nx,Ny}^{n+1} B \end{bmatrix} = \begin{bmatrix} \widetilde{V}_{1} p_{1}^{n} - q_{sc,1}^{n+1} B \\ \widetilde{V}_{2} p_{2}^{n} - q_{sc,2}^{n+1} B \\ \vdots \\ \widetilde{V}_{m} p_{m}^{n} - q_{sc,m}^{n+1} B \\ \vdots \\ \widetilde{V}_{NxNy} p_{NxNy}^{n} - q_{sc,NxNy}^{n+1} B \end{bmatrix}$$

(based on *i* and *j* Indices)

(based on block index *m*)

$$A\vec{p}^{\,n+1} = \vec{d}^{\,n}$$

2D Flow Problem/Block-Centered Grid



x represents non-zero element

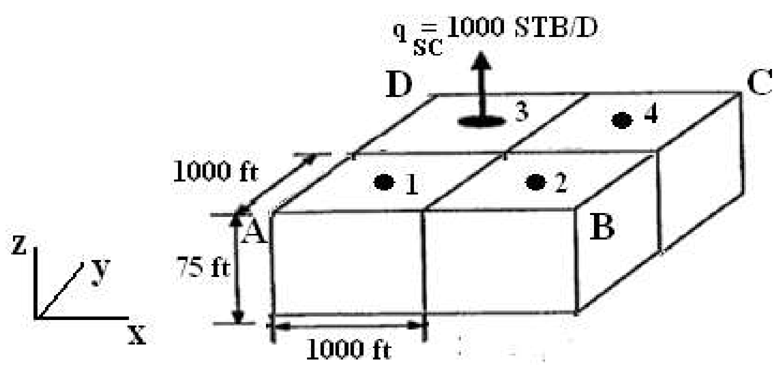
Matrix Structure for a 3 × 4 × 1 Areal Reservoir Problem.

Note that the matrix is 12×12, and is a sparse, symmetric, 5-band matrix

Example

A block-centered grid system in a horizontal reservoir is shown in Figure 1. The initial reservoir pressure is P=3500 psia. The pressure at the boundary AB is maintained at the initial reservoir pressure. The boundaries BC, CD and DA are closed boundaries. The rock and fluid properties for this reservoir are c_t = 3.5×10^{-6} psi^{-1}, φ = 0.18, kx = ky= 200 md, Δx = Δy =1000 ft, h = 75 ft, B = 1 RB/STB, μ = 10 cp. Determine the pressure distribution after 10 Days of production using the implicit approximation method (***Gauss-Seidel method and Gaussian Elimination method***) with a time-step size of Δt = 10 Days. The flow system is governed by the following partial differential equation (PDE):

$$1.127\times10^{-3}\left[\frac{\partial}{\partial x}\left(\frac{k_x}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{k_y}{\mu}\frac{\partial p}{\partial y}\right)\right]-\frac{q_{sc}(x,y,t)B}{V_b}=\frac{\phi c_t}{5.615}\frac{\partial p}{\partial t}$$



Solution

$$1.127\times10^{-3}\left[\frac{\partial}{\partial x}\left(\frac{k_x}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{k_y}{\mu}\frac{\partial p}{\partial y}\right)\right]-\frac{q_{sc}(x,y,t)B}{V_b}=\frac{\phi c_t}{5.615}\frac{\partial p}{\partial t}$$

Applying implicit finite difference approximation to this PDE yields:

$$1.127\times10^{-3}\left[\frac{\lambda_{x,i+1/2,j}\left(\frac{p_{i+1,j}^{n+1}-p_{i,j}^{n+1}}{x_{i+1}-x_i}\right)-\lambda_{x,i-1/2,j}\left(\frac{p_{i,j}^{n+1}-p_{i-1,j}^{n+1}}{x_i-x_{i-1}}\right)}{x_{i+1/2}-x_{i-1/2}}+\frac{\lambda_{y,i,j+1/2}\left(\frac{p_{i,j+1}^{n+1}-p_{i,j}^{n+1}}{y_{j+1}-y_j}\right)-\lambda_{y,i,j-1/2}\left(\frac{p_{i,j}^{n+1}-p_{i,j-1}^{n+1}}{y_j-y_{j-1}}\right)}{y_{j+1/2}-y_{j-1/2}}\right]-\frac{q_{sc,i,j}^{n+1}B}{\Delta x_i\Delta y_j h}$$

$$=\frac{(\phi c_t)_{i,j}}{5.615}\left(\frac{p_{i,j}^{n+1}-p_{i,j}^{n}}{\Delta t^{n+1}}\right)$$

- Multiply both sides by the bulk volume of the block, $V_b = \Delta x_i \Delta y_j h$

$$1.127\times10^{-3}\Delta x_i \Delta y_j h \left[\frac{\lambda_{x,i+1/2,j}\left(\frac{p_{i+1,j}^{n+1}-p_{i,j}^{n+1}}{x_{i+1}-x_i}\right)-\lambda_{x,i-1/2,j}\left(\frac{p_{i,j}^{n+1}-p_{i-1,j}^{n+1}}{x_i-x_{i-1}}\right)}{x_{i+1/2}-x_{i-1/2}} + \frac{\lambda_{y,i,j+1/2}\left(\frac{p_{i+1,j}^{n+1}-p_{i,j}^{n+1}}{y_{j+1}-y_j}\right)-\lambda_{y,i,j-1/2}\left(\frac{p_{i,j}^{n+1}-p_{i-1,j}^{n+1}}{y_j-y_{j-1}}\right)}{y_{j+1/2}-y_{j-1/2}} \right] - q_{sc,i,j}^{n+1}B$$

$$= \frac{\Delta x_i \Delta y_j h(\phi c_t)_{i,j}}{5.615}\left(\frac{p_{i,j}^{n+1}-p_{i,j}^{n}}{\Delta t^{n+1}}\right)$$

Note that $\Delta x_i = x_{i+1/2} - x_{i-1/2}$ and $\Delta y_j = y_{j+1/2} - y_{j-1/2}$

- Define transmissibility in x and y directions and the accumulation term:

$$T_{x,i+1/2,j}\left(p_{i+1,j}^{n+1}-p_{i,j}^{n+1}\right)-T_{x,i-1/2,j}\left(p_{i,j}^{n+1}-p_{i-1,j}^{n+1}\right)$$
$$+T_{y,i,j+1/2}\left(p_{i,j+1}^{n+1}-p_{i,j}^{n+1}\right)-T_{y,i,j-1/2}\left(p_{i,j}^{n+1}-p_{i,j-1}^{n+1}\right)-q_{sc,i,j}^{\;n+1}B=\widetilde{V}_{i,j}\left(p_{i,j}^{n+1}-p_{i,j}^{n}\right)$$

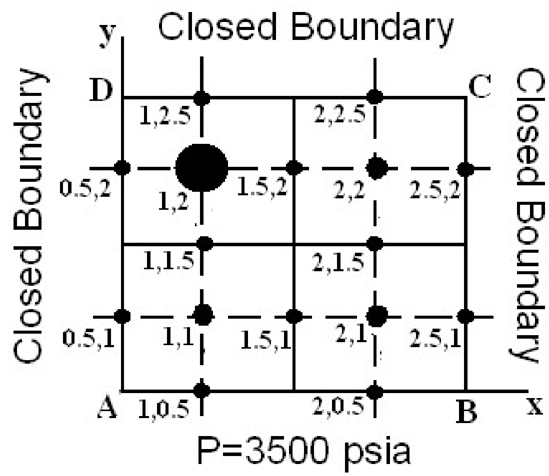
………………..(1)

where

$$T_{x,i+1/2,j}=1.127\times10^{-3}\frac{\lambda_{x,i+1/2,j}\Delta y_j h}{\left(x_{i+1}-x_i\right)},\;\; T_{x,i-1/2,j}=1.127\times10^{-3}\frac{\lambda_{x,i-1/2,j}\Delta y_j h}{\left(x_i-x_{i-1}\right)},$$

$$T_{y,i,j+1/2}=1.127\times10^{-3}\frac{\lambda_{y,i,j+1/2}\Delta x_i h}{\left(y_{j+1}-y_j\right)},\;\; T_{y,i,j-1/2}=1.127\times10^{-3}\frac{\lambda_{y,i,j-1/2}\Delta x_i h}{\left(y_j-y_{j-1}\right)}$$

$$and\;\;\widetilde{V}_{i,j}=\frac{\left(\phi c_t\right)_{i,j}\Delta x_i\Delta y_j h}{5.615\Delta t^{n+1}},\quad \lambda_{x,i\pm1/2,j}=\frac{k_{x,i\pm1/2,j}}{\mu_{i\pm1/2,j}},\quad \lambda_{y,i,j\pm1/2}=\frac{k_{y,i,j\pm1/2}}{\mu_{i,j\pm1/2}}$$



Since Δx=Δy, thus the accumulation term for all blocks:

$$\widetilde{V}_{i,j} = \frac{(\phi c_t)_{i,j}\,\Delta x_i \Delta y_j h}{5.615\Delta t^{n+1}} = \frac{(0.18)(3.5\times 10\text{-}6)(1000)(1000)(75)}{5.615\,(10)} = 0.841496\,B/(D.psi)$$

Since Δx=Δy and kx=ky , thus the transmissibility for all the interior boundaries (T_{ib}) between the blocks:

$$T_{ib} = 1.127\times 10^{-3}\,\frac{(k/\mu)\,\Delta y\,h}{(\Delta x)} = 1.127\times 10^{-3}\,\frac{(200/10)\,(1000)(75)}{(1000)} = 1.6905\,B/(D.psi)$$

The transmissibility for all the outer closed boundaries = 0

The transmissibility for all the outer constant pressure boundaries ($\widetilde{T}_{ocp}$) :

$$\widetilde{T}_{ocp} = 2 \times 1.127 \times 10^{-3} \frac{(k/\mu)\,\Delta y\, h}{\Delta x}$$

$$= 2 \times 1.127 \times 10^{-3} \frac{(200/10)\,(1000)(75)}{(1000)} = 3.381\, B/(D.psi)$$

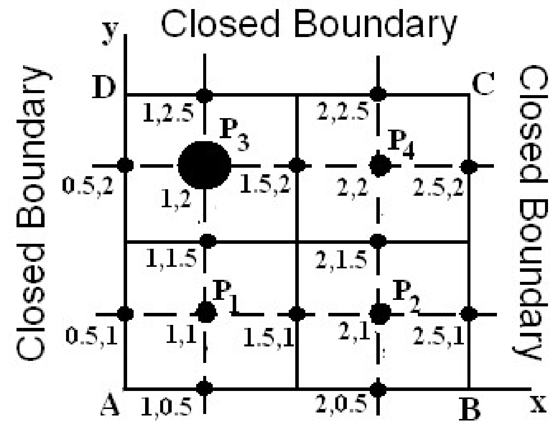
Using ***Gauss-Seidel method*** to find the pressure distribution:
From Eq. 1:

$$T_{x,i+1/2,j}\left(p_{i+1,j}^{n+1} - p_{i,j}^{n+1}\right) - T_{x,i-1/2,j}\left(p_{i,j}^{n+1} - p_{i-1,j}^{n+1}\right)$$
$$+ T_{y,i,j+1/2}\left(p_{i,j+1}^{n+1} - p_{i,j}^{n+1}\right) - T_{y,i,j-1/2}\left(p_{i,j}^{n+1} - p_{i,j-1}^{n+1}\right) - q_{sc,i,j}^{\;n+1} B = \widetilde{V}_{i,j}\left(p_{i,j}^{n+1} - p_{i,j}^{n}\right)$$

……………(1)

$$p_{i,j}^{n+1} = [1/(T_{x,i+1/2,j} + T_{x,i-1/2,j} + T_{y,i,j+1/2} + T_{y,i,j-1/2} + \widetilde{V}_{i,j})]$$
$$[\widetilde{V}_{i,j}\, p_{i,j}^{n} + T_{x,i+1/2,j}\, p_{i+1,j}^{n+1} + T_{x,i-1/2,j}\, p_{i-1,j}^{n+1} + T_{y,i,j+1/2}\, p_{i,j+1}^{n+1} + T_{y,i,j-1/2}\, p_{i,j-1}^{n+1} - q_{sc,i,j}^{\;n+1} B]$$

$\widetilde{V}$ =0.841496 B/(D.psi)
T internal boundary=1.6905 B/(D.psi)
T closed boundary= 0
T constant pressure boundary= 3.381 B/(D.psi)

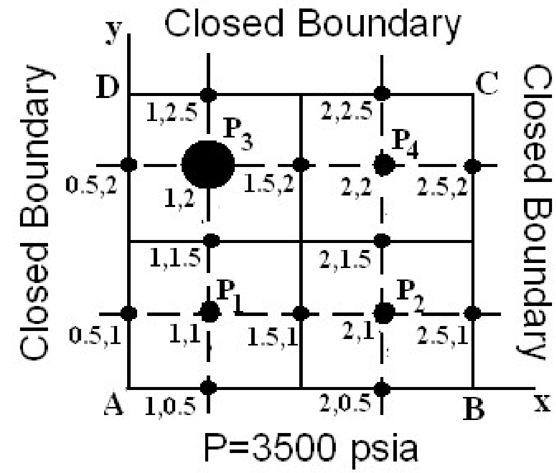


$$p_{i,j}^{n+1} = [1/(T_{x,i+1/2,j} + T_{x,i-1/2,j} + T_{y,i,j+1/2} + T_{y,i,j-1/2} + \widetilde{V}_{i,j})]$$
$$[\widetilde{V}_{i,j}\ p_{i,j}^{n} + T_{x,i+1/2,j}\ p_{i+1,j}^{n+1} + T_{x,i-1/2,j} p_{i-1,j}^{n+1} + T_{y,i,j+1/2}\ p_{i,j+1}^{n+1} + T_{y,i,j-1/2}\ p_{i,j-1}^{n+1} - q_{sc,i,j}^{\ n+1} B]$$

$$For\ p_1^{n+1}\ (i=1,\ j=1):$$
$$p_1^{n+1} = [1/(1.6905 + 0 + 1.6905 + 3.381 + 0.841496)]$$
$$[0.841496\, p_1^{n} + 1.6905\, p_2^{n+1} + 0 + 1.6905\, p_3^{n+1} + 3.381(3500) - 0]$$

$$For\ p_2^{n+1}\ (i=2,\ j=1):$$
$$p_2^{n+1} = [1/(0 + 1.6905 + 1.6905 + 3.381 + 0.841496)]$$
$$[0.841496\, p_2^{n} + 0 + 1.6905\, p_1^{n+1} + 0 + 1.6905\, p_4^{n+1} + 3.381(3500) - 0]$$

$\widetilde{V}$=0.841496 B/(D.psi)
T internal boundary=1.6905 B/(D.psi)
T closed boundary= 0
T constant pressure boundary= 3.381 B/(D.psi)



$$p_{i,j}^{n+1} = [1/(T_{x,i+1/2,j} + T_{x,i-1/2,j} + T_{y,i,j+1/2} + T_{y,i,j-1/2} + \widetilde{V}_{i,j})]$$

$$[\widetilde{V}_{i,j}\, p_{i,j}^{n} + T_{x,i+1/2,j}\, p_{i+1,j}^{n+1} + T_{x,i-1/2,j}\, p_{i-1,j}^{n+1} + T_{y,i,j+1/2}\, p_{i,j+1}^{n+1} + T_{y,i,j-1/2}\, p_{i,j-1}^{n+1} - q_{sc,i,j}^{\;n+1} B]$$

$For\ p_3^{n+1}\ (i=1, j=2):$

$$p_3^{n+1} = [1/(1.6905 + 0 + 0 + 1.6905 + 0.841496)]$$

$$[0.841496\, p_3^{n} + 1.6905\, p_4^{n+1} + 0 + 0 + 1.6905\, p_1^{n+1} - 1000(1)]$$

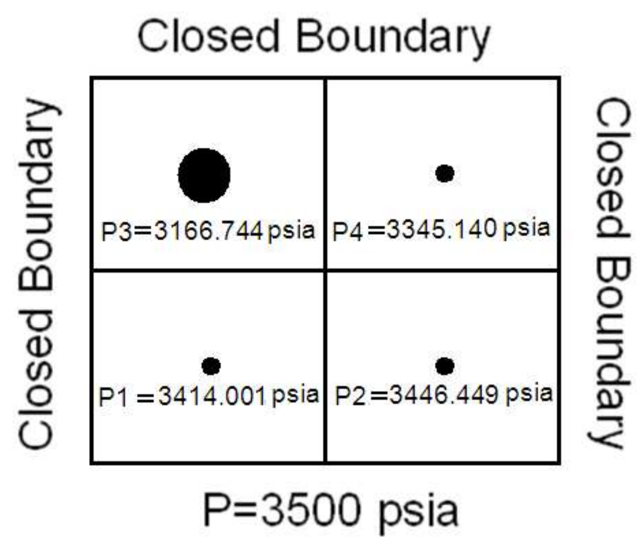
$For\ p_4^{n+1}\ (i=2, j=2):$

$$p_4^{n+1} = [1/(0 + 1.6905 + 0 + 1.6905 + 0.841496)]$$

$$[0.841496\, p_4^{n} + 0 + 1.6905\, p_3^{n+1} + 0 + 1.6905\, p_2^{n+1} - 0]$$

Pressure distribution in the reservoir after 10 days using ***Gauss-Seidel method***

Iteration No.	0	1	2	3	....	13	14	15
P1, psia	3500	3500	3447.346	3426.93	....	3414.001	3414.001	3414.001
P2, psia	3500	3500	3467.213	3454.5	....	3446.45	3446.449	3446.449
P3, psia	3500	3263.173	3204.133	3181.241	....	3166.745	3166.744	3166.744
P4, psia	3500	3405.185	3368.422	3354.167	....	3345.14	3345.14	3345.14



Using ***Gaussian Elimination method*** to find the pressure distribution:
From Eq. 1:

$$T_{x,i+1/2,j}\left(p_{i+1,j}^{n+1}-p_{i,j}^{n+1}\right)-T_{x,i-1/2,j}\left(p_{i,j}^{n+1}-p_{i-1,j}^{n+1}\right)$$
$$+T_{y,i,j+1/2}\left(p_{i,j+1}^{n+1}-p_{i,j}^{n+1}\right)-T_{y,i,j-1/2}\left(p_{i,j}^{n+1}-p_{i,j-1}^{n+1}\right)-q_{sc,i,j}^{\;n+1}B=\widetilde{V}_{i,j}\left(p_{i,j}^{n+1}-p_{i,j}^{n}\right)$$

……………(1)

Re-arrange Eq.1 as follows:

$$p_{i,j}^{n+1}\,(T_{x,i+1/2,j}+T_{x,i-1/2,j}+T_{y,i,j+1/2}+T_{y,i,j-1/2}+\widetilde{V}_{i,j})-$$
$$p_{i+1,j}^{n+1}T_{x,i+1/2,j}-p_{i-1,j}^{n+1}T_{x,i-1/2,j}-p_{i,j+1}^{n+1}T_{y,i,j+1/2}-p_{i,j-1}^{n+1}T_{y,i,j-1/2}=p_{i,j}^{n}\widetilde{V}_{i,j}-q_{sc,i,j}^{\;n+1}B$$

……………(2)

Use Eq. 2 to find the matrix-vector equation:

$$For\ p_1\ (i=1,\ j=1):$$
$$p_1^{n+1}\,(1.6905+0+1.6905+3.381+0.841496)-$$
$$p_2^{n+1}(1.6905)-0-p_3^{n+1}(1.6905)-3500(3.381)=p_1^{n}(0.841496)-0$$
$$p_1^{n+1}\,(7.603496)-p_2^{n+1}(1.6905)-p_3^{n+1}(1.6905)=14778.736$$

……………(3)

$$For\ p_2(i=2,\ j=1):$$

$$p_2^{n+1}(0+1.6905+1.6905+3.381+0.841496)-$$

$$p_1^{n+1}(1.6905)-0-p_4^{n+1}(1.6905)-3500(3.381)=p_2^n(0.841496)-0$$

$$p_2^{n+1}(7.603496)-p_1^{n+1}(1.6905)-p_4^{n+1}(1.6905)=14778.736 \quad \ldots\ldots\ldots\ldots\ldots(4)$$

$$For\ p_3(i=1,\ j=2):$$

$$p_3^{n+1}(1.6905+0+0+1.6905+0.841496)-$$

$$p_4^{n+1}(1.6905)-0-0-p_1^{n+1}(1.6905)=p_3^n(0.841496)-1000(1)$$

$$p_3^{n+1}(4.222496)-p_4^{n+1}(1.6905)-p_1^{n+1}(1.6905)=1945.236 \quad \ldots\ldots\ldots\ldots\ldots(5)$$

$$For\ p_4(i=2,\ j=2):$$

$$p_4^{n+1}(0+1.6905+0+1.6905+0.841496)-$$

$$0-p_3^{n+1}(1.6905)-0-p_2^{n+1}(1.6905)=p_4^n(0.841496)-0$$

$$p_4^{n+1}(4.222496)-p_3^{n+1}(1.6905)-p_2^{n+1}(1.6905)=2945.236 \quad \ldots\ldots\ldots\ldots\ldots(6)$$

To formulate the matrix-vector Eq. from Eqs. 3, 4, 5, and 6:

$$p_1^{n+1}(7.603496) - p_2^{n+1}(1.6905) - p_3^{n+1}(1.6905) = 14778.736 \quad \ldots\ldots\ldots\ldots\ldots(3)$$

$$p_2^{n+1}(7.603496) - p_1^{n+1}(1.6905) - p_4^{n+1}(1.6905) = 14778.736 \quad \ldots\ldots\ldots\ldots\ldots(4)$$

$$p_3^{n+1}(4.222496) - p_4^{n+1}(1.6905) - p_1^{n+1}(1.6905) = 1945.236 \quad \ldots\ldots\ldots\ldots\ldots(5)$$

$$p_4^{n+1}(4.222496) - p_3^{n+1}(1.6905) - p_2^{n+1}(1.6905) = 2945.236 \quad \ldots\ldots\ldots\ldots\ldots(6)$$

$$\begin{array}{cccc} P_1 & P_2 & P_3 & P_4 \end{array}$$

$$\begin{bmatrix} 7.603496 & -1.6905 & -1.6905 & 0 \\ -1.6905 & 7.603496 & 0 & -1.6905 \\ -1.6905 & 0 & 4.222496 & -1.6905 \\ 0 & -1.6905 & -1.6905 & 4.222496 \end{bmatrix} \begin{bmatrix} p_1^{n+1} \\ p_2^{n+1} \\ p_3^{n+1} \\ p_4^{n+1} \end{bmatrix} = \begin{bmatrix} 14778.736 \\ 14778.736 \\ 1945.236 \\ 2945.236 \end{bmatrix}$$

Solving the matrix-vector Eq. by ***Gaussian Elimination method*** yields:

$$\begin{bmatrix} 7.603496 & -1.6905 & -1.6905 & 0 \\ 0 & 7.227644 & -0.375852 & -1.6905 \\ 0 & 0 & 3.827099 & -1.778409 \\ 0 & 0 & 0 & 3.000693 \end{bmatrix} \begin{bmatrix} p_1^{n+1} \\ p_2^{n+1} \\ p_3^{n+1} \\ p_4^{n+1} \end{bmatrix} = \begin{bmatrix} 14778.736 \\ 18064.521 \\ 6170.412 \\ 10037.733 \end{bmatrix}$$

p_4^{n+1} =3345.138 psia

p_3^{n+1} =3166.742 psia

p_2^{n+1} =3446.449 psia

p_1^{n+1} =3414.000 psia

THANK YOU