Petroleum engineering

Third stage

Engineering analysis

Lecture -2-

2- Homogeneous differential equ

* The form of equation $\boxed{\frac{dy}{dx} = f\left(\frac{y}{x}\right)}$

* The diff equ is called homogeneous equ if each term of $f(x,y)$ have same degree.

*** working Rule**

① Put $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$

② seperate the veriables.

③ integerate both the sides.

④ Put $v = \frac{y}{x}$

Ex-1- solve the following diff equ:

$(x^2+y^2)\,dx + xy\,dy = 0$

Sol

$(x^2+y^2) + xy\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{(x^2+y^2)}{xy} \Longrightarrow \frac{dy}{dx} = -\left\{\frac{\cancel{x^2}^{x}}{\cancel{xy}} + \frac{\cancel{y^2}^{y}}{\cancel{xy}}\right\}$

$\frac{dy}{dx} = -\left[\frac{x}{y} + \frac{y}{x}\right]$

Put $y = vx \Longrightarrow v = \frac{y}{x}$, $\frac{x}{y} = \frac{1}{v}$, $\frac{y}{x} = v$

$\frac{dy}{dx} = -\left[\frac{1}{v} + v\right] \Longrightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = -\left[\frac{1}{v} + v\right]$

$$v + x\frac{dv}{dx} + \frac{1}{v} + v = 0$$

$$\frac{x}{dx}\,dv + \frac{1}{v} + 2v = 0$$ العلاقة بين dv

$$\frac{x}{dx} + \frac{\frac{1}{v} + 2v}{dv} = 0$$ ب الطرفين

$$\frac{dx}{x} + \frac{dv}{\frac{1}{v} + 2v} = 0 \Longrightarrow \frac{dx}{x} + \frac{v\,dv}{1+2v^2}$$

$$\int \frac{dx}{x} + \int \frac{v\,dv}{1+2v^2} = 0$$ * $\int \frac{1}{v}$ d

$$\ln x + \frac{1}{4}\ln(1+2v^2) = C$$ * Put v =

$$\ln x + \frac{1}{4}\ln\left(1+2\frac{y^2}{x^2}\right) = C$$

EX-2-

$$(2xy + x^2)\dot{y} = 3y^2 + 2xy$$

Sol.

$$(2xy + x^2)\frac{dy}{dx} = 3y^2 + 2xy \Longrightarrow \frac{dy}{dx} = \frac{3y}{2x}$$

Put $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{3v^2x^2 + 2x^2v}{2vx^2 + x^2} = \frac{x^2(3v^2+2v)}{x^2(2v+1)}$$

$$x\frac{dv}{dx} = \frac{3v^2+2v}{2v+1} - v = \frac{3v^2+2v-2v^2-v}{2v+1}$$

$$x\frac{dv}{dx}=\frac{v^2+v}{2v+1} \Longrightarrow \frac{x}{dx}=\left[\frac{v^2+v}{2v+1}\right]\cdot\frac{1}{dv}$$

$$\frac{dx}{x}=\left[\frac{2v+1}{v^2+v}\right]dv$$

$$\ln x=\ln(v^2+v)+C$$

$$\ln x=\ln\left(\frac{y^2}{x^2}+\frac{y}{x}\right)+C$$

EX-3-

$$\frac{dy}{dx}=\frac{y}{x}+x\sin\frac{y}{x}$$

Sol

$$y=vx \quad , \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=v+x\sin v \Longrightarrow \cancel{x}\frac{dv}{dx}=\cancel{x}\sin v$$

$$\frac{\cancel{dx}}{dx}=\frac{\sin v}{dv} \Longrightarrow \frac{dv}{\sin v}=dx$$

$$\int \operatorname{csec} v\ dv=dx$$

$$\ln\tan\frac{v}{2}=x+C \qquad \text{where } v=\frac{y}{x}$$

$$\ln\tan\frac{y}{2x}=x+C$$

* $\frac{1}{\sin v}=\operatorname{cosec} v$

* $\int \operatorname{csec} v\,dv=\ln\tan\frac{v}{2}$

Ex-4-

$$(xy+y^2)\,dx = (x^2+xy+y^2)\,dy$$

Sol

$$(xy+y^2) = (x^2+xy+y^2)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{xy+y^2}{x^2+xy+y^2} * \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$\frac{dy}{dx} = \frac{\frac{y}{x}+\frac{y^2}{x^2}}{1+\frac{y}{x}+\frac{y^2}{x^2}}$$

let $v = \frac{y}{x}$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{v+v^2}{1+v+v^2} \Rightarrow \frac{v+v^2}{1+v+v^2} - v = 0$$

$$\frac{dx}{x} + \frac{dv}{v-\left(\frac{v+v^2}{1+v+v^2}\right)} = 0$$

$$\frac{dx}{x} + \frac{(1+v+v^2)\,dv}{v^3} = 0$$

$$\frac{dx}{x} + \left\{\frac{1}{v^3} + \frac{x}{x^3 v^2} + \frac{v^2}{v^3}\right\} dv = 0$$

$$\frac{dx}{x} + \frac{1}{v^3}dv + \frac{1}{v^2}dv + \frac{1}{v}dv = 0$$

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$$\frac{1}{\quad} - \frac{1}{\quad} + \ln v = c$$

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H.W solve the diff eqn ans: $\frac{x}{y} = \ln x + c$

1- $(y^2 - xy)\,dx + x^2\,dy = 0$

2- $\frac{dy}{dx} + \frac{x - 2y}{2x - y} = 0$ ans: $(y - x) = c(x$

3- $(x^2 + y^2)\,dy = xy\,dx$ ans: $-\frac{x^2}{2y^2} + \ln y$

4. $(x \sin\frac{y}{x} - y\cos\frac{y}{x})\,dx + x\cos\frac{y}{x}\,dy = 0$

ans: $\ln x + \ln \sin\frac{y}{x} = c$