Petroleum engineering

Third stage

Engineering analysis

Lecture -1-

ordinary differential equations

Deferential equations

Partial differential Equations
(هي المعادلات اي تكون المتغير لأكثر من متغير)

EX:

① $\frac{\partial x}{\partial y} + \frac{\partial^2 x}{\partial t^2} + \cdots$

② $\frac{\partial x}{\partial y} + \frac{\partial x}{\partial P} + \cdots$

③ $\frac{\partial^2 v}{\partial p^2} + \frac{\partial^2 v}{\partial t^2}$

ordinary differential equations
هي المعادلات اي تكون المتغير فيها متغير واحد فقط

EX:

① $\frac{dy}{dx} + x\frac{dy}{dx}$

② $\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx}$

- order: (can be define) is the highest (الاعلى) differential coefficient present in the equation

degree : the degree of the highest derivative after removing the radical sign & fraction.

EX:

① $\cos x \frac{d^2y}{dx^2} + \sin x \left(\frac{dy}{dx}\right)^2 + 8y = \tan x$

Sol

order = 2 degree = 1

② $x^2\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^4 = 0$

ordinary differential equation

non-linear — linear

تسمى المعادلة التفاضلية الاعتيادية خطية عندما تكون الحالات الاتية :-

١- عدم ضرب المشتقة في نفسها او في مشتقة ثانية

ex:

① $y'y'' + 2y = 0$ ② $(y')^2 + 2xy = y''$

2x: بالمشتقة (y) (dependent variable) ٢- عدم ضرب المتغير المعتمد

① $yy'' + 2xy = 0$ ② $yy' + y'' = x$

٣) عدم ضرب المشتقة الثانية (order = 2) في دالة (x)

x:

① $\cos x \cdot y'' + 2y' = 0$

---

linear differential equation

- 1st order
  - separable
  - homogenous
  - exact
  - linear
  - Bernoulli's equ (non-linear)
- 2nd order
- higher order

2nd order differential equations

- Reducible to 1st order
- homogeneous: $a\ddot{y} + b\dot{y} + cy = 0$
- non-homo: $a\ddot{y} + b\dot{y} + cy = f(x)$

hiegher order D.E

- homogenous
- non homogenous

1st order differential equations:

~~D seperable:~~

① variables separable:

If the diff equa can be written in the form

$$F(y)\,dy = \phi(x)\,dx$$

we say that variabls are separable.
we get the solution by intgerting both sides

EX: 1/ Solve: $\frac{dy}{dx} = \frac{x\sqrt{1+y^2}}{2-3x^2}$

Sol $\frac{dy}{\sqrt{1+y^2}} = \frac{x\,dx}{(2-3x^2)}$

$\sinh^{-1} y = -\frac{1}{\quad} \ln|2-3x^2| + C$

where:

① $\int \frac{1}{\sqrt{1+u^2}} \frac{du}{dx} = \sinh^{-1} y +$

② $\int \frac{1}{u}\,du = \ln|u| + C$

EX-2- solve the equation

$$x(2y-3)\,dx + (x^2+1)\,dy = 0$$

Sol

تقسم المعادلة على $(2y-3)(x^2+1)$

$$\frac{x(2y-3)}{(2y-3)(x^2+1)}dx + \frac{(x^2+1)}{(2y-3)(x^2+1)}dy = 0$$

$$\frac{x}{(x^2+1)}dx + \frac{dy}{(2y-3)} = 0$$

by separable Method

$$\frac{x}{(x^2+1)}dx = -\frac{dy}{(2y-3)}$$

$$* \int \frac{1}{u}du = \ln|u| + C$$

$u_1 = x^2+1 \qquad du_1 = 2x$

$u_2 = 2y-3 \qquad du_2 = 2$

$$\frac{2x}{2(x^2+1)}dx = -\frac{2\,dy}{2(2y-3)}dy$$

by integerting both side

$$\frac{1}{2}\ln|x^2+1| = \frac{-1}{2}\ln|2y-3| + C$$

$$\ln|x^2+1| = -\ln|2y-3| + C$$

EX-3- solve the equ:

$$y' = \frac{e^x}{ey}$$

$$* \int e^u du = e^u + C$$

$$* \frac{d}{dx}e^u = e^u \frac{du}{dx}$$

Sol

$$\frac{dy}{dx} = \frac{e^x}{e^y}$$

$$e^y dy - e^x dx = 0$$

$$e^y - e^x = C$$

Ex-4- Solve the diff equation

$$\frac{dx}{dy}\sin x + \cosh 2y = 0$$

Sol

$$\frac{dx}{dy}\sin x = -\cosh 2y$$

$$dx \sin x = -\cosh 2y \, dy$$ integerate both sides

$$-\cos x = -\frac{1}{2}\sinh 2y + C$$

H.W solve diff eqn

① $y' \sqrt{2xy} = 1$

② $\frac{dx}{dy} \ln x = \frac{x}{y}$

③ $y\sqrt{2x^2+3}\, dy + x\sqrt{4-y^2}\, dx = 0$

④ $y' = e^{3x-2y} + x^2 e^{-2y}$