Al-Ayen University
College of Petroleum Engineering

Reservoir Engineering II

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Lecture 11: Pseudo-Steady State Flow of Reservoir Fluids (Part 1),
Ref.: Reservoir Engineering Handbook by Tarek Ahmed

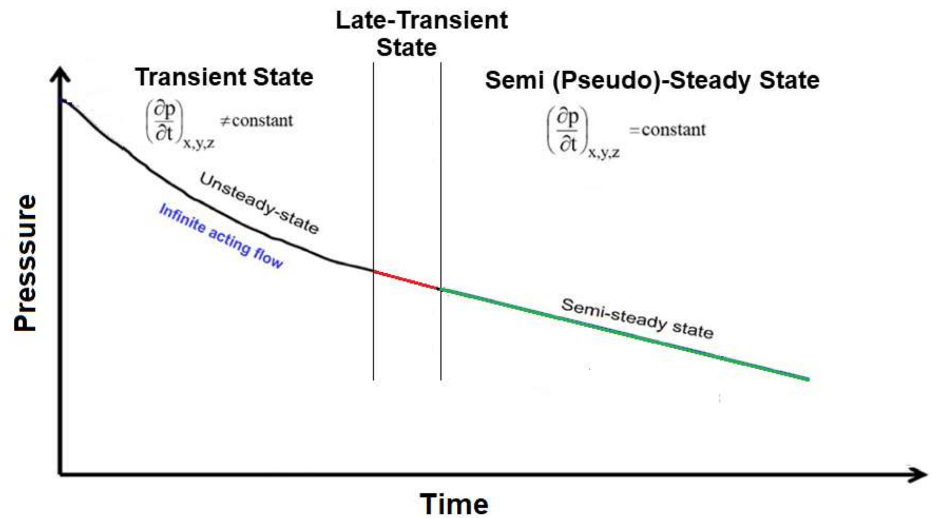
Outline

- Semi (Pseudo)-Steady State Flow
  - Well Drainage Area for Radial Flow at Pseudo-Steady State
  - Volumetric Average Reservoir Pressure
  - Pseudo-Steady State of Radial Flow for Slightly Compressible Fluids
- Summary
- Questions for Discussion

Semi (Pseudo)-Steady State Flow

- The arrival of the pressure disturbance at the well drainage boundary marks the end of the transient flow period and the beginning of the ***semi (pseudo)-steady state.***
- During this flow state, the reservoir boundaries and the shape of the drainage area influence the wellbore pressure response as well as the behavior of the pressure distribution throughout the reservoir.
- There is a short period of time that separates the transient state from the semisteady state that is called ***late-transient state.***
- During the semisteady-state flow, the change in pressure with time becomes the same throughout the drainage area. Mathematically, this important condition can be expressed as:

$$\left(\frac{\partial p}{\partial t}\right)_r = \text{constant}$$



Pressure Vs. time at any location in a volumetric reservoir

Well Drainage Area for Radial Flow at Pseudo-Steady State

At the pseudo-steady state:

$$\left(\frac{\partial p}{\partial t}\right)_r = \text{constant}$$

The definition of the compressibility: $c = \frac{-1}{V}\frac{dV}{dp}$

Arranging: $cVdp = -dV$

Differentiating with respect to time t: $cV\frac{dp}{dt} = -\frac{dV}{dt} = q$

This includes the assumption that the flow results from the compressibility only

Thus: $\frac{dp}{dt} = -\frac{q}{cV}$

Expressing the pressure decline rate dp/dt in the above relation in psi/hr gives:

$$\frac{dp}{dt} = -\frac{q}{24cV} = -\frac{Q_o B_o}{24cV}$$

where q = flow rate, bbl/day
Q_o = flow rate, STB/day
dp/dt = pressure decline rate, psi/hr
V = pore volume, bbl

$$\frac{dp}{dt} = -\frac{q}{24\,cV} = -\frac{Q_o\,B_o}{24\,cV}$$

For a radial drainage system, the pore volume is given by:

$$V = \frac{\pi\, r_e^2\, h\phi}{5.615} = \frac{Ah\phi}{5.615}$$, where A = drainage area, ft^2

Combining the Equations gives:

$$\frac{dp}{dt} = -\frac{0.23396\,q}{c_t\,\pi\, r_e^2\, h\phi} = \frac{-0.23396\,q}{c_t\,Ah\phi}$$

Examination of the above expression reveals the following important characteristics of the behavior of the pressure decline rate dp/dt during the semisteady-state flow:

- The reservoir pressure declines at a higher rate with an increase in the fluids production rate
- The reservoir pressure declines at a slower rate for reservoirs with higher total compressibility coefficients
- The reservoir pressure declines at a lower rate for reservoirs with larger pore volumes

Example

An oil well is producing at a constant oil flow rate of 1200 STB/day under a semisteady-state flow regime. Well testing data indicate that the pressure is declining at a constant rate of 4.655 psi/hr. The following additional data are available:

$h = 25\ ft$ $\quad \phi = 15\%$ $\quad B_o = 1.3\ bbl/STB$

$c_t = 12 \times 10^{-6}\ psi^{-1}$

Calculate the well drainage area.

Solution

$q = Q_o\ B_o = (1200)\ (1.3) = 1560\ bb/day$

Apply Equation :

$$\frac{dp}{dt} = \frac{-0.23396q}{c_t A h \phi}$$

$$-4.655 = -\frac{0.23396(1560)}{(12 \times 10^{-6})(A)(25)(0.15)}$$

$A = 1,742,344\ ft^2$ or $A = 1,742,400/43,560 = 40\ acres$

Volumetric Average Reservoir Pressure

$$\frac{dp}{dt} = -\frac{0.23396\,q}{c_t\,\pi\,r_e^2\,h\phi} = \frac{-0.23396\,q}{c_t\,Ah\phi}$$

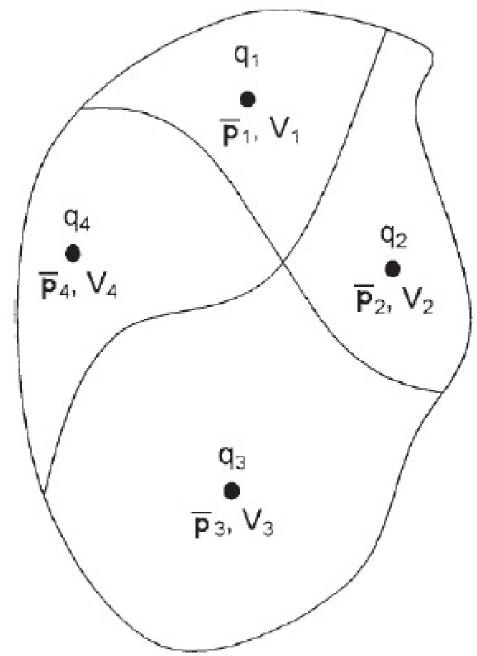
Or : $$\frac{(p_i - \bar{p}_r)}{t} = \frac{0.23396\,q}{c_t\,Ah\phi}$$

Thus: $$\bar{p}_r = p_i - \frac{0.23396\,q\,t}{c_t Ah\phi}$$

where **t** is approximately the elapsed time since the end of the transient flow regime to the time of interest.

- The volumetric average pressure of the entire reservoir can be determined from the individual well drainage properties as follows:

$$\bar{p}_r = \frac{\sum_i \bar{p}_{ri} V_i}{\sum_i V_i}$$



Pseudo-Steady State of Radial Flow for Slightly Compressible Fluids

The diffusivity equation for the transient flow regime is: $$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \left(\frac{\phi \mu c_t}{0.0002637k}\right)\frac{\partial p}{\partial t}$$

For the pseudo-steady state flow, the term (∂p/∂t) is constant and is expressed by Equation:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \left(\frac{\phi \mu c_t}{0.0002637k}\right)\left(\frac{-0.23396q}{c_t A h \phi}\right)$$

or

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{-887.22q\mu}{Ahk}$$

But: $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r}$$

Thus, for a circular drainage area, the equation can be expressed as: $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = -\frac{887.22q\mu}{(\pi r_e^2)hk}$$

Integrating the above equation gives: $$r\frac{\partial p}{\partial r} = -\frac{887.22q\mu}{(\pi r_e^2)hk}\left(\frac{r^2}{2}\right) + c_1$$

$$r\frac{\partial p}{\partial r} = -\frac{887.22 q\mu}{(\pi r_e^2)hk}\left(\frac{r^2}{2}\right) + c_1$$

Where c_1 is the constant of the integration and can be evaluated by imposing the outer no-flow boundary condition [i.e., $(\partial p/\partial r)_{re} = 0$] on the above relation to give:

$$c_1 = \frac{141.2 q\mu}{hk}$$

Combining the above two expressions gives:

$$\frac{\partial p}{\partial r} = \frac{141.2 q\mu}{hk}\left(\frac{1}{r} - \frac{r}{r_e^2}\right)$$

Integrating again:

$$\int_{p_{wf}}^{p_i} dp = \frac{141.2 q\mu}{hk}\int_{rw}^{re}\left(\frac{1}{r} - \frac{r}{r_e^2}\right)dr$$

Performing the above integration and assuming (r_w^2/r_e^2) is negligible gives:

$$(p_i - p_{wf}) = \frac{141.2\, q\mu}{kh}\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2}\right]$$

or

$$Q = \frac{0.00708\, kh\, (p_i - p_{wf})}{\mu B\left[\ln\left(\frac{r_e}{r_w}\right) - 0.5\right]}$$

Eq. of Pseudo-Steady State of Radial Flow for Slightly Compressible Fluids

where
Q = flow rate, STB/day
B = formation volume factor, bbl/STB
k = permeability, mD

Summary

- The arrival of the pressure disturbance to the drainage boundary marks the end of the transient flow period and the beginning of the ***semi (pseudo)-steady state.***

- There is a short period of time that separates the transient state from the semi-steady state that is called ***late-transient state.***

- Mathematically, the ***semi (pseudo)-steady state*** condition can be expressed as: $\left(\frac{\partial p}{\partial t}\right)_r = \text{constant}$

- The radial flow of slightly compressible fluids has been investigated under the condition of pseudo-steady state, and mathematical expressions have been derived to find the drainage area, volumetric average reservoir pressure, bottom-hole pressure, and flow rate.

Questions for Discussion

Q1) What are the reservoir flow stages of a well producing from:

a) a volumetric reservoir

b) a strong water-drive reservoir

Q2) What are the factors affecting the time periods of the flow stages in Q1 (above).

Q3) How it can be integrated the analysis presented in this lecture with calculations of the material balance equation.

THANK YOU