Al-Ayen University
College of Petroleum Engineering

Reservoir Engineering II

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Lecture 4: MATERIAL BALANCE APPLIED TO OIL RESERVOIRS (Part 2)

Refs.: (1) ***Reservoir Engineering Handbook*** by **Tarek Ahmed**, Fifth Edition, Ch. 11

(2) ***Fundamentals of Reservoir Engineering*** by **LP. DAKE**, Seventeenth impression 1998, Ch. 3

Outlines

- MBE as an Equation of a Straight Line
- Case 1. Volumetric Undersaturated-Oil Reservoirs
- Case 2. Volumetric Saturated-Oil Reservoirs (Without a Gas Cap)
- Case 3. Gas-Cap-Drive Reservoirs
- Case 4. Water-Drive Reservoirs

MBE as an Equation of a Straight Line

$$N = \frac{N_p\left[B_o + (R_p - R_s)B_g\right] - (W_e - W_p B_w) - G_{inj}B_{ginj} - W_{inj}B_{wi}}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + m B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{oi}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}$$

GENERAL FORM OF THE MBE

- There are essentially three unknowns in the general form of MBE:
  1. The original oil in place ***N***
  2. The cumulative water influx ***We***
  3. The original size of the gas cap as compared to the oil zone size ***m***

To find N, We and m, Havlena and Odeh (1963) expressed the MBE in the following form:

$$N_p\left[B_o + (R_p - R_s)B_g\right] + W_pB_w = N\left[(B_o - B_{oi}) + (R_{si} - R_s)B_g\right] + m\,N B_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) + N(1+m)B_{oi}\left[\frac{c_wS_{wi} + c_f}{1 + S_{wi}}\right]\Delta p + W_e + W_{inj}B_w + G_{inj}B_{ginj}$$

Or

$$F = N\left[E_o + m E_g + E_{f,w}\right] + \left(W_e + W_{inj}B_w + G_{inj}B_{ginj}\right)$$

Assuming no pressure maintenance by gas or water injection, the equation becomes:

$$F = N\left[E_o + m E_g + E_{f,w}\right] + W_e$$

$$F = N_p\left[B_o + (R_p - R_s)B_g\right] + W_pB_w$$

$$E_o = (B_o - B_{oi}) + (R_{si} - R_s)B_g$$

$$E_g = B_{oi}\left[(B_g/B_{gi}) - 1\right]$$

$$E_{f,w} = (1+m)B_{oi}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta p$$

In terms of the two-phase formation volume factor B_t:

$$F = N_p\left[B_t + (R_p - R_{si})B_g\right] + W_pB_w$$

$$E_o = B_t - B_{ti}$$

$$E_g = B_{ti}\left[(B_g/B_{gi}) - 1\right]$$

Case 1. Volumetric Undersaturated-Oil Reservoirs

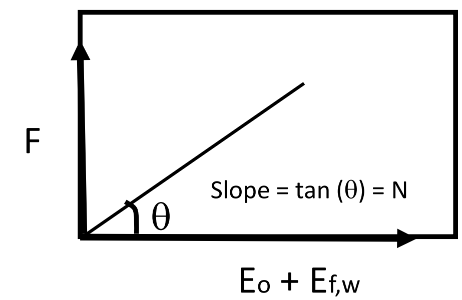
Assuming no water or gas injection, the linear form of the MBE as expressed by:

$$F = N\left[E_o + m\,E_g + E_{f,w}\right] + W_e$$

- $W_e = 0$, since the reservoir is volumetric
- $m = 0$, since the reservoir is undersaturated
- $R_s = R_{si} = R_p$, since all produced gas is dissolved in the oil

$$F = N(E_o + E_{f,w})$$

A straight-line equation with a slope =N



Or

$$N = \frac{F}{E_o + E_{f,w}}$$

$$F = N_p B_o + W_p B_w$$

$$E_o = B_o - B_{oi}$$

$$E_{f,w} = B_{oi}\left[\frac{c_w S_w + c_f}{1 - S_{wi}}\right]\Delta p$$

$$\Delta p = p_i - \overline{p}_r$$

N = initial oil in place, STB

p_i = initial reservoir pressure

$\overline{p}_r$ = volumetric average resevoir pressure

Determination of the reservoir drive mechanism from MBE:

Dake in 1994 suggested that plotting the term $\frac{F}{E_o + E_{f,w}}$ versus cumulative production (***Np)*** or ***time***, as shown in the **Figure,** will indicate type of the drive mechanism.

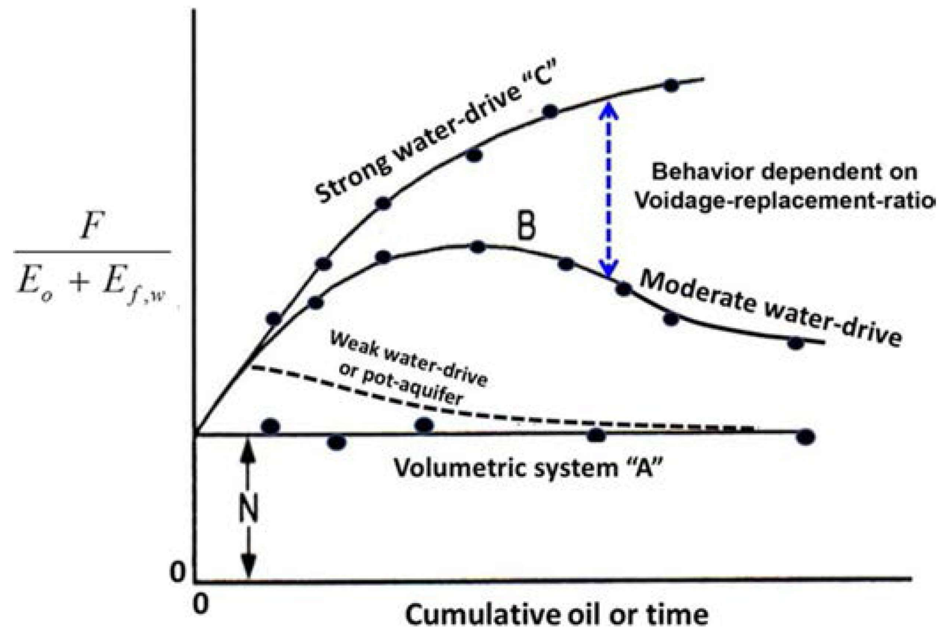
- Line A in the plot implies that the reservoir can be classified as a *volumetric reservoir*. This defines a purely *depletion-drive reservoir* whose energy derives solely from the *expansion of the rock, connate water, and the oil*.
- The curves B and C, indicating that the reservoir has been energized by water influx, abnormal pore compaction, or a combination of these two.

For volumetric undersaturated-oil reservoirs

$$F = N_p B_o + W_p B_w$$

$$E_o = (B_o - B_{oi})$$

$$E_{f,w} = (1+\not{m}) B_{oi} \left[\frac{c_w S_{wi} + c_f}{1 - S_{wi}}\right] \Delta p$$



Determination of the reservoir drive mechanism from MBE

Example

An oil field is a volumetric undersaturated reservoir. The initial reservoir pressure is 3685 psi. The following additional data are available:

$S_{wi} = 24\%$ $c_w = 3.62 \times 10^{-6}\ psi^{-1}$ $c_f = 4.95 \times 10^{-6}\ psi^{-1}$

$B_w = 1.0\ bbl/STB$ $p_b = 1500\ psi$

The field production and PVT data are summarized below:

Volumetric Average Pressure	No. of producing wells	B_o bbl/STB	N_p MSTB	W_p MSTB
3685	1	1.3102	0	0
3680	2	1.3104	20.481	0
3676	2	1.3104	34.750	0
3667	3	1.3105	78.557	0
3664	4	1.3105	101.846	0
3640	19	1.3109	215.681	0
3605	25	1.3116	364.613	0
3567	36	1.3122	542.985	0.159
3515	48	1.3128	841.591	0.805
3448	59	1.3130	1273.530	2.579
3360	59	1.3150	1691.887	5.008
3275	61	1.3160	2127.077	6.500
3188	61	1.3170	2575.330	8.000

Calculate the initial oil in place.

Solution

Step 1. Calculate Ef,w:

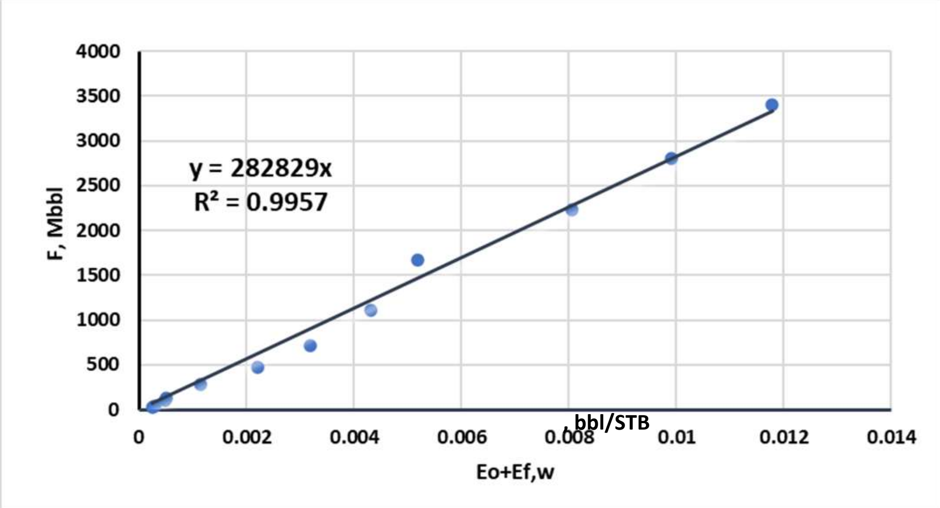
$$E_{f,w} = B_{oi}\left[\frac{c_w S_w + c_f}{1 - S_{wi}}\right]\Delta p$$

$$\Delta p = p_i - \bar{p}_r$$

$$E_{f,w} = 1.00313 \times 10^{-5}(3685 - \bar{p}_r)$$

Step 2. Construct the following table:

$\bar{p}_r$, psi	F, Mbbl $F = N_pB_o + W_pB_w$	E_o, bbl/STB $E_o = B_o - B_{oi}$	Δp	$E_{f,w}$ bbl/STB	$E_o + E_{f,w}$ bbl/STB
3685					
3680	26.8383	0.0002	5	5.02E-05	0.00025
3676	45.5364	0.0002	9	9.03E-05	0.00029
3667	102.9489	0.0003	18	0.000181	0.000481
3664	133.4692	0.0003	21	0.000211	0.000511
3640	282.7362	0.0007	45	0.000451	0.001151
3605	478.2264	0.0014	80	0.000803	0.002203
3567	712.6639	0.002	118	0.001184	0.003184
3515	1105.646	0.0026	170	0.001705	0.004305
3448	1674.724	0.0028	237	0.002377	0.005177
3360	2229.839	0.0048	325	0.00326	0.00806
3275	2805.733	0.0058	410	0.004113	0.009913
3188	3399.71	0.0068	497	0.004986	0.011786



Step 3. Plot the underground withdrawal term F against the expansion term ($E_o + E_{f,w}$) on a Cartesian scale, as shown in Figure.

Step 4. Draw the best straight line through the points and determine the slope of the line and the volume of the active initial oil in place as: N= 283 MMSTB

Note: The initial oil in place as determined from the MBE is referred to as the effective or active initial oil in place. This value is usually smaller than that of the volumetric estimate due to oil being trapped in undrained fault compartments or low permeability regions of the reservoir.

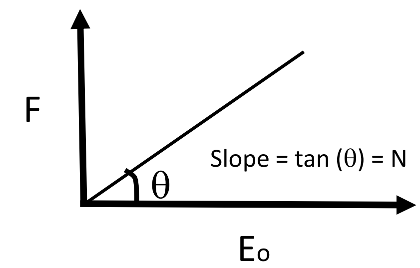
Case 2. Volumetric Saturated-Oil Reservoirs (Without a Gas Cap)

- The main driving mechanism in this type of reservoir results from the liberation and expansion of the solution gas as the pressure drops below the bubble-point pressure.

- Assuming that the water and rock expansion term $E_{f,w}$ is negligible in comparison with the expansion of solution gas, MBE can be simplified as:

$$F = N\,E_o$$

A straight-line equation with a slope =N



Where:

$$F = N_p\left[B_t + \left(R_p - R_{si}\right)B_g\right] + W_pB_w$$

$$E_o = B_t - B_{ti}$$

- The above interpretation technique is useful in that, if the actual plot turns out to be nonlinear, then this deviation can itself be diagnostic in determining the actual drive mechanisms in the reservoir. For instance, it may turn out to be nonlinear because there is an unsuspected water influx into the reservoir helping to maintain the pressure.

Case 3. Gas-Cap-Drive Reservoirs

- For a reservoir in which the *expansion of the gas-cap gas is the predominant* driving mechanism and *assuming* that the *natural water influx is negligible (We = 0)*, the effect of *water and pore compressibilities can be considered negligible*.
- Under these conditions, the Havlena-Odeh material balance can be expressed as:

$$F = N\left[E_o + m\,E_g\right]$$

Where:

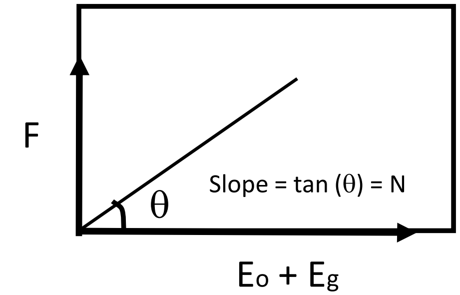
$$E_g = B_{oi}\left[\left(B_g/B_{gi}\right) - 1\right]$$

There are three possible unknowns:

- *N* is unknown, *m* is known
- *m* is unknown, ***N*** is known
- *N* and *m* are unknown

a. *N* is unknown, *m* is known

$$F = N\left[E_o + m\,E_g\right]$$ ⟷ A straight-line equation with a slope =N



$$F = N_p\left[B_o + (R_p - R_s)B_g\right] + W_pB_w$$

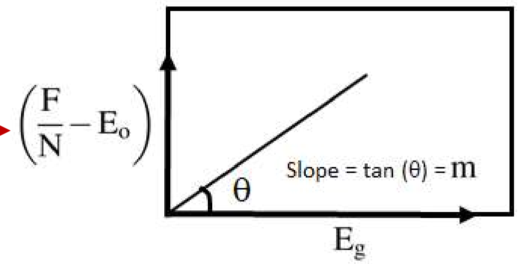
$$E_o = (B_o - B_{oi}) + (R_{si} - R_s)B_g$$

$$E_g = B_{oi}\left[(B_g/B_{gi}) - 1\right]$$

b. *m* is unknown, ***N*** is known

The equation above can be rearranged to give:

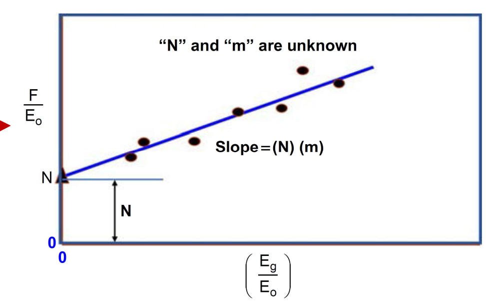
$$\left(\frac{F}{N} - E_o\right) = m\,E_g$$ ⟷ A straight-line equation with a slope =m



c. N and *m* are unknown

The equation above can be rearranged to give:

$$\frac{F}{E_o} = N + m\,N\left(\frac{E_g}{E_o}\right)$$ ⟷ A straight-line equation with a slope =mN and an intercept = N.



Homework

The production history and the PVT data of a gas-cap-drive reservoir are given below:

Date	$\bar{p}$ psi	N_p MSTB	G_p Mscf	B_t bbl/STB	B_g bbl/scf
5/1/89	4415	—	—	1.6291	0.00077
1/1/91	3875	492.5	751.3	1.6839	0.00079
1/1/92	3315	1015.7	2409.6	1.7835	0.00087
1/1/93	2845	1322.5	3901.6	1.9110	0.00099

The initial gas solubility R_{si} is 975 scf/STB. ***Estimate the initial oil and gas in place***.

Note: Show your results on a graph paper and the method used to calculate the slope and intercept.

Case 4. Water-Drive Reservoirs

- In a water drive reservoir, identifying the type of the aquifer and characterizing its properties are perhaps the most challenging tasks involved in conducting a reservoir engineering study.
- The full MBE can be expressed as:

$$F = N\left(E_o + m\,E_g + E_{f,w}\right) + W_e$$

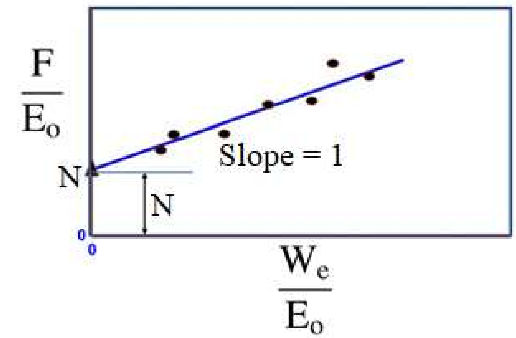
- Dake (1978) pointed out that the term *$E_{f,w}$ can frequently be neglected in water-drive reservoirs*. This is not only for the usual reason that *the water and pore compressibilities are small*, but also because a *water influx helps to maintain the reservoir pressure* and, therefore, the *Δp appearing in the $E_{f,w}$ term is reduced*. If, in addition, the reservoir has no an initial gas-cap, then the equation becomes:

$$F = N\,E_o + W_e$$

The equation can be rearranged and expressed as:

$$\frac{F}{E_o} = N + \frac{W_e}{E_o}$$

A straight-line equation with a slope =1 and an intercept = N.



- Different models can be used to calculate the water influx (We).

Example

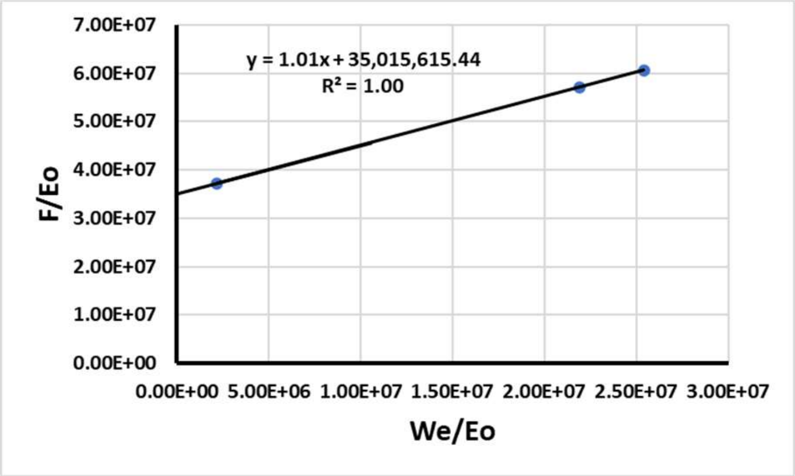
An undersaturated reservoir with data given below. Assuming that the rock and water compressibilities are negligible, calculate the initial oil-in-place (N).

p, psi	F, bbl	Eo, bbl/STB	We, bbl
3500	-	-	
3488	2.04E+06	0.0548	119800.8
3162	8.77E+06	0.154	3374389
2782	1.71E+07	0.282	7168081

Solution

P, psi	F/Eo, STB	We/Eo, STB
3500		
3488	**37226277**	**2186145.985**
3162	**56948052**	**21911616.88**
2782	**60638298**	**25418726.95**

From graph, N= 35.02 MMSTB



THANK YOU