Al-Ayen University
College of Petroleum Engineering

Reservoir Engineering II

Lecturer: Dr. Mohammed Idrees Al-Mossawy

Lecture 3: MATERIAL BALANCE APPLIED TO OIL RESERVOIRS (Part 1)

Refs.: (1) ***Reservoir Engineering Handbook*** by **Tarek Ahmed**, Fifth Edition, Ch. 11

(2) ***Fundamentals of Reservoir Engineering*** by **LP. DAKE**, Seventeenth impression 1998, Ch. 3

Outlines

- INTRODUCTION
- BASIC ASSUMPTIONS OF THE MBE
- GENERAL FORM OF THE MBE
- RESERVOIR DRIVE INDICES
- EXAMPLE

INTRODUCTION

- The material balance equation (MBE) has long been recognized as one of the basic tools of reservoir engineers for interpreting and predicting reservoir performance.
- The general form of the material balance equation was first presented by Schilthuis in 1941.
- The MBE is considered as a zero-dimensional model.
- The MBE, when properly applied, can be used to:
  - Estimate initial hydrocarbon volumes in place
  - Predict future reservoir performance
  - Predict ultimate hydrocarbon recovery under various types of primary driving mechanisms
- In the simplest form of MBE , the equation can be written on volumetric basis as:

Initial volume = volume remaining + volume removed

BASIC ASSUMPTIONS OF THE MBE

- ***Constant temperature:*** Pressure-volume changes in the reservoir are assumed to occur without any temperature changes.
- ***Pressure equilibrium:*** All parts of the reservoir have the same pressure, and fluid properties are therefore constant throughout. Minor variations in the vicinity of the well bores may usually be ignored.
- ***Constant reservoir volume:*** Reservoir volume is assumed to be constant except for those conditions of rock and water expansion or water influx that are specifically considered in the equation.
- ***Reliable production data:*** All production data should be recorded with respect to the same time period. There are essentially three types of production data:
    - *Oil-production data*
    - *Gas-production data*
    - *Water-production data*

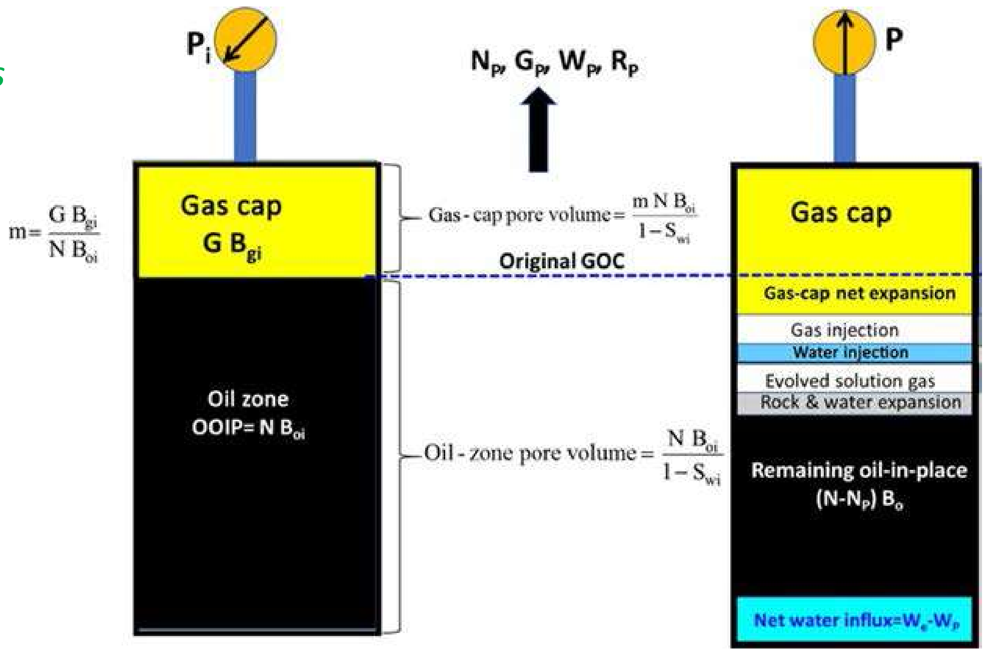
GENERAL FORM OF THE MBE

Initial volume = Volume remaining + Volume removed

Initial volume = *Pore volume occupied by the oil initially in place at pi* + *Pore volume occupied by the gas in the gas cap at pi*

Volume remaining + *Volume removed* =

Pore volume occupied by the remaining oil at p +
Pore volume occupied by the gas in the gas cap at p +
Pore volume occupied by the evolved solution gas at p +
Pore volume occupied by the net water influx at p+
Change in pore volume due to connate water expansion and pore volume reduction due to rock expansion +
Pore volume occupied by the injected gas at p +
Pore volume occupied by the injected water at p



The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties.

Pore volume occupied by the oil initially in place at pi:

Volume occupied by initial oil in place = N Boi

Where:
N = oil initially in place, STB
Boi = oil formation volume factor at initial reservoir pressure pi, bbl/STB

Pore volume occupied by the gas in the gas cap at pi:

Volume of gas cap= m N Boi

where m is a dimensionless parameter and defined as the ratio of gas-cap volume to the oil zone volume.

$$m = \frac{\text{Initial volume of gas cap}}{\text{Volume of oil initially in place}} = \frac{G\,B_{gi}}{N\,B_{oi}}$$

G = Initial volume of gas-cap gas, scf

Bgi = Initial gas formation volume factor at pi, bbl/scf

Pore volume occupied by the remaining oil at p:

$$\text{Volume of the remaining oil} = (N - N_p)\, B_o$$

Where:

N_p = cumulative oil production, STB
B_o = oil formation volume factor at reservoir pressure p, bbl/STB

Pore volume occupied by the gas in the gas cap at p:

- As the reservoir pressure drops to a new level p, the gas in the gas cap expands and occupies a larger volume.
- Assuming no gas is produced from the gas cap during the pressure decline, the new volume of the gas cap can be determined as:

$$\text{Volume of the gas cap at p} = \left[\frac{m\ N\ B_{oi}}{B_{gi}}\right] B_g$$

Where:

B_{gi} = gas formation volume factor at initial reservoir pressure, bbl/scf
B_g = current gas formation volume factor, bbl/scf

Pore volume occupied by the evolved solution gas at p:

$$\begin{bmatrix}\text{volume of the evolved} \\ \text{solution gas at p}\end{bmatrix} = \begin{bmatrix}\text{volume of gas initially} \\ \text{in solution}\end{bmatrix} - \begin{bmatrix}\text{volume of gas} \\ \text{produced}\end{bmatrix} - \begin{bmatrix}\text{volume of gas} \\ \text{remaining in solution}\end{bmatrix}$$

$$\begin{bmatrix}\text{volume of the evolved} \\ \text{solution gas at p}\end{bmatrix} = \left[N R_{si} - N_p R_p - (N - N_p) R_s\right] B_g$$

Where:

N_p = cumulative oil produced, STB
R_p = net cumulative produced gas-oil ratio, scf/STB → $R_p = \frac{G_p}{N_p}$
R_s = current gas solubility factor, scf/STB
B_g = current gas formation volume factor, bbl/scf
R_{si} = gas solubility at initial reservoir pressure, scf/STB
G_p = cumulative gas produced, scf

Pore volume occupied by the net water influx at p:

$$\text{net water influx} = W_e - W_p B_w$$

Where:

W_e = cumulative water influx, bbl
W_p = cumulative water produced, STB
B_w = water formation volume factor, bbl/STB

Change in pore volume due to connate water expansion and pore volume reduction due to rock expansion

- The component describing the reduction in the hydrocarbon pore volume due to the expansion of initial (connate) water and the reservoir rock *cannot be neglected for an undersaturated oil reservoir*.
- The effect of these two components (cw and cf), however, *can be generally neglected for gas-cap-drive reservoir* or when the *reservoir pressure drops below the bubble-point pressure*.

The compressibility coefficient, $c = \frac{-1}{V}\frac{\partial V}{\partial p}$ → $\Delta V = Vc\,\Delta p$

Where ΔV represents the net changes or expansion of the material as a result of changes in the pressure Δp.

$$\text{Connate water expansion} = [(\text{pore volume})\, S_{wi}]\, c_w\, \Delta p \;\rightarrow\; \text{Connate water expansion} = [\frac{N\,B_{oi}(1+m)}{1-S_{wi}} S_{wi}]\, c_w\, \Delta p$$

$$\text{Change in pore volume} = (\text{pore volume})\, c_f\, \Delta p \;\rightarrow\; \text{Change in pore volume} = \frac{N\,B_{oi}(1+m)}{1-S_{wi}}\, c_f\, \Delta p$$

To find the por volume (P.V):

The total volume of the hydrocarbon system = Initial oil volume + initial gas cap volume = (P.V) $(1 - S_{wi})$

$$N\,B_{oi} + m\,N\,B_{oi} = (P.V)(1-S_{wi})$$

$$P.V = \frac{N\,B_{oi}(1+m)}{1-S_{wi}}$$

$$\text{Total changes} = \text{Connate water expansion} + \text{Change in pore volume} = N\,B_{oi}(1+m)\left(\frac{S_{wi}c_w + c_f}{1-S_{wi}}\right)\Delta p$$

Pore volume occupied by the injected gas at p **+** *Pore volume occupied by the injected water at p*

Assuming that G_{inj} volumes of gas and W_{inj} volumes of water have been injected for pressure maintenance, the total pore volume occupied by the two injected fluids is given by:

$$\text{Total volume of the injected gas and water} = G_{inj}\, B_{ginj} + W_{inj}\, B_w$$

Where:

G_{inj} = cumulative gas injected, scf
B_{ginj} = injected gas formation volume factor, bbl/scf
W_{inj} = cumulative water injected, STB
B_w = water formation volume factor, bbl/STB

Combining Equations of the nine terms composing the MBE gives:

$$N\,Boi + m\,N\,Boi = (N - N_p)\,B_o + \left[\frac{m\,N\,B_{oi}}{B_{gi}}\right]B_g + [N R_{si} - N_p R_p - (N - N_p) R_s] B_g + W_e - W_p\,B_w + N\,B_{oi}(1+m)\left(\frac{S_{wi} c_w + c_f}{1 - S_{wi}}\right)\Delta p + G_{inj}\,B_{ginj} + W_{inj}\,B_w$$

Rearranging MBE gives:

$$N = \frac{N_p\left[B_o + (R_p - R_s)B_g\right] - (W_e - W_p\,B_w) - G_{inj}B_{ginj} - W_{inj}B_{wi}}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + m\,B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{oi}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}$$

GENERAL FORM OF THE MBE

For the total formation volume factor (Bt): $B_t = B_o + (R_{si} - R_s)B_g$

Introducing Bt into MBE and assuming, for sake of simplicity, no water or gas injection gives:

$$N = \frac{N_p\left[B_t + (R_p - R_{si})B_g\right] - (W_e - W_p B_w)}{(B_t - B_{ti}) + m\,B_{ti}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{ti}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}$$

RESERVOIR DRIVE INDICES

$$N = \frac{N_p\left[B_t + (R_p - R_{si})B_g\right] - (W_e - W_pB_w)}{(B_t - B_{ti}) + m\,B_{ti}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{ti}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}$$

Let $A = N_p\left[B_t + (R_p - R_{si})B_g\right]$

Thus:

$$N\left((B_t - B_{ti}) + mB_{ti}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{ti}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p\right) = A - (W_e - W_pB_w)$$

Dividing both sides of the equation by A yields:

$$\frac{N(B_t - B_{ti})}{A} + \frac{NmB_{ti}\left[\frac{B_g}{B_{gi}} - 1\right]}{A} + \frac{NB_{ti}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}{A} = 1 - \frac{(W_e - W_pB_w)}{A}$$

$$\frac{N(B_t - B_{ti})}{A} + \frac{NmB_{ti}\left[\frac{B_g}{B_{gi}} - 1\right]}{A} + \frac{NB_{ti}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}{A} + \frac{(W_e - W_pB_w)}{A} = 1$$

$$DDI + SDI + EDI + WDI = 1.0$$

$$DDI = \frac{N(B_t - B_{ti})}{N_p\left[B_t + (R_p - R_{si})B_g\right]}$$

$$SDI = \frac{NmB_{ti}\left[\frac{B_g}{B_{gi}} - 1\right]}{N_p\left[B_t + (R_p - R_{si})B_g\right]}$$

$$EDI = \frac{NB_{ti}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}{N_p\left[B_t + (R_p - R_{si})B_g\right]}$$

$$WDI = \frac{(W_e - W_pB_w)}{N_p\left[B_t + (R_p - R_{si})B_g\right]}$$

Where:
DDI = depletion-drive index
SDI = segregation (gas-cap)-drive index
EDI = expansion (rock and fluid)-drive index
WDI = water-drive index

EXAMPLE

A combination-drive reservoir contains 10 MMSTB of oil initially in place. The ratio of the original gas-cap volume to the original oil volume, i.e., m, is estimated as 0.25. The initial reservoir pressure is 3000 psia at 150°F. The reservoir produced 1 MMSTB of oil, 1100 MMscf of 0.8 specific gravity gas, and 50,000 STB of water by the time the reservoir pressure dropped to 2800 psi. The following PVT is available:

	3000 psi	**2800 psi**
B_o, bbl/STB	1.58	1.48
R_s, scf/STB	1040	850
B_g, bbl/scf	0.00080	0.00092
B_t, bbl/STB	1.58	1.655
B_w, bbl/STB	1.000	1.000

The following data are also available:

$S_{wi} = 0.20 \qquad c_w = 1.5 \times 10^{-6}\ psi^{-1} \qquad c_f = 1 \times 10^{-6}\ psi^{-1}$

Calculate:

a. Cumulative water influx

b. Net water influx

c. Primary driving indices at 2800 psi

Solution

Because the reservoir contains a gas cap, the rock and fluid expansion can be neglected., i.e., set cf and cw = 0. For illustration purposes, however, the rock and fluid expansion term will be included in the calculations.

Part A. Cumulative water influx

$$R_p = \frac{G_p}{N_p} = \frac{1100 \times 10^6}{1 \times 10^6} = 1100 \text{ scf/STB}$$

$$N = \frac{N_p\left[B_t + (R_p - R_{si})B_g\right] - (W_e - W_p B_w)}{(B_t - B_{ti}) + m B_{ti}\left[\frac{B_g}{B_{gi}} - 1\right] + B_{ti}(1+m)\left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right]\Delta p}$$

$$W_e = N_p\left[B_t + (R_p - R_{si})B_g\right] - N\left[(B_t - B_{ti}) + mB_{ti}\left(\frac{B_g}{B_{gi}} - 1\right) + B_{ti}(1+m)\left(\frac{S_{wi}c_w + c_f}{1 - S_{wi}}\right)\Delta p\right] + W_p B_{wp}$$

$$W_e = 10^6[1.655 + (1100 - 1040)0.00092] - 10^7\left[(1.655 - 1.58) + 0.25(1.58)\left(\frac{0.00092}{0.00080} - 1\right) + 1.58(1 + 0.25)\left(\frac{0.2(1.5 \times 10^{-6}) + 1 \times 10^{-6}}{1 - 0.2}\right)(3000 - 2800)\right] + 50,000$$

We = 411,281 bbl

Part B. Net water influx

$$\text{Net water influx} = W_e - W_p B_w = 411,281 - 50,000 = 361,281 \text{ bbl}$$

Part C. Primary recovery indices at 2800 psi

$$A = N_p \left[B_t + \left(R_p - R_{si} \right) B_g \right]$$

$$A = 10^6 \left[1.655 + (1100 - 1040)\, 0.00092 \right] = 1{,}710{,}200 \text{ bbl}$$

$$DDI = N(B_t - B_{ti}) / A$$

$$DDI = \frac{10 \times 10^6 (1.655 - 1.58)}{1{,}710{,}200} = 0.4385$$

$$SDI = \left[Nm\, B_{ti} \left(\frac{B_g}{B_{gi}} - 1 \right) \right] / A$$

$$SDI = \frac{10 \times 10^6 (0.25)(1.58)\left(\frac{0.00092}{0.0008} - 1 \right)}{1{,}710{,}200} = 0.3465$$

$$WDI = \left(W_e - W_p\, B_w \right) / A$$

$$WDI = \frac{411{,}281 - 50{,}000}{1{,}710{,}200} = 0.2113$$

$$EDI = 1 - 0.4385 - 0.3465 - 0.2113 = 0.0037$$

These calculations show that the 43.85% of the recovery was obtained by depletion drive, 34.65% by gas-cap drive, 21.13% by water drive, and only 0.37% by connate water and rock expansion.

THANK YOU