Al-Ayen University
College of Petroleum Engineering

Reservoir Engineering II

Lecturer: Dr. Mohammed Idrees Al-Mossawy

2020/2021

Lecture 9: Unsteady-State Flow of Reservoir Fluids (Part 3)
Ref.: Reservoir Engineering Handbook by Tarek Ahmed

Outlines

❖ *Unsteady-State Flow*

❑ **Solution of the Diffusivity Equation**

- ➢ **Radial Flow of the Compressible Fluids**
  - ✓ **The m(p) Solution-Mothed (Exact-Solution)**
    - ❖ **Example**
  - ✓ **The Pressure-Squared Approximation Method (p2-method)**
    - ❖ **Example**

Unsteady-State Flow

Solution of the Diffusivity Equation:

Radial Flow of the Compressible Fluids

$$\frac{\partial^2 m(p)}{\partial r^2} + \frac{1}{r}\frac{\partial m(p)}{\partial r} = \frac{\phi \mu c_t}{0.000264\,k}\frac{\partial m(p)}{\partial t}$$

The radial diffusivity equation for compressible fluids

In general, there are three forms of the mathematical solution to the diffusivity equation:

- The m(p)-Solution Method (Exact Solution)
- The Pressure-Squared Method (p^2-Approximation Method)
- The Pressure Method (p-Approximation Method)

The m(p)-Solution Method (Exact-Solution)

Imposing the constant-rate condition as one of the boundary conditions, Al-Hussainy, et al. (1966) proposed the following exact solution to the diffusivity equation:

$$m(p_{wf}) = m(p_i) - 57895.3\left(\frac{p_{sc}}{T_{sc}}\right)\left(\frac{Q_g T}{kh}\right)\left[\log\left(\frac{kt}{\phi \mu_i c_{ti} r_w^2}\right) - 3.23\right]$$

where p_{wf} = bottom-hole flowing pressure, psi
p_e = initial reservoir pressure
Q_g = gas flow rate, Mscf/day
t = time, hr
k = permeability, md
p_{sc} = standard pressure, psi
T_{sc} = standard temperature, °R
T = reservoir temperature
r_w = wellbore radius, ft
h = thickness, ft
μ_i = gas viscosity at the initial pressure, cp
c_{ti} = total compressibility coefficient at p_i, psi^{-1}
ϕ = porosity

When p_{sc} = 14.7 psia and T_{sc} = 520 °R :

$$m(p_{wf}) = m(p_i) - \left(\frac{1637 Q_g T}{kh}\right)\left[\log\left(\frac{kt}{\phi \mu_i c_{ti} r_w^2}\right) - 3.23\right]$$

In terms of the dimensionless time t_D as:

$$m(p_{wf}) = m(p_i) - \left(\frac{1637 Q_g T}{kh}\right)\left[\log\left(\frac{4 t_D}{\gamma}\right)\right]$$

$$t_D = \frac{0.000264\, kt}{\phi \mu_i c_{ti} r_w^2}$$

$$\gamma = e^{0.5772} = 1.781$$

- The radial gas diffusivity equation can be expressed in a dimensionless form in terms of the dimensionless real gas pseudopressure drop ψ_D.

$$m(p_{wf}) = m(p_i) - \left(\frac{1422\, Q_g\, T}{kh} \right) \psi_D$$

where Q_g = gas flow rate, Mscf/day
k = permeability, md

The dimensionless pseudopressure drop ψ_D can be determined as a function of t_D by using the appropriate expression *of Equations used for Infinite-Acting Reservoir*. Example, when $t_D > 100$, the ψ_D can be calculated by applying Equation: $\psi_D = 0.5\,[\ln(t_D) + 0.80907]$

Example

A gas well with a wellbore radius of 0.3 ft is producing at a constant flow rate of 2000 Mscf/day under transient flow conditions. The initial reservoir pressure (shut-in pressure) is 4400 psi at 140°F. The formation permeability and thickness are 65 md and 15 ft, respectively. The porosity is recorded as 15%. It is given the properties of the gas as well as values of m(p) as a function of pressures in the Table below. Assuming that the initial total isothermal compressibility is 0.0003 1/psi, calculate the bottom-hole flowing pressure after 1.5 hours.

p	μ_g (cp)	z	m(p), psi^2/cp
0	0.01270	1.000	0.000
400	0.01286	0.937	13.2×10^6
800	0.01390	0.882	52.0×10^6
1200	0.01530	0.832	113.1×10^6
1600	0.01680	0.794	198.0×10^6
2000	0.01840	0.770	304.0×10^6
2400	0.02010	0.763	422.0×10^6
2800	0.02170	0.775	542.4×10^6
3200	0.02340	0.797	678.0×10^6
3600	0.02500	0.827	816.0×10^6
4000	0.02660	0.860	950.0×10^6
4400	0.02831	0.896	1089.0×10^6

Solution

$$t_D = \frac{0.000264kt}{\phi\mu c_t r_w^2}$$

$$t_D = \frac{(0.000264)(65)(1.5)}{(0.15)(0.02831)(3\times10^{-4})(0.3^2)} = 224,498.6$$

$$m(p_{wf}) = m(p_i) - \left(\frac{1637\,Q_g\,T}{kh}\right)\left[\log\left(\frac{4t_D}{\gamma}\right)\right]$$

$$m(p_{wf}) = 1089\times10^6 - \frac{(1637)(2000)(600)}{(65)(15)}\left[\log\left(\frac{(4)224498.6}{e^{0.5772}}\right)\right]$$

$$= 1077.5(10^6)\ psi^2/cp$$

From the given PVT data, interpolate using the value of $m(p_{wf})$ to give a corresponding p_{wf} of 4367 psia.

Also, the example can be solved by applying the ψD:

$$\psi_D = 0.5[\ln(t_D) + 0.80907]$$

$$\psi_D = 0.5[\ \ln(224498.6) + 0.8090] = 6.565$$

$$m(p_{wf}) = m(p_i) - \left(\frac{1422\,Q_g\,T}{kh}\right)\psi_D$$

$$m(p_{wf}) = 1089\times10^6 - \left(\frac{1422(2000)(600)}{(65)(15)}\right)(6.565)$$

$$= 1077.5\times10^6\ psi^2/cp$$

From the given PVT data, interpolate using the value of $m(p_{wf})$ to give a corresponding p_{wf} of 4367 psia.

The Pressure-Squared Approximation Method (p2-method)

The first approximation to the exact solution is to remove the pressure-dependent term (μz) outside the integral that defines $m(p_{wf})$ and $m(p_i)$ to give:

$$m(p_i) - m(p_{wf}) = \frac{2}{\bar{\mu}\,\bar{z}} \int^{p_i} p\,dp$$

or

$$m(p_i) - m(p_{wf}) = \frac{p_i^2 - p_{wf}^2}{\bar{\mu}\;\bar{z}}$$

The bars over μ and z represent the values of the gas viscosity and deviation factor as evaluated at the average pressure. This average pressure is given by:

$$\bar{p} = \sqrt{\frac{p_i^2 + p_{wf}^2}{2}}$$

Combining with Equations of the exact solution gives:

$$p_{wf}^2 = p_i^2 - \left(\frac{1637 Q_g T \bar{\mu}\,\bar{z}}{kh}\right)\left[\log\left(\frac{kt}{\phi \mu_i c_{ti} r_w^2}\right) - 3.23\right]$$

or

$$p_{wf}^2 = p_i^2 - \left(\frac{1637 Q_g T \bar{\mu}\,\bar{z}}{kh}\right)\left[\log\left(\frac{4 t_D}{\gamma}\right)\right]$$

or, equivalently:

$$p_{wf}^2 = p_i^2 - \left(\frac{1422\, Q_g\, T \bar{\mu}\,\bar{z}}{kh}\right)\psi_D$$

The above approximation solution is limited to the reservoir pressures < 2000. It should be pointed out that when this method is used to determine p_{wf} it is perhaps sufficient to use μ and z at the initial pressure.

Example

A gas well is producing at a constant rate of 7454.2 Mscf/day under transient flow conditions. The following data are available:

$k = 50$ md $h = 10$ ft $\phi = 20\%$ $p_i = 1600$ psi
$T = 600^{\circ}R$ $r_w = 0.3$ ft $c_{ti} = 6.25 \times 10^{-4}$ psi^{-1}

The gas properties are tabulated below:

p	μ_g, cp	z	m(p), psi^2/cp
0	0.01270	1.000	0.000
400	0.01286	0.937	13.2×10^6
800	0.01390	0.882	52.0×10^6
1200	0.01530	0.832	113.1×10^6
1600	0.01680	0.794	198.0×10^6

Calculate the bottom-hole flowing pressure after 4 hours using:

a. The m(p)-method

b. The p2-method

Solution

a. The m(p)-method

$$t_D = \frac{0.000264kt}{\phi\mu c_t r_w^2}$$

$$t_D = \frac{0.000264(50)(4)}{(0.2)(0.0168)(6.25 \times 10^{-4})(0.3^2)} = 279,365.1$$

Since $t_D > 100$, the ψ_D can be calculated by applying Equation: $\psi_D = 0.5\,[\ln(t_D) + 0.80907]$

$$\psi_D = 0.5[\mathrm{Ln}(279365.1) + 0.80907] = 6.6746$$

$$m(p_{wf}) = m(p_i) - \left(\frac{1422\,Q_g\,T}{kh}\right)\psi_D$$

$$m(p_{wf}) = (198 \times 10^6) - \left[\frac{1422(7454.2)(600)}{(50)(10)}\right]6.6746 = 113.1 \times 10^6$$

The corresponding value of p_{wf} = 1200 psia

b. The p2-method

$$t_D = \frac{0.000264kt}{\phi\mu c_t r_w^2}$$

$$t_D = \frac{0.000264(50)(4)}{(0.2)(0.0168)\left(6.25 \times 10^{-4}\right)\left(0.3^2\right)} = 279,365.1$$

Since $t_D > 100$, the ψ_D can be calculated by applying Equation: $\psi_D = 0.5\,[\ln (t_D) + 0.80907]$

$$\psi_D = 0.5[\text{Ln}(279365.1) + 0.80907] = 6.6746$$

$$p_{wf}^2 = p_i^2 - \left(\frac{1422 Q_g T\overline{\mu z}}{kh}\right)\psi_D$$

$$p_{wf}^2 = 1600^2 - \left[\frac{(1422)(7454.2)(600)(0.0168)(0.794)}{(50)(10)}\right] \times 6.6747 = 1,427,491 \text{ psia2}$$

Pwf = 1195 psia

The absolute average error is 0.42%

THANK YOU