Al-Ayen University
College of Petroleum Engineering

Reservoir Engineering II

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Lecture 10: Unsteady-State Flow of Reservoir Fluids (Part 4),
Ref.: Reservoir Engineering Handbook by Tarek Ahmed

Outlines

❖ *Unsteady-State Flow*

- **Solution of the Diffusivity Equation**
  - **Radial Flow of the Compressible Fluids**
    - ✓ **The Pressure-Approximation Method**
      - ❖ **Example**

Unsteady-State Flow

Solution of the Diffusivity Equation

Radial Flow of the Compressible Fluids

$$\frac{\partial^2 m(p)}{\partial r^2} + \frac{1}{r}\frac{\partial m(p)}{\partial r} = \frac{\phi \mu c_t}{0.000264\,k}\frac{\partial m(p)}{\partial t}$$

The radial diffusivity equation for compressible fluids

Imposing the constant-rate condition as one of the boundary conditions, it has been shown that the *exact solution* to this equation is:

$$m(p_{wf}) = m(p_i) - \left(\frac{1637\,Q_g\,T}{kh}\right)\left[\log\left(\frac{kt}{\phi \mu_i c_{ti} r_w^2}\right) - 3.23\right]$$

In terms of the dimensionless time t_D as:

$$m(p_{wf}) = m(p_i) - \left(\frac{1637\,Q_g\,T}{kh}\right)\left[\log\left(\frac{4t_D}{\gamma}\right)\right]$$

$$t_D = \frac{0.000264\,kt}{\phi \mu_i c_{ti} r_w^2}$$

$$\gamma = e^{0.5772} = 1.781$$

The Pressure-Approximation Method

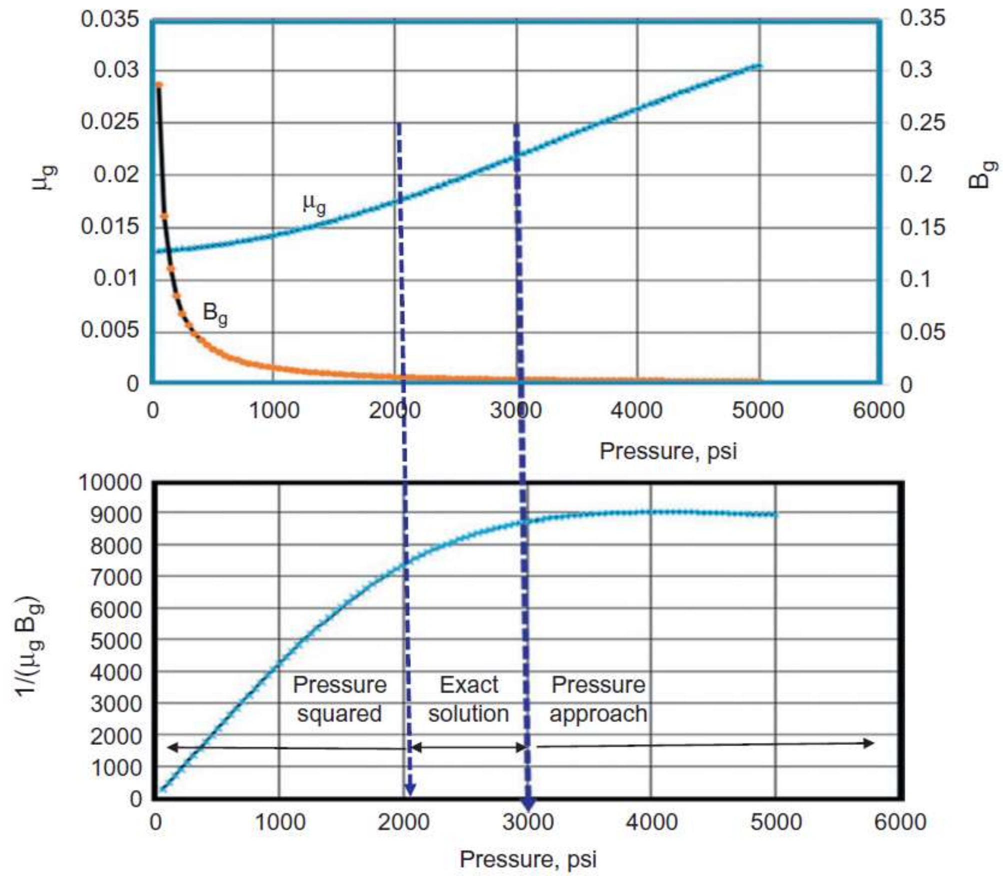
- The second method of approximation (*the first approximation was the Pressure-Squared Method*) to the exact solution of the radial flow of gases is to treat the gas as a *pseudoliquid.*
- Recalling the gas formation volume factor B_g as expressed in bbl/scf is given by: $$B_g = \left(\frac{p_{sc}}{5.615T_{sc}}\right)\left(\frac{zT}{p}\right)$$
- Solving the above expression for p/z gives: $$\frac{p}{z} = \left(\frac{Tp_{sc}}{5.615T_{sc}}\right)\left(\frac{1}{B_g}\right)$$
- The difference in the real gas pseudopressure is given by: $$m(p_i) - (p_{wf}) = \int_{p_{wf}}^{P_i} \frac{2p}{\mu z} dp$$
- Combining the above two expressions gives: $$m(p_i) - m(p_{wf}) = \frac{2Tp_{sc}}{5.615T_{sc}} \int_{p_{wf}}^{p_i} \left(\frac{1}{\mu \ B_g}\right) dp$$

$$m(p_i) - m(p_{wf}) = \frac{2Tp_{sc}}{5.615T_{sc}} \int_{p_{wf}}^{p_i} \left(\frac{1}{\mu \ B_g}\right) dp$$

- Fetkovich (1973) suggested that at high pressures (p > 3000), $1/\mu B_g$ is nearly constant.

- Imposing Fetkovich's condition on the above equation and integrating gives:

$$m(p_i) - m(p_{wf}) = \frac{2Tp_{sc}}{5.615T_{sc}\overline{\mu}\overline{B}_g}(p_i - p_{wf})$$



$1/(\mu_g B_g)$ vs. pressure

$$m(p_i) - m(p_{wf}) = \frac{2Tp_{sc}}{5.615T_{sc}\bar{\mu}\bar{B}_g}(p_i - p_{wf})$$

- Combining with Equations of the exact solution gives:

$$p_{wf} = p_i - \left(\frac{162.5 \times 10^3 Q_g \bar{\mu} \bar{B}_g}{kh}\right)\left[\log\left(\frac{kt}{\phi \bar{\mu} \bar{c}_t r_w^2}\right) - 3.23\right]$$

or

$$p_{wf} = p_i - \left(\frac{162.5\,(10^3)\,Q_g \bar{\mu} \bar{B}_g}{kh}\right)\left[\log\left(\frac{4\,t_D}{\gamma}\right)\right]$$

or equivalently in terms of dimensionless pressure drop:

$$p_{wf} = p_i - \left(\frac{141.2\,(10^3)\,Q_g \bar{\mu} \bar{B}_g}{kh}\right) p_D$$

where Q_g = gas flow rate, Mscf/day
k = permeability, md
$\bar{B}_g$ = gas formation volume factor, bbl/scf
t = time, hr
p_D = dimensionless pressure drop
t_D = dimensionless time

It should be noted that the gas properties, i.e., μ, B_g, and c_t, are evaluated at pressure $\bar{p}$ as defined below:

$$\bar{p} = \frac{p_i + p_{wf}}{2}$$

- Again, this method is only limited to applications above 3000 psi.
- *When solving for p_{wf}, it might be sufficient to evaluate the gas properties at p_i.*

Example

A gas well with a wellbore radius of 0.3 ft is producing at a constant flow rate of 2000 Mscf/day under transient flow conditions. The initial reservoir pressure (shut-in pressure) is 4400 psia at 140°F. The formation permeability and thickness are 65 md and 15 ft, respectively. The porosity is recorded as 15%. It is given that at the initial reservoir pressure: μ= 0.02831 cp, z= 0.896 and the initial total isothermal compressibility is 0.0003 1/psi. Calculate the bottom-hole flowing pressure after 1.5 hours.

Solution

$$t_D = \frac{0.000264kt}{\phi \mu c_t r_w^2}$$

$$t_D = \frac{(0.000264)(65)(1.5)}{(0.15)(0.02831)\left(3 \times 10^{-4}\right)\left(0.3^2\right)} = 224,498.6$$

Since $t_D > 100$, the p_D can be calculated by applying Equation: $p_D = 0.5[\ln(t_D) + 0.80907]$

$$p_D = 0.5[\ln(224498.6) + 0.80907] = 6.565$$

$$B_g = \left(\frac{p_{sc}}{5.615 T_{sc}}\right)\left(\frac{zT}{p}\right) = 0.00503\left(\frac{zT}{p}\right)$$

$$B_g = 0.00503\ \frac{(0.896)(600)}{4400} = 0.000615 \quad \text{bbl/scf}$$

$$p_{wf} = p_i - \left(\frac{141.2(10^3)Q_g\bar{\mu}\bar{B}_g}{kh}\right)p_D$$

$$p_{wf} = 4400 - \left[\frac{141.2 \times 10^3 (2000)(0.02831)(\ 0.000615\ )}{(65)(15)}\right]6.565$$

$$p_{wf} = 4367 \text{ psia}$$

THANK YOU