

## Lecture Four

### Mathematical and physical principles for pressure drop calculations

#### 4.1 Introduction

When two or more phase flows simultaneously in pipes, the flow behavior is much more complex than for single phase flow. The phases tend to separate because of differences in density. Shear stresses at the pipe wall different for each phase as a result of their different densities and viscosities. Expansion of the highly compressible gas phase with decreasing pressure increases the in-situ volumetric flow rate of the gases. As a result, the gas and liquid phases normally do not travel at the same velocity in the pipe. For upward flow, the less dense, more compressible, less viscous gas phase tends to flow at higher velocity than the liquid phase, causing a phenomenon known as slippage. However, for downward flow, the liquid often flows faster than the gas.

Perhaps the most distinguishing aspect of multiphase flow is the variation in the physical distribution of the phases in the flow conduit, a characteristic known as flow pattern or flow regime. During multiphase flow through pipes, the flow pattern that exists depends on the relative magnitude of the forces that act on the fluids. Buoyancy, turbulence, inertia, and surface-tension forces vary significantly with flow rates, pipe diameter, inclination and fluid properties of the phases. Several different flow patterns can exist in a given well as a result of the large pressure and temperature changes the fluids encounter. Especially important is the significant variation in pressure gradient with flow pattern. Thus, the ability to predict flow pattern as a function of the flow parameters is of primary concern.

Analytical solutions are available for many single phase flow problem. Even when empirical correlations for necessary (i.e. for turbulent flow friction

factors), the accuracy of prediction were excellent. The increased complexity of multiphase flow logically resulted in a higher degree of empiricism for predicting flow behavior. Many empirical correlations have been developed to predict flow pattern, slippage between phases, friction factors and other such parameters for multiphase flow in pipes. Virtually all the existing standard design methods rely on these empirical correlations. However, since the mid-1970, dramatic advances have taken place that improves understanding of the fundamental mechanisms that govern multiphase flow. These have resulted in new predictive methods that rely much less on empirical correlations.

## **4.2 Oil Models**

Two approaches have been used to simulate mass transfer for hydrocarbon systems: the "black- oil" or constant composition model and the (variable) compositional model. Each is described in the following sections.

### **4.2.1 Black-Oil Model**

The term black oil is a misnomer and refers to any liquid phase that contains dissolved gas, such as hydrocarbons produced from oil reservoirs. These oils are typically dark in color, have gravities less than 40° API and undergo relatively small changes in composition within the two phase envelope. A better description of the fluid system is constant- composition model.

For black oils with associated gas, a simplified parameter has been defined to account for gas that dissolves (condenses) or evolves (boils) from solution in the oil. This parameter  $R_s$  (gas-oil ratio in solution), second parameter called oil formation volume factor,  $B_o$ , also has been defined to describe the shrinkage or expansion of the oil phase. Once the black oil model parameters are known, oil density and other physical properties of the two phases can be calculated.

### 4.2.2 Compositional model

For volatile oils and condensate fluid, vapor/liquid equilibrium (VLE) or flash calculations are more accurate to describe mass transfer than black-oil model parameters.

Given the composition of a fluid mixture or "feed" a VLE calculation will determine the amount of the feed that exists in the vapor and liquid phases and composition of each phase. From these results, it is possible to determine the quality or mass fraction of gas in the mixture.

VLE calculations are considered more rigorous than black-oil model parameters to describe mass transfer. However, they also are much more difficult to perform. If a detailed composition is available for a gas/oil system, it is possible to generate black-oil parameters from VLE calculations. However, the nearly constant composition those results for the liquid phase and the increased computation requirements make the black-oil model more attractive for nonvolatile oils.

### 4.2.1 Volumetric Flow Rates

After mass transfer calculations are completed, it is possible to calculate the in-situ volumetric flow rates of each phase. For the black-oil model, volumetric flow rates are determined from

$$q_o = q_{o_{sc}} B_o \dots\dots\dots (4.1)$$

$$q_w = q_{w_{sc}} B_w \dots\dots\dots (4.2)$$

and

$$q_g = \left( q_{g_{sc}} - q_{o_{sc}} R_s - q_{w_{sc}} R_{sw} \right) B_g \dots\dots\dots (4.3)$$

Where  $B_g$  is derived from the engineering equation of state to be

$$B_g = p_{sc} ZT / p Z_{sc} T_{sc} \dots\dots\dots (4.4)$$

For the compositional model, volumetric flow rates are calculated from

$$q_L = \frac{w_i(1 - x_g)}{\rho_L} \dots\dots\dots (4.5)$$

and

$$q_g = w_i x_g / \rho_g \dots\dots\dots (4.6)$$

Where  $x_g$  is the no-slip quality or gas mass fraction and is obtained from the results of a VLE calculation as follows.

$$x_g = \frac{VM_g}{VM_g + LM_L} \dots\dots\dots (4.7)$$

If free water exists when the compositional model is used, the water flow rate must be added to  $q_L$  to account for all the liquid.

#### Example 4.1: Compositional-Model Flow Rates,

A gas-condensate well is flowing at a rate of 500,000 lbm/D. At a given location in the pipe, a VLE calculation is performed on the gas composition, yielding

$$L = 0.05 \left( \frac{\text{mole liquid}}{\text{mole feed}} \right) \quad V = 0.95 \left( \frac{\text{mole vapor}}{\text{mole feed}} \right).$$

$$M_L = 100 \left( \frac{\text{lbm}}{\text{mole liquid}} \right) \quad M_{gV} = 20 \left( \frac{\text{lbm}}{\text{mole vapor}} \right).$$

and

$$\rho_L = 50 \text{ lbm/ft}^3 \quad \rho_g = 5.0 \text{ lbm/ft}^3.$$

#### Solution:

From Eq 4-7

$$\begin{aligned} x_g &= \frac{(0.95)(20.0)}{(0.95)(20.0) + (0.05)(100.0)} \\ &= 0.792 \text{ lbm vapor/lbm mixture.} \end{aligned}$$

With Eq 4-5

$$q_L = \frac{(500,000)(1 - 0.792)}{(86,400)(50)} = 0.024 \text{ ft}^3/\text{sec.}$$

With Eq. 4-6

$$q_g = \frac{(500,000)(0.792)}{(86,400)(5.0)} = 0.917 \text{ ft}^3/\text{sec.}$$

## 4.3 Definition of variables

When performing multiphase-flow calculations, single-phase-flow equations often are modified to account for the presence of a second phase. This involves defining mixture expressions for velocities and fluid properties that use weighting factors based on either Volume or mass fractions. The choice of variables and weighting factors often depends on the predicted flow pattern. The following sections discuss each parameter,

### 4.3.1 Weighting factors

When gas and liquid flow simultaneously up a well, the higher mobility of the gas phase tends to make the lower density and viscosity of the gas. Under steady-state conditions, this results in a reduced area fraction for the gas phase and an expanded area fraction for the liquid phase. Thus, slippage of gas past liquid results in larger in-situ liquid volume fractions than would be present if the two phases flowed at the same velocity. **Liquid holdup** can be defined as the fraction situ liquid volume fraction of a pipe cross section or volume increment that is occupied by the liquid phase's. It can be estimated from a variety of equations developed from experimental data for a limited range of flow conditions. A more common procedure is to develop empirical correlations to predict liquid Holdup,  $H_L$ , for a broad range of flow conditions.

For the case of equal phase velocities, or no-slip conditions, the volume fraction of liquid in the pipe can be calculated analytically from knowledge of the in situ volumetric flow rates given in the previous sections. Thus,

$$\lambda_L = q_L / (q_L + q_g) \dots\dots\dots (4.8)$$

Where  $q_L$  is the sum of oil and water flow rates for the black oil model, or is given by Eq. 4.5 for the compositional model. If free water exists when the

compositional model is used, the water flow rate must be added to the oil or condensate flow rate to account for all the liquid. Because the no-slip liquid holdup can be determined rigorously, it commonly is used as a correlating parameter to predict other multiphase-flow parameters, such as  $H_L$ .

When oil and water flow simultaneously in pipes, with or without gas, it is possible for slippage to occur between the oil and water phases. This type of slippage is normally small compared with the slippage that can occur between gas and liquid. However, slippage can be important when velocities are low, especially in horizontal wells where stratified flow can occur. Assuming no slippage, the oil fraction in a liquid phase is calculated from

$$f_o = \frac{q_o}{q_o + q_w} \dots\dots\dots (4.9)$$

The water cut,  $f_w$ , based on in-situ rather than stock-tank flow rates, is simply (1- $f_o$ ).

### 4.3.2 Velocities

Velocities, individual phase velocities normally are quite different. Only for the cases of the highly turbulent, Dispersed –bubble – flow pattern and high velocity annular flow pattern in which the fluids exist as a homogeneous mixture are the phase velocities essentially equal. For all other cases significant slippage can occur between the gas and liquid .under steady state flow condition, slippage will result in a disproportion amount of the slower phase being present at any given location in the well. The in-situ volume fraction of liquid,  $H_L$  must be determined from empirical correlations or mechanism models. **Superficial velocities** which assume a given phase occupies the entire pipe area are important correlating parameter. Thus

$$V_{SL} = q_L / A_P \dots\dots\dots (4.10)$$

And

$$V_{Sg} = q_g / A_p \dots\dots\dots (4.11)$$

A total or mixture velocity then can be defined as

$$V_m = V_{SL} + V_{Sg} \dots\dots\dots (4.12)$$

**if there were no slip between phases**, both the gas and liquid would flow at the mixture velocity .because of the slip between phase, the liquid typically flow at a velocity less than mixture velocity, while the gas flow at a velocity greater than the mixture velocity time and space averaged velocity for each phase can be calculated from a knowledge of the time and space averaged liquid hold up obtained from the empirical correlation. Thus

$$V_L = V_{SL} / H_L \dots\dots\dots (4.13)$$

And

$$V_g = V_{sg} / (1 - H_L) \dots\dots\dots (4.14)$$

A slip of other velocity can be defined as the difference between the actual phase velocities.

$$V_s = V_g - V_L \dots\dots\dots (4.15)$$

### **Example 4.2: Superficial Velocities: Black-Oil Model.**

An oil well is flowing 10,000 STBO/D with a producing gas/oil ratio of 1,000 scf/STBO or a gas-production rate of 10 MMscf/D. At a location in the tubing where the pressure and temperature are 1,700 psia and 180°F, calculate the in-situ volumetric flow rates and superficial velocities of the liquid and gas phases. Also calculate the mixture velocity and the no-slip liquid holdup. The following is known from the pressure/volume/temperature (PVT)

$$\begin{aligned}
 B_o &= 1.197 \text{ bbl/STBO} \\
 B_g &= 0.0091 \text{ ft}^3/\text{scf} \\
 R_s &= 281 \text{ scf/STBO} \\
 d &= 6.0 \text{ in.}
 \end{aligned}$$

**Solution:**

$$A_p = \frac{\pi d^2}{4} = \frac{\pi (6)^2}{4} = 0.196 \text{ ft}^2.$$

With Eq. 4.1

$$\begin{aligned}
 q_o &= \frac{(10,000 \text{ STBO/D})(1.197 \text{ bbl/STBO})(5.614 \text{ ft}^3/\text{bbl})}{86,400 \text{ sec/D}} \\
 &= 0.778 \text{ ft}^3/\text{sec}.
 \end{aligned}$$

With Eq. 4.10

$$v_{SL} = \frac{q_L}{A_p} = \frac{0.778}{0.196} = 3.97 \text{ ft/sec.}$$

With Eq. 4.3

$$q_g = \frac{[10 \times 10^6 - (10,000)(281)](0.0091)}{86,400} = 0.757 \text{ ft}^3/\text{sec.}$$

With Eq. 4.11

$$v_{Sg} = q_g/A_p = 0.757/0.196 = 3.86 \text{ ft/sec.}$$

With Eq. 4.12

$$v_m = v_{SL} + v_{Sg} = 3.97 + 3.86 = 7.83 \text{ ft/sec.}$$

With Eq. 4.8

$$\lambda_L = \frac{q_L}{q_L + q_g} = \frac{0.778}{0.778 + 0.757} = 0.507.$$

**4.3.3 Fluid properties**

Regardless of whether the black-oil or the compositional model is used, it is possible to calculate the density and viscosity of each phase (oil, water, gas) and gas/oil and gas/water surface tension. A variety of methods have been used to define mixture fluid properties.



**Oil/Water Mixture:**

For the black-oil-model case, and for the compositional-model case when free water exists, oil and water properties are combined under the assumption that there is no slip- page between the oil and water.

$$\rho_L = \rho_o f_o + \rho_w f_w \dots\dots\dots (4.16)$$

$$\sigma_L = \sigma_o f_o + \sigma_w f_w \dots\dots\dots (4.17)$$

and

$$\mu_L = \mu_o f_o + \mu_w f_w \dots\dots\dots (4.18)$$

**Gas/liquid mixture:**

Numerous equations have been proposed to describe the physical properties of gas/liquid mixture. In general these equations are referred to as "slip "or "no slip" properties depending on whether  $H_L$  and  $\lambda_L$  is used as the volumetric weighting factor. Thus, for the two phase viscosity,

$$\mu_s = \mu_L H_L + \mu_g (1 - H_L) \dots\dots\dots (4.19)$$

Or

$$\mu_s = (\mu_L^{HL}) * (\mu_g^{(1-HL)}) \dots\dots\dots (4.20)$$

And

$$\mu_n = \mu_L \lambda_L + \mu_g (1 - \lambda_L) \dots\dots\dots (4.21)$$

Although Eq. 4.19 has never been used in multiphase flow design correlation, Eq.4.20 is used by the Hagedorn and Brown correlation. All other empirical correlation use Eq.4.21

The following expressions have been used to calculated multiphase flow mixture densities.

$$\rho_s = \rho_L H_L + \rho_g (1 - H_L) \dots\dots\dots (4.22)$$

$$\rho_n = \rho_L \lambda_L + \rho_g (1 - \lambda_L) \dots\dots\dots (4.23)$$

### 4.3.4 Liquid holdup

Liquid holdup ( $H_L$ ) is defined as the fraction of a pipe cross section or volume increment that is occupied by the liquid phase. The value of  $H_L$  ranges from 0 (total gas) to 1 (total liquid). The liquid holdup is defined by

$$H_L = A_L / A_p \dots\dots\dots (4.24)$$

Where  $A_L$  is the pipe area of the liquid occupied by the liquid phase and  $A_p$  is pipe cross sectional area.