Al-Ayen University

College of Petroleum Engineering

Numerical Methods and Reservoir Simulation

Lecturer: Dr. Mohammed Idrees Al-Mossawy

Lecture 3: Basic Equations of Fluid Flow in Porous Media (Part 1)

Outlines

- Introduction
- Types of fluids
- Flow Regimes
- Flow geometry
- Number of flowing fluids in the reservoir

Introduction

To formulate a mathematical model of reservoir simulation, the primary reservoir characteristics that need to be considered include:

- Types of fluids in the reservoir
- Flow regimes
- Flow geometry
- Number of flowing fluids in the reservoir

Types of fluids

- The isothermal compressibility coefficient is essentially the controlling factor in identifying the type of the reservoir fluid.

- The isothermal compressibility coefficient (c) is described mathematically by the following two equivalent expressions:

In terms of fluid volume: $$c = \frac{-1}{V}\frac{\partial V}{\partial p} \qquad \text{……………………..(1)}$$

In terms of fluid density: $$c = \frac{1}{\rho}\frac{\partial \rho}{\partial p} \qquad \text{……………………..(2)}$$

In general, reservoir fluids are classified into three groups:

1. **Incompressible fluids**

$$\frac{\partial V}{\partial p} = 0 \quad \text{and} \quad \frac{\partial \rho}{\partial p} = 0$$

Incompressible fluids do not exist; this behavior, however, may be assumed in some cases to simplify the derivation and the final form of many flow equations.

2. **Slightly compressible fluids**

These "slightly" compressible fluids exhibit small changes in volume, or density, with changes in pressure. Integrating Equation 1 gives:

$$-c \int_{p_{ref}}^{p} dp = \int_{V_{ref}}^{V} \frac{dV}{V}$$

$$e^{c(p_{ref} - p)} = \frac{V}{V_{ref}}$$

$$V = V_{ref}\, e^{c(p_{ref} - p)}$$

where p = pressure, psia

V = volume at pressure p, ft^3

p_{ref} = initial (reference) pressure, psia

V_{ref} = fluid volume at initial (reference) pressure, psia

The e^x may be represented by a series expansion as:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$

Because the exponent x [which represents the term $c\,(p_{ref} - p)$] is very small for slightly compressible fluids, the e^x term can be approximated by truncating Equation to:

$$e^x = 1 + x$$

$$V = V_{ref}\,[1 + c\,(p_{ref} - p)]$$

A similar derivation is applied to density:

$$\rho = \rho_{ref}\,[1 - c\,(p_{ref} - p)]$$

3. Compressible Fluids

The isothermal compressibility of any compressible fluid is described by the following expression:

$$c_g = \frac{1}{p} - \frac{1}{z}\left(\frac{\partial z}{\partial p}\right)_T$$

Figures 1 and 2 show schematic illustrations of the volume and density changes as a function of pressure for the three types of fluids.

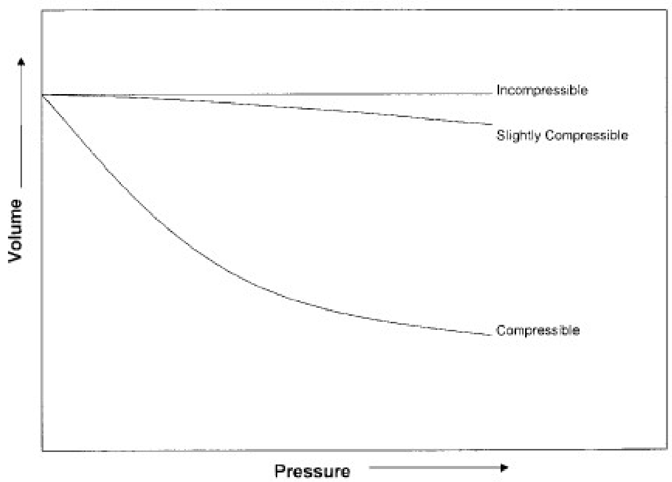


Figure 1. Pressure-volume relationship.

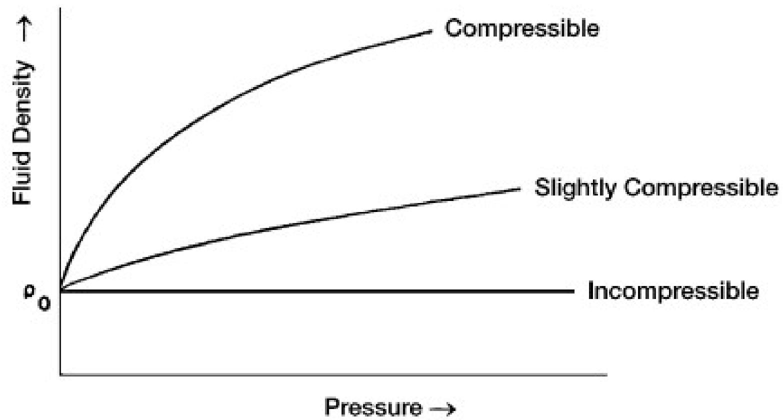


Figure 2. Fluid density versus pressure for different fluid types.

Flow Regimes

There are three flow regimes:

1. **Steady-State Flow**

$$\left(\frac{\partial p}{\partial t}\right)_i = 0$$

2. **Unsteady-State Flow (frequently called *transient flow*)**

$$\left(\frac{\partial p}{\partial t}\right) = f(i,t)$$

3. **Pseudo steady-State Flow (or semisteady-state flow)**

$$\left(\frac{\partial p}{\partial t}\right)_i = \text{constant}$$

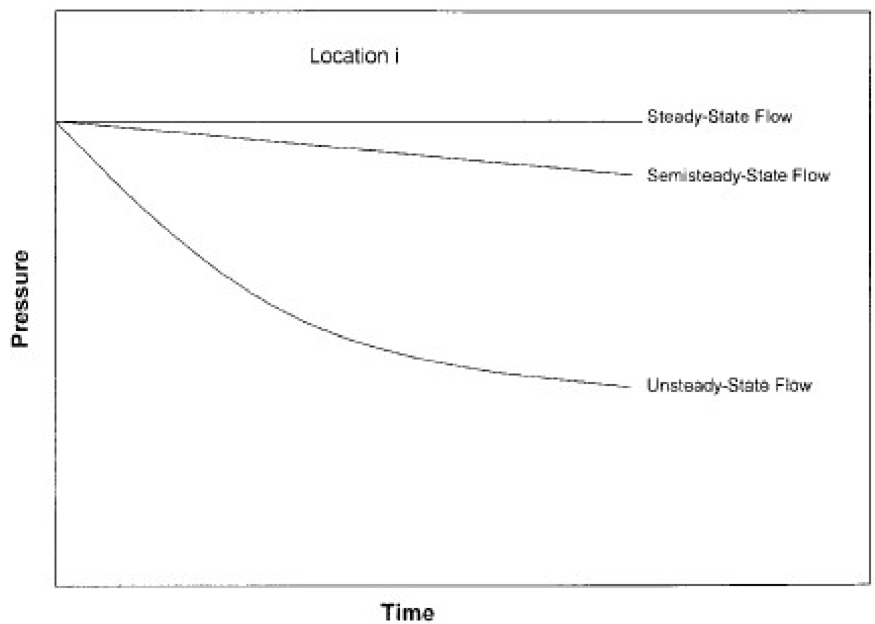
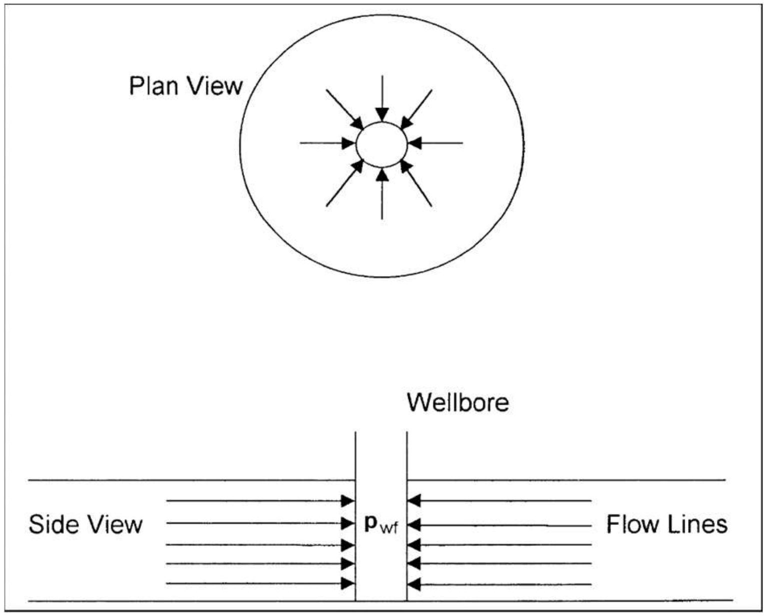


Figure 3. Flow regimes.

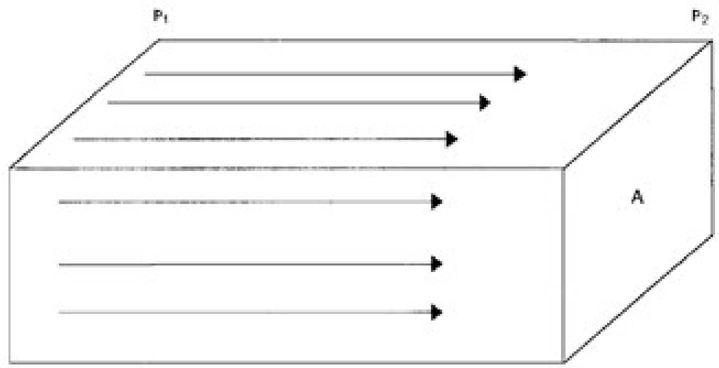
Flow geometry

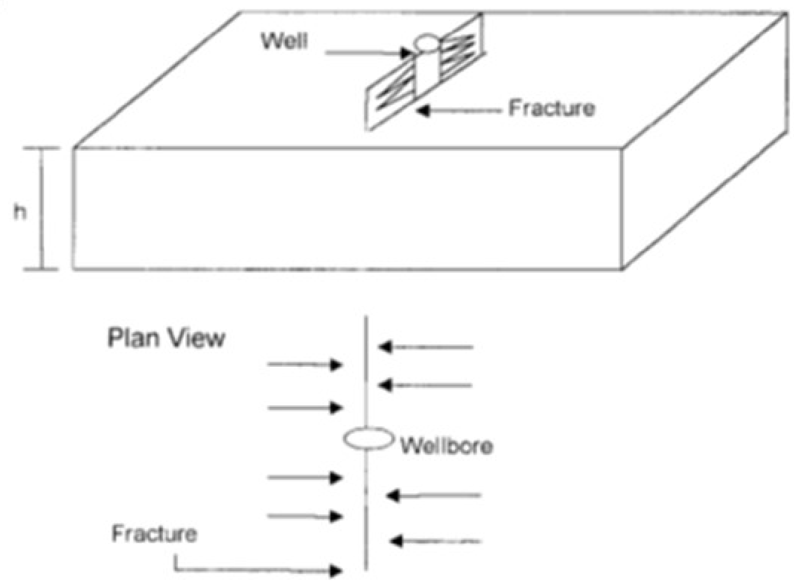
The actual flow geometry may be represented by one of the following flow geometries:

1. Radial Flow

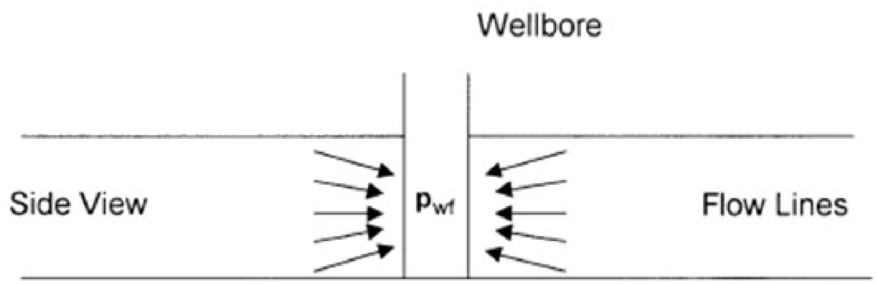


2. Linear Flow

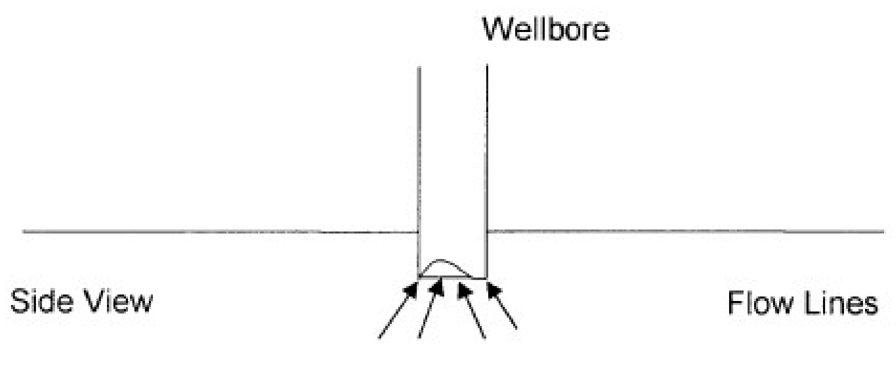




3. Spherical and Hemispherical Flow



Spherical flow due to limited entry



Hemispherical flow in a partially penetrating well.

Number of flowing fluids in the reservoir

There are generally three cases of flowing systems:

- Single-phase flow (oil, water, or gas)
- Two-phase flow (oil-water, oil-gas, or gas-water)
- Three-phase flow (oil, water, and gas)

The description of fluid flow and subsequent analysis of pressure data becomes more difficult as the number of mobile fluids increases.

THANK YOU