Al-Ayen University
College of Petroleum Engineering

Numerical Methods and Reservoir Simulation

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L14: (1) Introduction to Grid Systems & Boundary Conditions
(2) Incorporation of Dirichlet Boundary Conditions to Block-Centered Grids

Outline

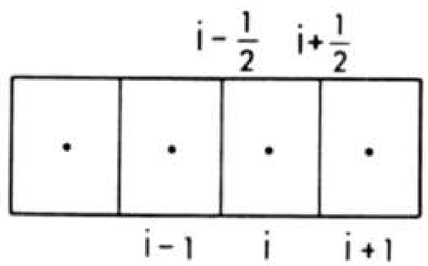
- Introduction
  - Grid Systems: Block-Centered and Point-Centered (or Point-Distributed) Grids
  - Types of Boundary Conditions:
    - Dirichlet Boundary Conditions
    - Neumann Boundary Conditions
- Incorporation of Boundary Conditions/Block-Centered Grid
  - Dirichlet Boundary Conditions

Introduction

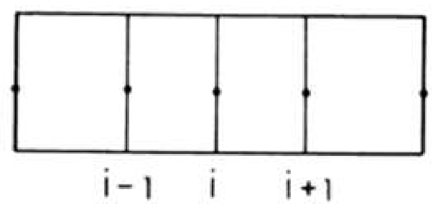
Grid Systems: Block-Centered and Point-Centered (or Point-Distributed) Grids

Block-Centered and Point-Centered (or Point-Distributed) Grids

- The only difference between Point-Centered (or Point-Distributed) and Block-Centered Grids is in the treatment of boundary conditions.
  - Block-centered grid has its boundaries one-half grid block away from the exterior boundaries.
  - Point-centered (or point-distributed) grid has its boundaries coincident with the exterior boundaries of the system.



Block-Centered



Point-Centered

- Note that we do not need to consider equally spaced grid-points or grid-blocks. Actually, it is more appropriate to use smaller grid spacing near the source/sink locations.

Types of Boundary Conditions

- Dirichlet Boundary Conditions
- Neumann Boundary Conditions

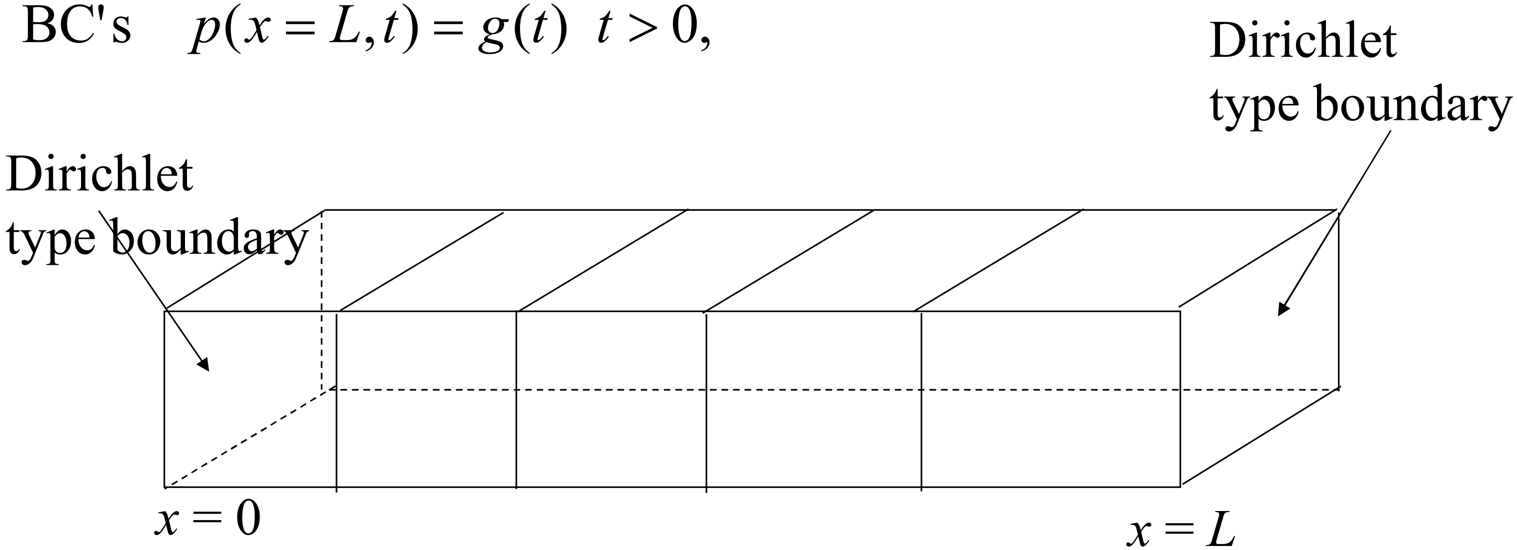
Dirichlet Boundary Conditions

- <u>Dirichlet Boundary Conditions:</u> pressure specified at the boundaries).

$$\text{IC} \quad p(x,0) = f(x) \; 0 \le x \le L,$$

$$\text{BC's} \quad p(x=0,t) = h(t) \;\; t > 0,$$

$$\text{BC's} \quad p(x=L,t) = g(t) \;\; t > 0,$$



Neumann Boundary Conditions

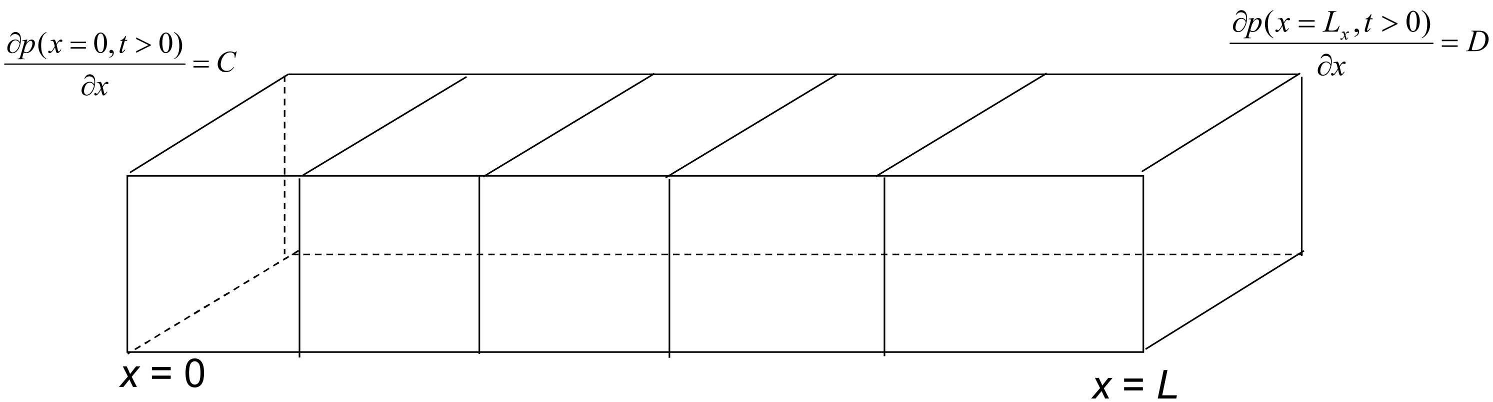
- Neumann Boundary Conditions: pressure gradients specified at the boundaries.

$$\text{IC} \quad p(x,0) = f(x) \; 0 \le x \le L,$$

$$\text{BC} \quad \frac{\partial p(x=0, t>0)}{\partial x} = C$$

$$\text{BC} \quad \frac{\partial p(x=L_x, t>0)}{\partial x} = D$$

$$\textit{Note}: \text{if C} = 0 \text{ and/or D} = 0\text{, then we have no} - \text{flow (closed) boundary.}$$



Applications of Finite Difference Approximation

The applications of finite difference approximations in this course will include the following modelling systems:

1. **1-D Block-Centered Grids:**
   a. Dirichlet Boundary Conditions
   b. Neumann Boundary Conditions

2. **1-D Point-Centered Grids:**
   a. Dirichlet Boundary Conditions
   b. Neumann Boundary Conditions

3. **2-D Systems:**
   a. Block-Centered Grids
   b. Point-Centered Grids

4. **Well modelling**

5. **Introduction to the Two-Phase Flow in a 1D Linear-Reservoir**

Incorporation of Boundary Conditions/Block-Centered Grid

(Spatial discretization with **Variable** grid block sizes)

Dirichlet Boundary Conditions

Incorporation of Boundary Conditions/Block-Centered Grid

- Dirichlet Boundary Conditions (pressure specified at the boundaries). For generality consider a heterogeneous reservoir with sources/sinks.

PDE $$1.127\times10^{-3}\frac{\partial}{\partial x}\left(\frac{k_x}{\mu}\frac{\partial p}{\partial x}\right)-\frac{q_{sc}(x,t)B}{V_b}=\frac{\phi c_t}{5.615}\frac{\partial p}{\partial t},\ 0<x<L_x,\ t>0$$

For instance:

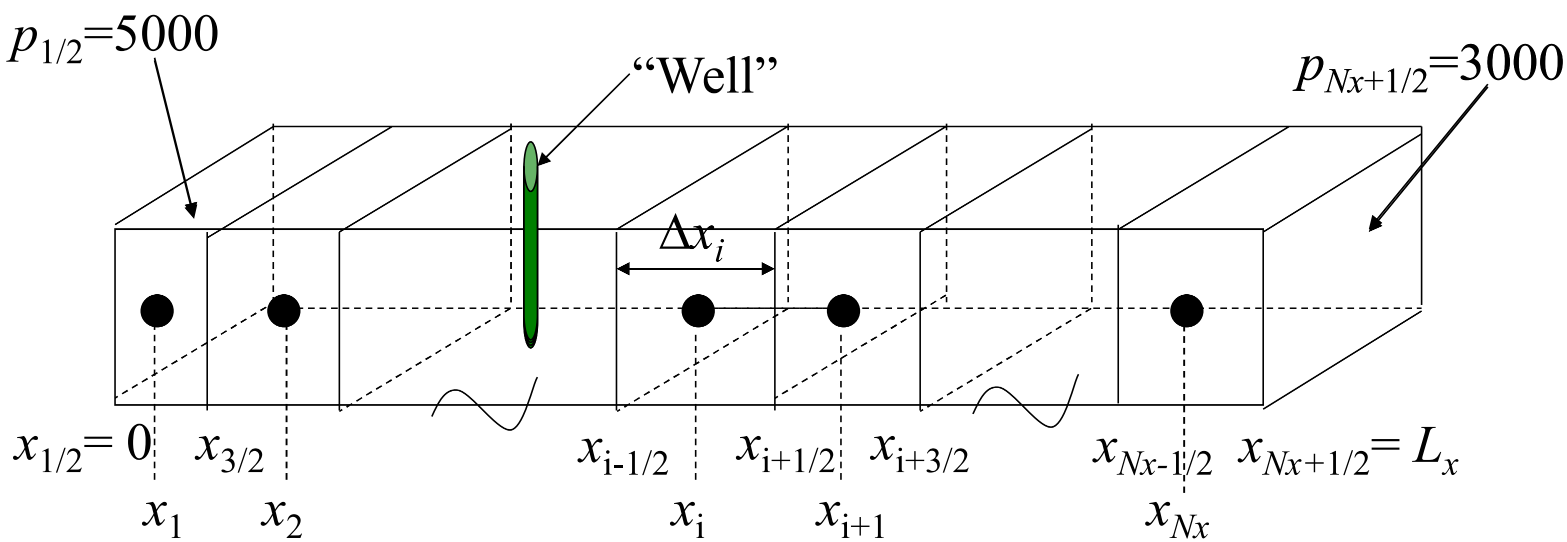
IC $p(x,0)=3000,\ 0\le x\le L_x,$

BC $p(x=0,t>0)=5000$

BC $p(x=L_x,t>0)=3000$

Incorporation of Boundary Conditions/Block-Centered Grid

- Dirichlet Boundary Conditions (pressure specified at the boundaries)



- General Implicit Difference Equation:

$$-T_{x,i-1/2}p_{i-1}^{n+1}+\left(T_{x,i+1/2}+T_{x,i-1/2}+\tilde{V}_i\right)p_i^{n+1}-T_{x,i+1/2}p_{i+1}^{n+1}=-q_{sc,i}^{n+1}B+\tilde{V}_i p_i^n$$

- For *i* =1,

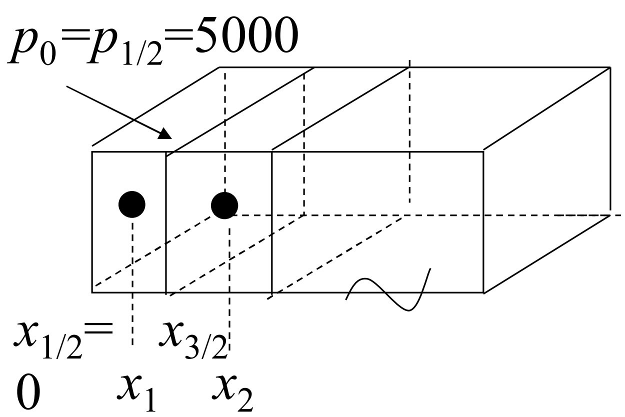
$$-T_{x,1/2}p_0^{n+1}+\left(T_{x,3/2}+T_{x,1/2}+\tilde{V}_1\right)p_1^{n+1}-T_{x,3/2}p_2^{n+1}=-q_{sc,1}^{n+1}B+\tilde{V}_1 p_1^n$$

$$T_{x,1/2}=2\times 1.127\times 10^{-3}\frac{\lambda_{x,1/2}wh}{\left(\Delta x_1+\Delta x_0\right)}$$

Modify $T_{x,1/2}$ as:

$$\tilde{T}_{x,1/2}=1.127\times 10^{-3}\frac{\lambda_{x,1/2}wh}{\left(x_1-x_{1/2}\right)}$$

$$=2\times 1.127\times 10^{-3}\frac{\lambda_{x,1/2}wh}{\Delta x_1}$$



- For i = 1, with this modification we have:

$$-\widetilde{T}_{x,1/2}p_0^{n+1}+\left(T_{x,3/2}+\widetilde{T}_{x,1/2}+\widetilde{V}_1\right)p_1^{n+1}-T_{x,3/2}p_2^{n+1}=-q_{sc,1}^{n+1}B+\widetilde{V}_1p_1^n$$

Because p_0 is known (BC), then for i = 1,

$$\left(T_{x,3/2}+\widetilde{T}_{x,1/2}+\widetilde{V}_1\right)p_1^{n+1}-T_{x,3/2}p_2^{n+1}=-q_{sc,1}^{n+1}B+\widetilde{V}_1p_1^n+\widetilde{T}_{x,1/2}p_0^{n+1}$$

where we evaluate $\widetilde{T}_{x,1/2}$ from:

$$\widetilde{T}_{x,1/2}=2\times1.127\times10^{-3}\frac{\lambda_{x,1/2}wh}{\Delta x_1}$$

- For i = 2,3,…,N_x-1

$$-T_{x,i-1/2}p_{i-1}^{n+1}+\left(T_{x,i+1/2}+T_{x,i-1/2}+\widetilde{V}_i\right)p_i^{n+1}-T_{x,i+1/2}p_{i+1}^{n+1}=-q_{sc,i}^{n+1}B+\widetilde{V}_i p_i^n$$

where we evaluate *transmissibilities* from:

$$T_{x,i\mp1/2}=2\times1.127\times10^{-3}\frac{\lambda_{x,i\mp1/2}wh}{\Delta x_i+\Delta x_{i\mp1}}$$

- General Implicit Difference Equation:

$$-T_{x,i-1/2}p_{i-1}^{n+1}+\left(T_{x,i+1/2}+T_{x,i-1/2}+\widetilde{V}_i\right)p_i^{n+1}-T_{x,i+1/2}p_{i+1}^{n+1}=-q_{sc,i}^{n+1}B+\widetilde{V}_i p_i^n$$

- For $i = N_x$,

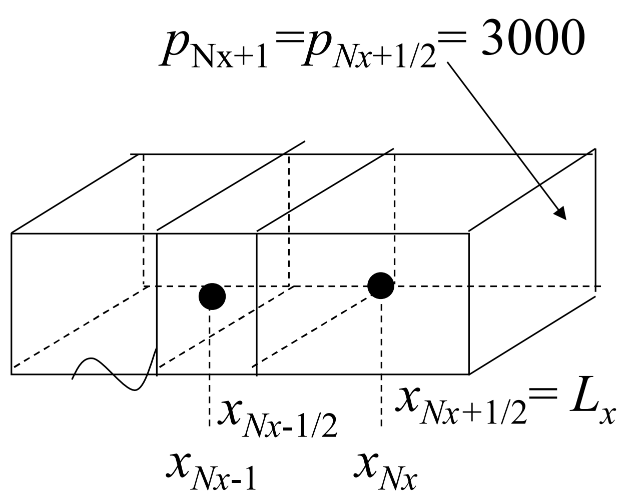
$$-T_{x,N_x-1/2}p_{N_x-1}^{n+1}+\left(T_{x,N_x+1/2}+T_{x,N_x-1/2}+\widetilde{V}_{N_x}\right)p_{N_x}^{n+1}-T_{x,N_x+1/2}p_{N_x+1}^{n+1}=-q_{sc,N_x}^{n+1}B+\widetilde{V}_{N_x}p_{N_x}^n$$

$$T_{x,N_x+1/2}=2\times1.127\times10^{-3}\frac{\lambda_{x,Nx+1/2}wh}{\left(\Delta x_{N_x+1}+\Delta x_{N_x}\right)}$$

Modify $T_{x,Nx+1/2}$ as:

$$\widetilde{T}_{x,N_x+1/2}=1.127\times10^{-3}\frac{\lambda_{x,Nx+1/2}wh}{\left(x_{N_x+1/2}-x_{N_x}\right)}$$

$$=2\times1.127\times10^{-3}\frac{\lambda_{x,Nx+1/2}wh}{\Delta x_{N_x}}$$



- For $i = N_x$, with this modification we have:

$$-T_{x,Nx-1/2}\,p_{Nx-1}^{n+1} + \left(\widetilde{T}_{x,Nx+1/2} + T_{x,Nx-1/2} + \widetilde{V}_{Nx}\right)p_{Nx}^{n+1} - \widetilde{T}_{x,Nx+1/2}\,p_{Nx+1}^{n+1} = -q_{sc,Nx}^{n+1}B + \widetilde{V}_{Nx}\,p_{Nx}^{n}$$

Because p_{Nx+1} is known (BC), then for $i = N_x$,

$$-T_{Nx-1/2}\,p_{Nx-1}^{n+1} + \left(\widetilde{T}_{x,Nx+1/2} + T_{x,Nx-1/2} + \widetilde{V}_{Nx}\right)p_{Nx}^{n+1} = -q_{sc,Nx}^{n+1}B + \widetilde{V}_{Nx}\,p_{Nx}^{n} + \widetilde{T}_{x,Nx+1/2}\,p_{Nx+1}^{n+1}$$

where we evaluate $\widetilde{T}_{x,Nx+1/2}$ from:

$$\widetilde{T}_{x,Nx+1/2} = 2\times 1.127\times 10^{-3}\,\frac{\lambda_{x,Nx+1/2}\,wh}{\Delta x_{Nx}}$$

Summary

- It has been presented two types of grid systems: Block-Centered and Point-Centered (or Point-Distributed) Grids.
- The only difference between Point-Centered and Block-Centered Grids is in the treatment of boundary conditions.
- There are two types of boundary conditions: Dirichlet Boundary Conditions and Neumann Boundary Conditions.
- Dirichlet Boundary Conditions: pressure specified at the boundaries.
- Neumann Boundary Conditions: pressure gradients specified at the boundaries.

Exercise

After discretization of the following PDE by implicit finite difference with the initial and boundary conditions shown below, write a generalized matrix-vector form ($A\vec{p}^{\,n+1} = \vec{d}^{\,n}$) for this problem.

$$\text{PDE} \quad 1.127\times10^{-3}\frac{\partial}{\partial x}\left(\frac{k_x}{\mu}\frac{\partial p}{\partial x}\right) - \frac{q_{sc}(x,t)B}{V_b} = \frac{\phi c_t}{5.615}\frac{\partial p}{\partial t},\ 0 < x < L_x,\ t > 0$$

$$\text{IC} \quad p(x,0) = P_0,\ 0 \le x \le L_x,$$

$$\text{BC} \quad p(x=0, t>0) = P_0$$

$$\text{BC} \quad p(x=L_x, t>0) = P_L$$

THANK YOU