Magnetic Circuits

Magnetic phenomena are described using a fairly large number of terms that are often, at first, somewhat difficult to keep track of. One approach that may help is to describe analogies between electrical circuits, which are usually more familiar, and corresponding magnetic circuits. Consider the analogous magnetic circuit shown in Fig1. The electrical circuit consists of a voltage source, v, sending current i through an electrical load with resistance R. The electrical load consists of a long wire of length l, cross-sectional area A, and conductance ρ.

The resistance of the electrical load is given by (1):

$$R = \rho \frac{l}{A} \qquad (1)$$

The current flowing in the electrical circuit is given by Ohm's law:

$$i = \frac{v}{R} \qquad (2)$$

In the magnetic circuit the driving force, analogous to voltage, is called the magnetomotive force (mmf), and designated by ***F***. The magnetomotive force is created by wrapping ***N*** turns of wire, carrying current ***i***, around a toroidal core.

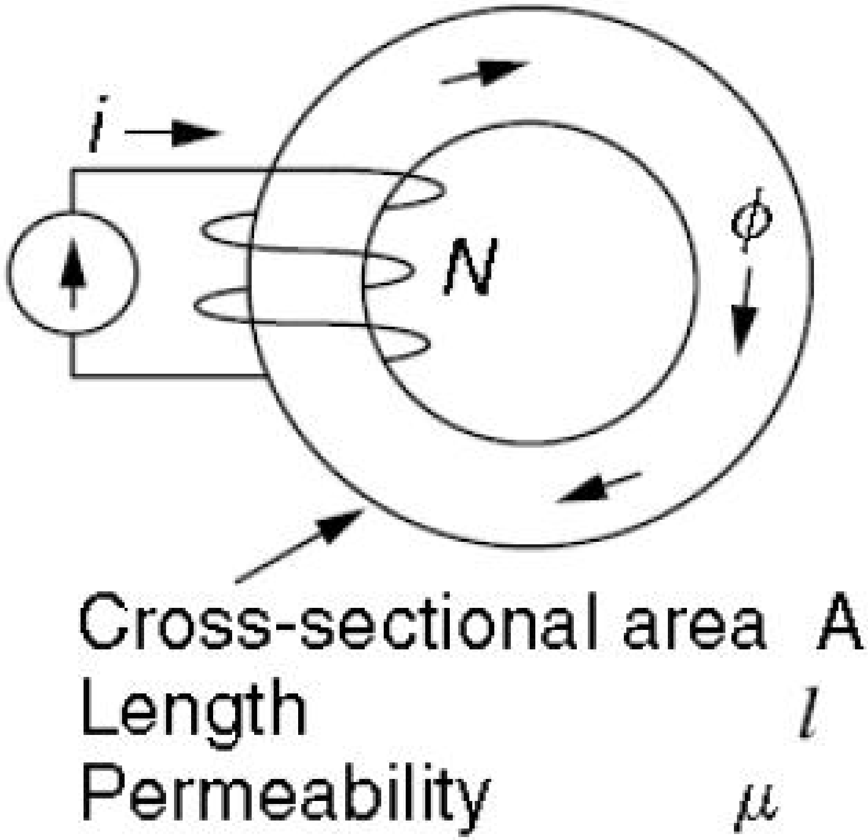

Figure 1 Analogous magnetic circuits.

By definition, the magnetomotive force is the product of **current x turns** , and has units of **ampere-turns**.

Magnetomotive force(mmf) $F = Ni$ **(ampere − turns)** (3)

The response to that mmf (analogous to current in the electrical circuit) is creation of magnetic flux ϕ, which has SI units of *webers* (Wb). The magnetic flux is proportional to the mmf driving force and inversely proportional to a quantity called *reluctance* **R**, which is analogous to electrical resistance, resulting in the "**Ohm's law**" of magnetic circuits given by.

$$F = R\,\phi \tag{4}$$

From (4), we can ascribe units for reluctance R as amp-turns per weber (A-t/Wb). Reluctance depends on the dimensions of the core as well as its materials:

$$R = \frac{l}{\mu A} \qquad \left(\frac{\text{A-t}}{\text{Wb}}\right) \tag{5}$$

Notice the similarity between (5) and the equation for resistance given in (1). The parameter in (5) that indicates how readily the core material accepts magnetic flux is the material's ***permeability μ***. There are three categories of magnetic materials: *diamagnetic*, in which the material tends to exclude magnetic fields; *paramagnetic*, in which the material is slightly magnetized by a magnetic field; and *ferromagnetic*, which are materials that very easily become magnetized. The vast majority of materials do not respond to magnetic fields, and their permeability is very close to that of free space. The materials that readily accept magnetic flux that is, ferromagnetic materials are principally iron, cobalt, and nickel and various alloys that include these elements. The units of permeability are weber per amp-turn-meter (*Wb/A-t-m*).

The permeability of free space is given by:

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ **Wb/A-t-m** (6)

Oftentimes, materials are characterized by their *relative permeability*, μ_r, which for ferromagnetic materials may be in the range of hundreds to hundreds of thousands. As will be noted later, however, the relative permeability is not a constant for a given material: It varies with the magnetic field intensity. In this regard, the magnetic analogy deviates from its electrical counterpart and so must be used with some caution.

$$\text{Relative permeability} - \mu_r - \frac{\mu}{\mu_0} \tag{7}$$

Another important quantity of interest in magnetic circuits is the magnetic flux density ***B***. As the name suggests, it is simply the "density" of flux given by the following:

$$B = \frac{\phi}{A} \quad \frac{\textbf{webers}}{\textbf{m2}} \textbf{ or tesla (T)} \tag{8}$$

When flux is given in weber (Wb) and area ***A*** is given in m^2, units for ***B*** are tesla (T). The analogous quantity in an electrical circuit would be the current density, given by:

$$J = \frac{i}{A} \tag{9}$$

The final magnetic quantity that we need to introduce is the magnetic field intensity, $\boldsymbol{H}$. The magnetic field intensity is defined as the magnetomotive force (mmf) per unit of length around the magnetic loop. With $\boldsymbol{N}$ turns of wire carrying current $\boldsymbol{i}$, the mmf created in the circuit is $\boldsymbol{Ni}$ ampere-turns. With $\boldsymbol{l}$ representing the mean path length for the magnetic flux, the magnetic field intensity is therefore:

$$\text{Magnetic field intensity } \boldsymbol{H} = \frac{Ni}{l} \text{ ampere-turns/meter} \qquad (10)$$

An analogous concept in electric circuits is the electric field strength, which is voltage drop per unit of length. In a capacitor, for example, the intensity of the electric field formed between the plates is equal to the voltage across the plates divided by the spacing between the plates.

Finally, if we combine (3), (4), (5), (8), and (10), we arrive at the following relationship between magnetic flux density $\boldsymbol{B}$ and magnetic field intensity $\boldsymbol{H}$:

$$B = \mu H \tag{11}$$

INDUCTANCE

Having introduced the necessary electromagnetic background, we can now address inductance. Inductance is, in some sense, a mirror image of capacitance. While capacitors store energy in an electric field, inductors store energy in a magnetic field. While capacitors prevent voltage from changing instantaneously, inductors, as we shall see, prevent current from changing instantaneously.

Where inductance ***L*** has been introduced and defined as:

$$\text{Inductance } L = \frac{N^2}{R} \tag{12}$$

Example 1. Inductance of a Core-and-Coil. Find the inductance of a core with effective length l = 0.1 m, cross-sectional area A = 0.001 m^2, and relative permeability μ_r somewhere between 15,000 and 25,000. It is wrapped with N= 10 turns of wire. What is the range of inductance for the core?

Solution. When the core's permeability is 15,000 times that of free space, it is

$\mu_{core} = \mu_r \mu_0 = 15{,}000 \times 4\pi \times 10^{-7} = 0.01885$ Wb/A-t-m

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so its reluctance is

$$R_{core} = \frac{l}{\mu_{\text{core}} A} = \frac{0.1 \text{ m}}{0.01885 \text{ (Wb/A-t-m)} \times 0.001 \text{ m}^2} = 5305 \text{ A-t/Wb}$$

And its inductance is

$$L = \frac{N^2}{R} = \frac{10^2}{5305} = 0.0188 \text{ henries} = 18.8 \text{ mH}$$

Similarly, when the relative permeability is 25,000 the inductance is

$$L = \frac{N^2}{R} = \frac{N^2 \mu_r \mu_0 A}{l} = \frac{10^2 \times 25{,}000 \times 4\pi \times 10^{-7} \times 0.001}{0.1}$$

$$= 0.0314 \text{ H} = 31.4 \text{ mH}$$