Lecture Five
Flow of natural gas in wells

5.1 Basic flow equation

The theoretical basis for most fluid flow equations is the general energy equation, an expression for the balance or conservation of energy between two points in a system. The energy equation is developed first, and using thermodynamic principles, is modified to a pressure gradient equation form.

The steady-state energy balance simply states that the energy of a fluid entering a control volume, plus any shaft work done on or by the fluid, plus any heat energy added to or taken from the fluid must equal the energy leaving the control volume.

Considering a steady-state system, the energy balance may be written as

$$U'_1 + p_1V_1 + \frac{mv_1^2}{2g_c} + \frac{m\,g\,h_1}{g_c} + q' + W'_s = U'_2 + p_2V_2 + \frac{m\,v_2^2}{2\,g_c} + \frac{m\,g\,h_2}{g_c} \quad \text{.......... (5-1)}$$

where

U' = internal energy,
pV = energy of expansion or compression,
$\frac{mv^2}{2g_c}$ = kinetic energy,
$\frac{m\,gh}{g_c}$ = potential energy,
q' = heat energy added to fluid, and
W'_s = work done on the fluid by the surroundings.

Dividing Equation 5-1 by m to obtain energy per unit mass balance and writing in differential form gives:

$$dU = d\left(\frac{p}{\rho}\right) + \frac{vdv}{g_c} + \frac{g}{g_c}dh + dq + d\,W_s = 0. \quad \text{......... (5-2)}$$

This form of the energy balance equation is difficult to apply because of the internal energy term and assuming no work is done on or by the fluid,

$$\frac{dp}{\rho} + \frac{v\,dv}{g_c} + \frac{g}{g_c}dh + dL_w = 0 \qquad \text{.............. (5-3)}$$

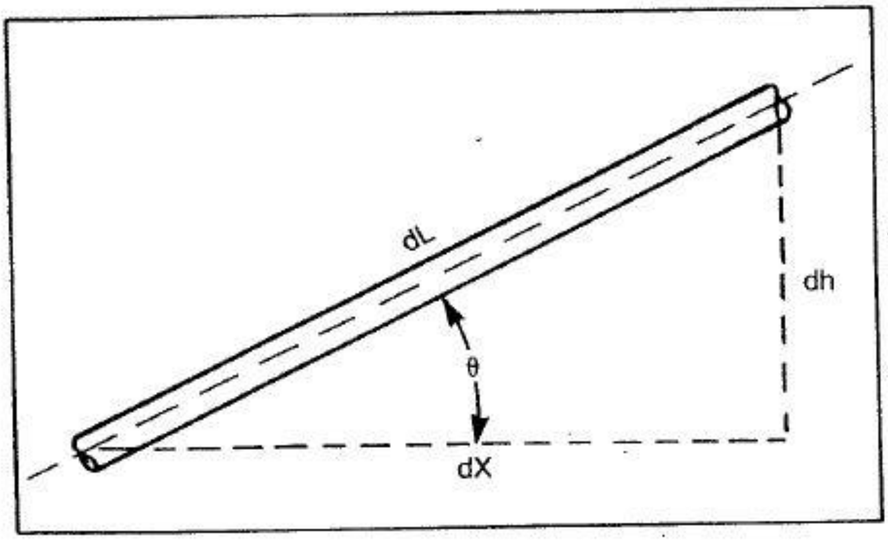

Figure 5-1: Flow geometry

If we consider a pipe inclined at some angle θ to the horizontal, as in Figure 5-1, since dh = dL sin θ

$$\frac{dp}{\rho} + \frac{v\,dv}{g_c} + \frac{g}{g_c}dL\sin\Theta + dL_w = 0.$$

Multiplying the equation by ρ/dL gives

$$\frac{dp}{dL} + \frac{\rho\,v\,dv}{g_c\,dL} + \frac{g}{g_c}\rho\sin\Theta + \rho\frac{dL_w}{dL} = 0. \qquad \text{................. (5-4)}$$

Equation 5-4 can be solved for pressure gradient, and if we consider a pressure drop as being positive in the direction of flow

$$\frac{dp}{dL} = \frac{g}{g_c}\rho\sin\Theta + \frac{\rho\,v\,dv}{g_c\,dL} + \left(\frac{dp}{dL}\right)_f, \qquad \text{............. (5-5)}$$

where

$$\left(\frac{dp}{dL}\right)_f = \rho\frac{d\,L_w}{dL}$$

is the pressure gradient due to viscous shear or friction losses.

In horizontal pipe flow the energy losses or pressure drops are caused by change in kinetic energy and friction losses only. Since most of the viscous shear occurs at the pipe wall, the ratio of wall shear stress (T_w.) to kinetic energy per unit volume (ρ $v^2/2$ gc) reflects the relative importance of wall shear stress to the total losses. This ratio forms a dimensionless group and defines a friction factor.

$$f' = \frac{\tau_w}{\rho v^2/2g_c} = \frac{2\tau_w g_c}{\rho v^2} \qquad \ldots\ldots\ldots\ldots (5\text{-}6)$$

To evaluate the wall shear stress, a force balance between pressure forces and wall shear stress can be formed. Referring to Figure 5-2,

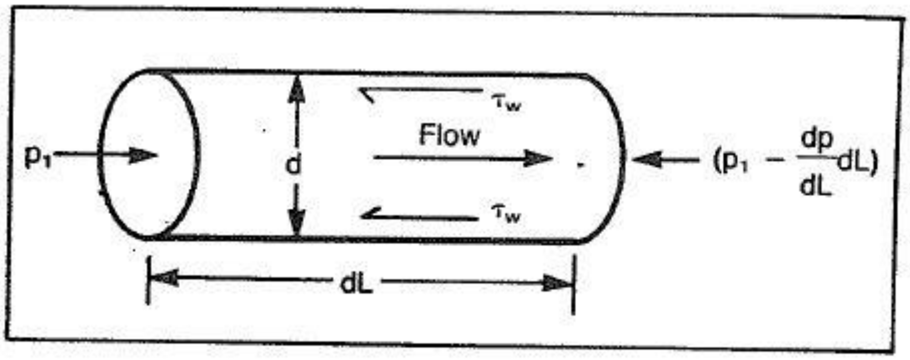

Figure 5-2: Force balance

$$\left[p_1 - \left(p_1 - \frac{dp}{dL}dL\right)\right]\frac{\pi d^2}{4} = \tau_w (\pi d) dL$$

$$\tau_w = \frac{d}{4}\left(\frac{dp}{dL}\right)_f. \qquad \ldots\ldots\ldots\ldots (5\text{-}7)$$

Substituting Equation 5-7 into Equation 5-6 and solving for the pressure gradient due to friction gives

$$\left(\frac{dp}{dL}\right)_f = \frac{2f'\rho v^2}{g_c d},$$

Which is the well-known Fanning equation, In terms of a Darcy-Weisbach or Moody friction factor, $f = 4f'$, and

$$\left(\frac{dp}{dL}\right)_f = \frac{f\rho v^2}{2 g_c d} \quad \text{........... (5-9)}$$

5.1.1 Laminar Single-Phase Flow

The friction factor for laminar flow can be determined analytically by combining Equation 5-9 with the **Hagen- Poiseuille** equation for laminar flow

$$v = \frac{d^2 g_c}{32\mu}\left(\frac{dp}{dL}\right)_f$$

or

$$\left(\frac{dp}{dL}\right)_f = \frac{32\mu v}{g_c d^2}.$$

Equating the expressions for frictional pressure gradient gives

$$\frac{32\mu v}{g_c d^2} = \frac{f\rho v^2}{2 g_c d}$$

or

$$f = \frac{64\mu}{\rho v d} = \frac{64}{N_{Re}}.$$

The dimensionless group, $\mathbf{N_{Re} = \rho v\, d/\mu}$ is the ratio of fluid momentum forces to viscous shear forces and is known as the Reynolds number. It is used as a parameter to distinguish between laminar and turbulent fluid flow. For engineering calculations, the dividing point between laminar and turbulent flow can be assumed to occur at a Reynolds number of **2100** for flow in a circular pipe. Using units of lbm/ft^3, ft/sec, ft and centipoise, the Reynolds number equation is

$$N_{Re} = \frac{1488\,\rho v d}{\mu}$$

5.1.2Turbulent Single-Phase Flow

The ability to predict flow behavior under turbulent flow conditions is a direct result of extensive experimental studies of velocity profiles and pressure gradients. These studies have shown that both velocity pro- file and pressure gradient are very sensitive to characteristics of the pipe wall. A logical approach to defining friction factors is to begin with the simplest case, i.e., the smooth-wall pipe, proceed to the partially rough wall, and finally to the fully rough wall. Only the most accurate empirical equations available for friction factors are presented here.

- ***Smooth-Wall Pipe***

For smooth-wall pipes, several equations have been developed, each valid over different ranges of Reynolds numbers. The most commonly used equation-since it is explicit in f and also covers a wide range of Reynolds numbers ($3000 < N_{Re} < 3 \times 10^6$) was presented by **Drew, Koo, and McAdams** in 1932.

$$f = 0.0056 + 0.5N_{Re}^{-0.32} \quad \text{.............. (5-10)}$$

An equation proposed by **Blasius** may be used for Reynolds numbers up to 100,000 for smooth pipes.

$$f = 0.316N_{Re}^{-0.25} \quad \text{................ (5-11)}$$

- ***Rough-Wall Pipe***

The inside wall of a pipe is not normally smooth, and in turbulent flow, the roughness can have a definite effect on the friction factor, and thus the pressure gradient. Wall roughness is a function of the pipe material, the method of manufacture, and the environment to which it has been exposed.

From a microscopic sense, wall roughness is not uniform. Individual protrusions, indentations, etc. vary in height, width, length, shape, and distribution. The absolute roughness of a pipe, €, is the mean protruding height of relatively uniformly distributed and sized, tightly packed sand grains that would give the same pressure gradient behavior as the actual pipe.

Dimensional analysis suggests that the effect of roughness is not due to its absolute dimensions, but rather to its dimensions relative to the inside diameter of the pipe, €/d. In turbulent flow, the effect of wall roughness has been found to be dependent on both the relative roughness and on the Reynolds number. If the laminar sub- layer that exists within the boundary layer is thick enough, the behavior is similar to a smooth pipe. The sublayer thickness is directly related to the Reynolds number.

Nikuradse's famous sand grain experiments formed the basis for friction factor data from rough pipes. His correlation for fully rough-wall pipe is still the best one available. The friction factor may be calculated explicitly from

$$\frac{1}{\sqrt{f}} = 1.74 - 2\,\text{Log}\left(\frac{2\,\epsilon}{d}\right). \qquad \text{…………… (5-12)}$$

The equation that is used as the basis for modern friction factor charts was proposed by **Colebrook and White** in 1939.

$$\frac{1}{\sqrt{f}} = 1.74 - 2\,\text{Log}\left(\frac{2\,\epsilon}{d} + \frac{18.7}{N_{Re}\sqrt{f}}\right) \qquad \text{…………… (5-13)}$$

The friction factor cannot be extracted readily from the Colebrook equation. By rearranging the equation as follows, a trial-and-error procedure may be used to solve the equation for friction factor.

$$f_c = \left\{\frac{1}{1.74 - 2\,\text{Log}\left(\frac{2\epsilon}{d} + \frac{18.7}{N_{Re}\sqrt{f_g}}\right)}\right\}^2$$

Values of f_g are estimated and then f_c is calculated until fg and fc. agree to an acceptable tolerance. Using the Drew, Koo, and McAdams equation as an initial guess is recommended. After each unsuccessful iteration, the calculated value becomes the assumed value for the next iteration. Also, if more than one pressure loss calculation is to be made as in the case of the iterative procedures discussed in later sections, then the "converged" value of the previous calculation should be used for the initial guess in the next calculation. Convergence using this method is rapid, normally taking only two or three iterations. The variation of single-phase friction factor with Reynolds number and relative roughness is shown graphically in Figure 5-3. The Colebrook equation may be applied to flow problems in the smooth, transition, and fully rough zones of turbulent flow. For large values of Reynolds number, it degenerates to the Nikuradse equation.

An explicit friction factor equation was proposed by **Jain** and compared in accuracy to the Colebrook equation. Jain found that for a range of relative roughness between 10^{-6} and 10^{-2} and a range of Reynolds number between 5 x 10^3 and 10^8 the errors were within (± 1.0%) when compared with the Colebrook equation. The equation gives a maximum error of 3% for Reynolds numbers as low as 2000. The equation is

$$\frac{1}{\sqrt{f}} = 1.14 - 2\,\mathrm{Log}\left(\frac{\epsilon}{d} + \frac{21.25}{N_{Re}^{0.9}}\right). \quad \ldots\ldots\ldots\ldots\ldots\ldots (5\text{-}14)$$

Equation 5-14 is recommended for all calculations requiring a friction factor determination for turbulent flow. It is much easier to use than Equation 5-13 and, since the value of e will usually not be known to any high degree of accuracy, will give satisfactory results.

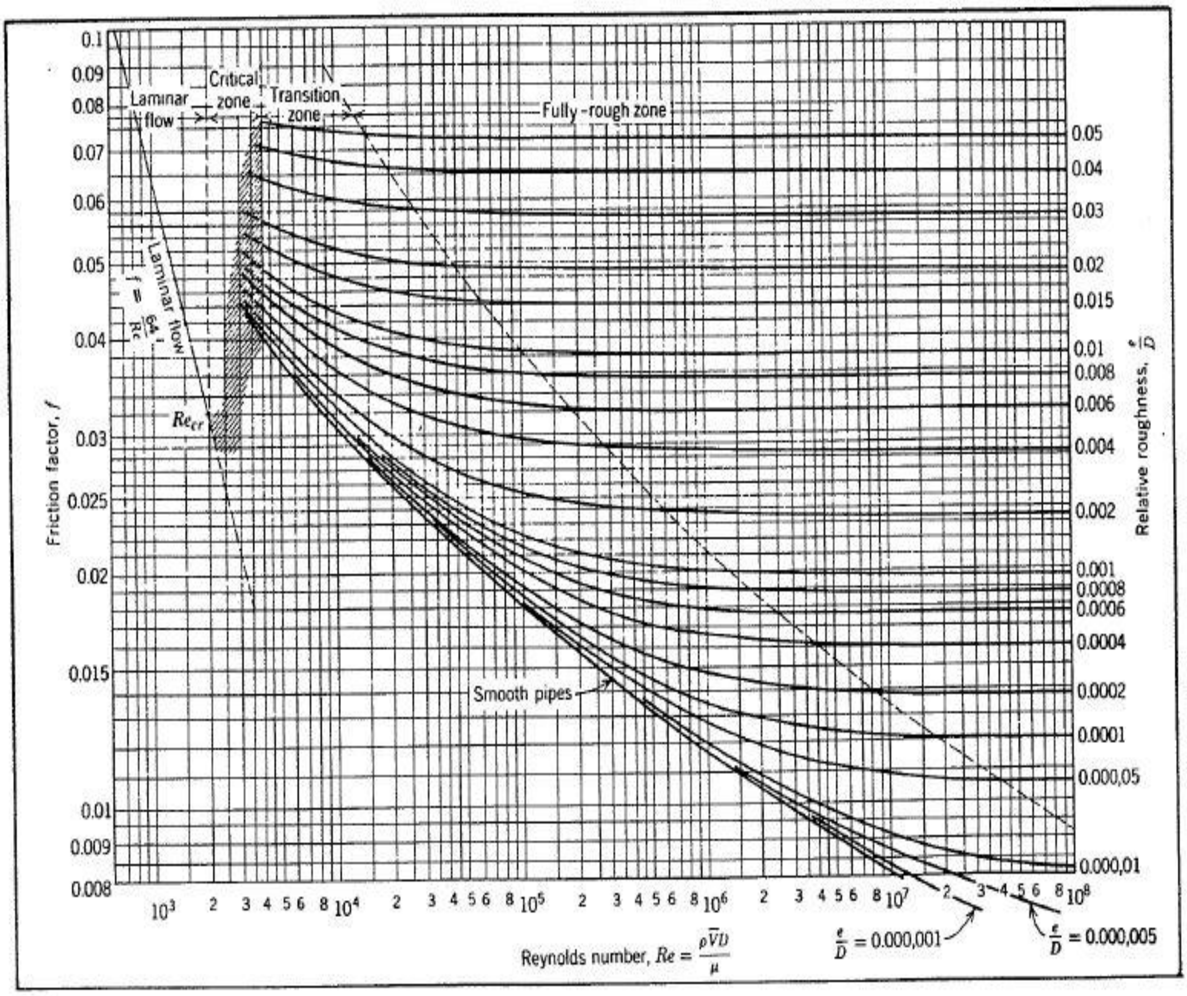

Figure 5-3: friction factor for fully developed flow in circular pipes

The determination of the value to use for pipe wall roughness in the friction factor equations is sometimes difficult. It is important to emphasize that e is not a property that is physically measured. Rather, it is the sand grain roughness that would result in the same friction factor. The only way this can be evaluated is by comparison of the behavior of a normal pipe with one that is sand-roughened. Moody has done this, and his results, given in Figure 5-4, are still the accepted values. These values should not be considered inviolate and could change significantly by such things as paraffin deposition, erosion, or corrosion. Thus, if measured pressure gradients are available, a friction factor and Reynolds number can be calculated, and an effective €/d obtained from the Moody diagram. This value of €/d should then be used for future predictions until updated again. If no information on roughness is available, a value of € = 0.0006 ft is recommended for tubing and line pipe that has been in service for some time.

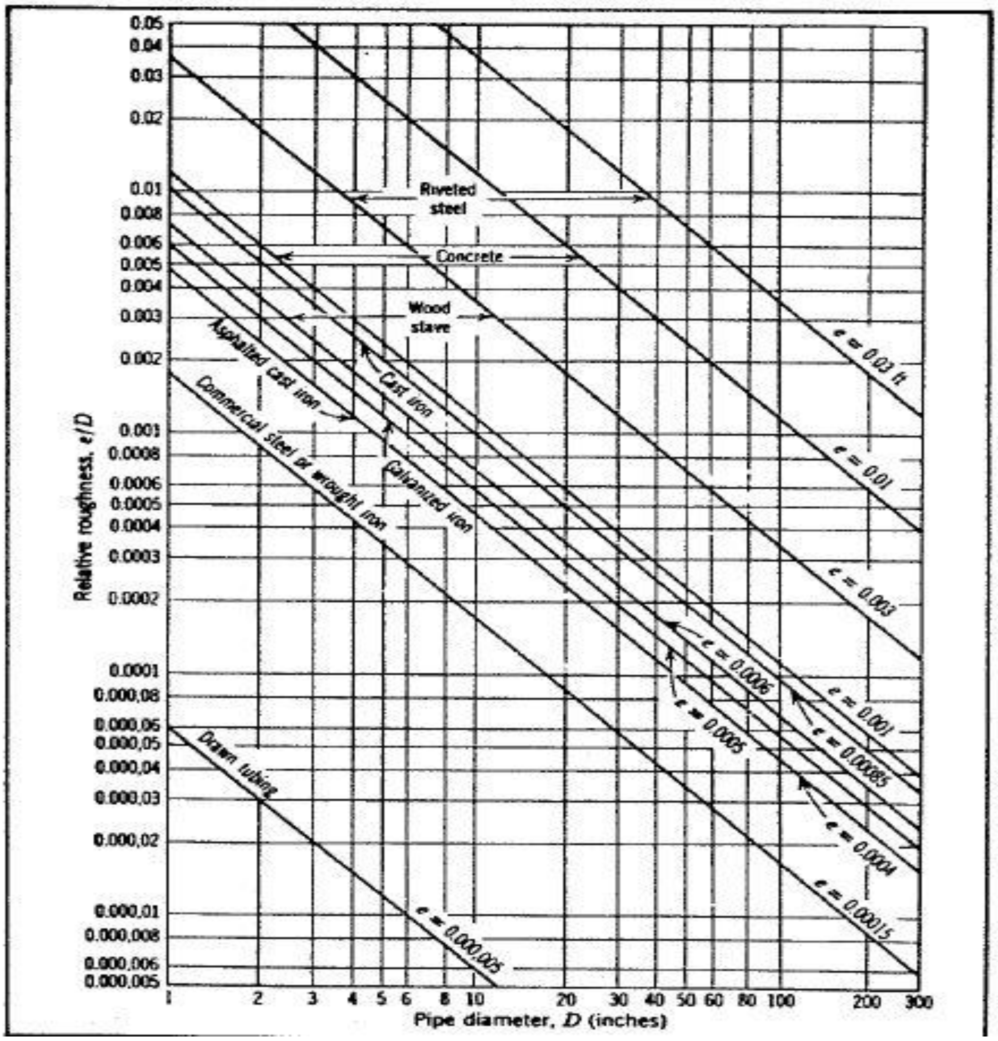

Figure 5-4: Relative roughness values for pipes of common engineering materials.

Example 5-1:

A liquid of specific gravity 0.82 and viscosity of 3 cp (.003 kg/m-sec) flows in a 4 in. (101.6 mm) diameter pipe at a velocity of 30 ft/sec (9.14 m/sec). The pipe material is new commercial steel. Calculate the friction factor using both the **Colebrook** equation and the **Jain** equation.

Solution:

From Figure 5-4 for commercial steel, €/d = 0.00045 Colebrook Solution: Use the Drew, Koo and McAdams equation for a first guess.

$N_{Re} = \rho\ v\ d/\mu = (820)(9.14)(0.1016)/0.003 = 253{,}824$

$$f_g = 0.0056 + 0.5\ N_{Re}^{-0.32} = 0.0056 + 0.5\ (253{,}824)^{-0.32}$$

$$f_g = 0.015$$

$$f_c = \left[1.74 - 2\ \text{Log}\left(\frac{2\epsilon}{d} + \frac{18.7}{N_{Re}\sqrt{f_g}}\right)\right]^{-2}$$

$$f_c = \left[1.74 - 2\ \text{Log}\left(2(.00045) + \frac{18.7}{253{,}824\ \sqrt{.015}}\right)\right]^{-2}$$

$$f_c = 0.0183$$

This value is not close enough to f_g; therefore another trial is required using $f_g = 0.0183$.

$$f_c = \left[1.74 - 2\ \text{Log}\left(2(.00045) + \frac{18.7}{253{,}824\ \sqrt{.0183}}\right)\right]^{-2}$$

$$f_c = 0.0182$$

A third trial using $f_g = 0.0182$ gives $f_c = 0.0182$.

Jain Solution:

$$f = \left[1.14 - 2\,\text{Log}\left(\frac{\epsilon}{d} + \frac{21.25}{N_{Re}^{0.9}}\right)\right]^{-2}$$

$$f = \left[1.14 - 2\,\text{Log}\left(0.00045 + \frac{21.25}{(253{,}824)^{0.9}}\right)\right]^{-2}$$

$$f = 0.0183$$

Combining Equations 5-5 and 5-9, the pressure gradient equation, which is applicable to any fluid at any pipe inclination angle, becomes

$$\frac{dp}{dL} = \frac{g}{g_c}\rho \sin\Theta + \frac{f\rho v^2}{2g_c d} + \frac{\rho\, v\, dv}{g_c\, dL}, \quad \ldots\ldots\ldots\ldots\ldots (5\text{-}15)$$

Where the friction factor, f, is a function of Reynolds number and pipe roughness. This relationship is shown in the Moody diagram (Fig. 5-3). The total pressure gradient can be considered to be composed of three distinct components; that is,

$$\frac{dp}{dL} = \left(\frac{dp}{dL}\right)_{el} + \left(\frac{dp}{dL}\right)_{f} + \left(\frac{dp}{dL}\right)_{acc}, \quad \ldots\ldots\ldots\ldots (5\text{-}16)$$

where

$$\left(\frac{dp}{dL}\right)_{el} = \frac{g}{g_c}\rho \sin\Theta$$

is the component due to potential energy or elevation change. It is also referred to as the hydrostatic component since it is the only component that would apply at conditions of no flow.

$$\left(\frac{dp}{dL}\right)_f = \frac{f\rho v^2}{2g_c d}$$

is the component due to friction losses.

$$\left(\frac{dp}{dL}\right)_{acc} = \frac{\rho\, v\, dv}{g_c\, dL}$$

is the component due to kinetic energy change or convective acceleration. Equation 5-15 applies for any fluid in steady-state, one-dimensional flow for which f, ρ, and v can be defined.