

Petroleum engineering

Third stage

Engineering analysis

Lecture -1-

ordinary differential equations

Differential equations

Partial differential Equations

(هي المعادلات التي تكون المشتقة لأكثر من متغير)

ordinary differential equations

(هي المعادلات التي تكون المشتقة فيها متغير واحد فقط)

EX:

① $\frac{\partial x}{\partial y} + \frac{\partial^2 x}{\partial t^2} + \dots$

② $\frac{\partial x}{\partial y} + \frac{\partial x}{\partial p} + \dots$

③ $\frac{\partial^2 v}{\partial p^2} + \frac{\partial^2 v}{\partial t^2}$

EX:

① $\frac{dy}{dx} + x \frac{dy}{dx}$

② $\left(\frac{d^2 y}{dx^2}\right)^3 + \frac{dy}{dx}$

can be define

order: ^{المرتبة} is the highest differential coefficient present in the equation

degree: ~~is~~ the degree of the highest derivative after removing the radical sign & fraction.

EX:

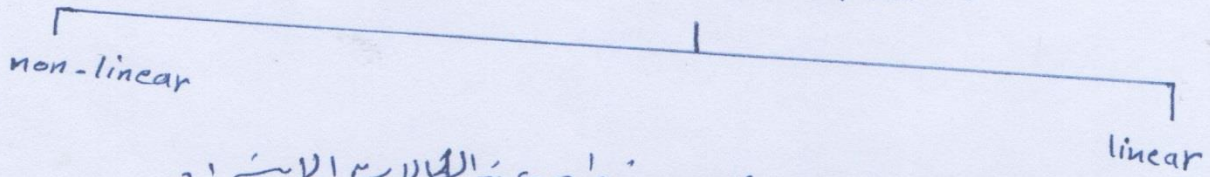
① $\cos x \frac{d^2 y}{dx^2} + \sin x \left(\frac{dy}{dx}\right)^2 + 8y = \tan x$

Sol

order = 2 degree = 1

② $x^2 \left(\frac{d^2 y}{dx^2}\right)^3 + y \left(\frac{dy}{dx}\right)^4 + y^4 = 0$

ordinary differential equation



تسمى المعادلة التفاضلية الاعيادية خطية عند الحالات الآتية :-
 1- عدم ضرب المشتقة في نفسها او في مشتقة ثابت

Ex:

$$D y \bar{y}'' + 2y = 0$$

$$② (\bar{y}')^2 + 2xy = \bar{y}$$

Ex: (y) بالمشتقة (dependent variable) -2 عدم ضرب المتغير المعتمد

$$D y \bar{y}'' + 2xy = 0$$

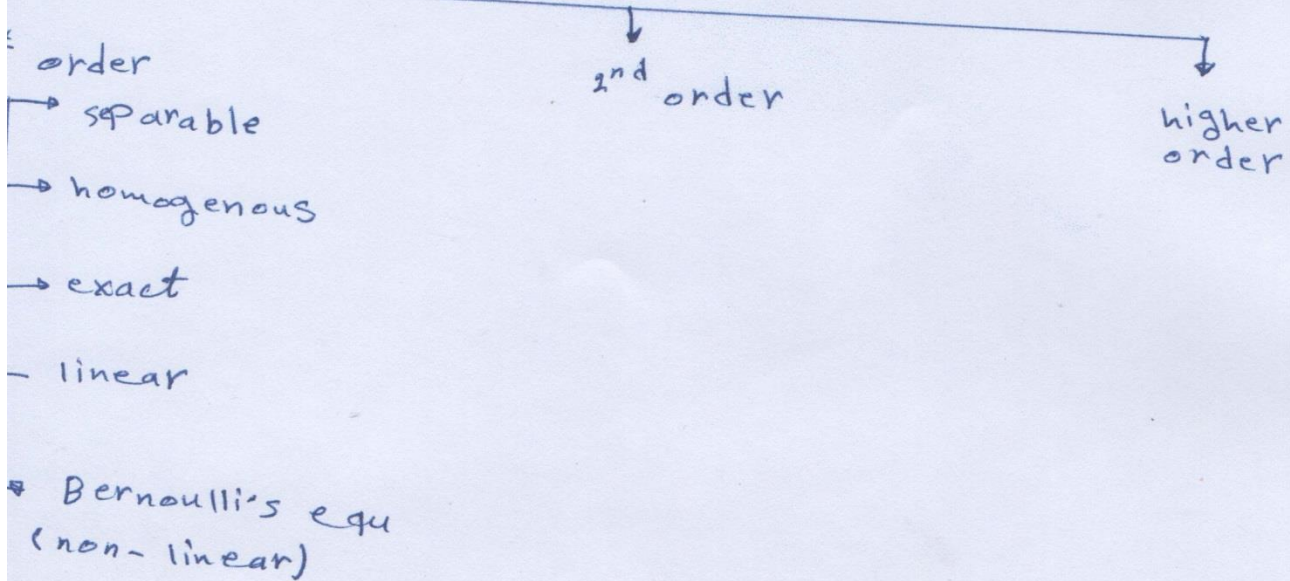
$$② y \bar{y}'' + \bar{y} = x$$

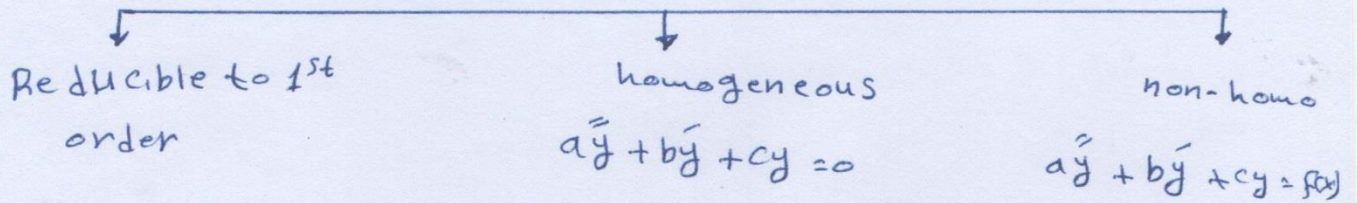
(3) عدم ضرب المشتقة الثابت (order = 2) في دالة (x)

Ex:

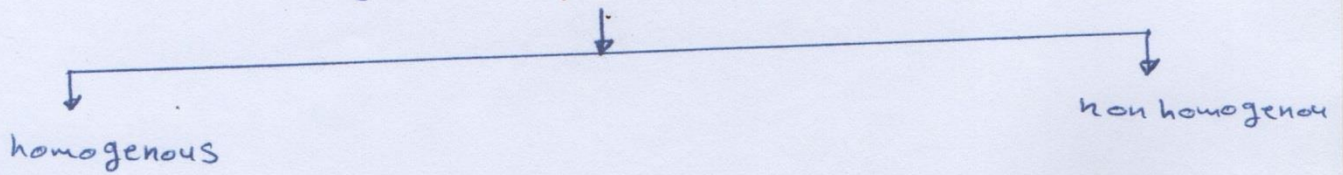
$$\cos x \cdot \bar{y}'' + 2\bar{y} = 0$$

linear differential equation



2nd order differential equations

higher order D.E

1st order differential equations:~~separable:~~

① variables separable:

If the diff equa can be written in the form

$$F(y) dy = \phi(x) dx$$

We say that variables are separable.

We get the solution by integrating both sides

EX: 1/

$$\text{Solve: } \frac{dy}{dx} = \frac{x \sqrt{1+y^2}}{2-3x^2}$$

$$\text{Sol } \frac{dy}{\sqrt{1+y^2}} = \frac{x dx}{(2-3x^2)}$$

$$\sinh^{-1} y = -\frac{1}{2} \ln|2-3x^2| + C$$

where:

$$\int \frac{1}{\sqrt{1+u^2}} \frac{du}{dx} = \sinh^{-1} y +$$

$$\int \frac{1}{u} du = \ln|u| + C$$

EX-2- solve the equation ~~XXXX~~

$$x(2y-3) dx + (x^2+1) dy = 0$$

sol

$$\frac{x(2y-3)}{(2y-3)(x^2+1)} dx + \frac{(x^2+1)}{(2y-3)(x^2+1)} dy = 0$$

$$\frac{x}{(x^2+1)} dx + \frac{dy}{(2y-3)} = 0$$

$$\frac{x}{(x^2+1)} dx = - \frac{dy}{(2y-3)}$$

$$u_1 = x^2 + 1 \quad du_1 = 2x$$

$$u_2 = 2y - 3 \quad du_2 = 2$$

$$\frac{2x}{2(x^2+1)} dx = - \frac{2 dy}{2(2y-3)}$$

$$\frac{1}{2} \ln|x^2+1| = -\frac{1}{2} \ln|2y-3| + C$$

$$\ln|x^2+1| = -\ln|2y-3| + C$$

تقسيم المتكاملات على (2y-3)(x^2+1)

by separable method

$$* \int \frac{1}{u} du = \ln|u| + C$$

by integrating both side

EX-3- solve the equ:

$$y' = \frac{e^x}{e^y}$$

sol

$$\frac{dy}{dx} = \frac{e^x}{e^y}$$

$$\cancel{e^y} e^y dy \leftarrow e^x dx = 0$$

$$e^y - e^x = C$$

$$* \int e^u du = e^u + C$$

$$* \frac{d}{dx} e^u = e^u \frac{du}{dx}$$

EX-4- Solve the diff equation

$$\frac{dx}{dy} \sin x + \cosh 2y = 0$$

sol

$$\frac{dx}{dy} \sin x = -\cosh 2y$$

$$dx \sin x = -\cosh 2y dy$$

integrate both sides

$$-\cos x = -\frac{1}{2} \sinh 2y + C$$

~~$$\frac{dx}{dy} \sin x = -\cosh 2y$$~~

H.w solve diff equ

① $y \sqrt{2xy} = 1$

② $\frac{dx}{dy} \ln x = \frac{x}{y}$

③ $y \sqrt{2x^2+3} dy + x \sqrt{4-y^2} dx = 0$

④ $y = e^{3x-2y} + x^2 e^{-2y}$