Lecture Six

5.2 Static and Flowing Bottom Hole Pressure

Often, the static or flowing pressure at the formation must be known in order to predict the productivity or absolute open flow potential of gas wells. The preferred method is to measure the pressure with a bottom-hole pressure gauge. It is often impractical or too expensive to measure static or flowing bottom-hole pressures with bottom-hole gauges. However, for many problems, a sufficiently precise value can be estimated from wellhead data (gas specific gravity, surface pressure and temperature, formation temperature, and well depth). Calculation of static (or shut-in) pressure amounts to evaluating the pressure difference equal to the weight of the column of gas. In the case of flowing wells, the gas column weight and friction effects must be evaluated and summed up.

Several methods are available for calculating static and flowing pressure drop in gas wells. The most widely used method is that of Cullender and Smith. All of the methods begin with Equation 5-15, with modifications for flow geometry. In most cases the acceleration gradient is ignored. Since it is frequently necessary to calculate the static bottom-hole pressure in a gas well, this procedure will be presented first.

5.2.1 Static Bottom-Hole Pressure

For a vertical (θ = 90°, sin θ = 1), shut-in (v = 0) gas well, Equation 5-15 becomes

$$\frac{dp}{dh} = \frac{g\,\rho_g}{g_c}, \qquad \ldots\ldots\ldots\ldots\ldots (5\text{-}17)$$

where

$$\rho_g = \frac{pM}{ZRT}.$$

Combining this with Equation (5.17),

$$\frac{dp}{p} = \frac{g\,M\,dh}{g_c Z\,R\,T}. \quad \ldots\ldots\ldots\ldots (5.18)$$

- **Average Pressure and Temperature Method:**

If Z is evaluated at average pressure and temperature in the in crement,

$$\int_{p_{ts}}^{p_{ws}} \frac{dp}{p} = \frac{g\,M}{g_c R\,\bar{Z}\,\bar{T}} \int_o^H dh,$$

from which

$$p_{ws} = p_{ts}\,\text{EXP}\left(\frac{g\,M\,H}{g_c R\,\bar{Z}\,\bar{T}}\right), \quad \ldots\ldots\ldots\ldots (5.19)$$

This equation holds for any consistent set of units. For conventional field units,

$$p_{ws} = p_{ts}\,\text{EXP}\,[(0.01875\,\gamma_g H)/(\bar{T}\bar{Z})] \quad \ldots\ldots\ldots\ldots (5.20)$$

where

p_{ws} = static or shut-in BHP, psia,
p_{ts} = static tubing pressure, psia,
γ_g = gas gravity (air = 1),
H = well depth, ft,
$\bar{T}$ = average temperature in the tubing, °R, and
$\bar{Z}$ = gas compressibility factor evaluated at $\bar{T}$, $\bar{p}$ = $(p_{ws} + p_{ts})/2$.

Evaluation of Ž makes the calculation iterative, and the procedure outlined previously can be used.

Example 5-2:

Using the following data, calculate p_{ws} with Equation 5-20.

H = 10,000 ft, γ_g = 0.6, p_{ts} = 4000 psia,
T_s = 70°F = 530°R, T_f = 220°F = 680°R

Solution:

A good first guess for P_{ws} can be obtained from

$$p^*_{ws} = p_{ts}\,(1 + 2.5 \times 10^{-5}\,H)$$

$$p^*_{ws} = 4000(1 + 2.5 \times 10^{-5}\,(10000)) = 5000 \text{ psia}$$

$$\bar{T} = \frac{T_s + T_f}{2} = \frac{530 + 680}{2} = 605°\text{R}$$

$$\bar{p} = \frac{p_{ts} + p^*_{ws}}{2} = \frac{4000 + 5000}{2} = 4500 \text{ psia}$$

$$Z = 0.932$$

$$p_{ws} = 4000 \text{ EXP } [(0.01875)(.6)(10000)/(605)Z]$$

$$p_{ws} = 4000 \text{ EXP } [0.18595/Z] = 4000 \text{ EXP } [0.18595/0.932]$$

$$p_{ws} = 4883$$

This is not close enough to the estimated value of 5000 psia. Set the calculated value of P_{ws} as the next estimated value and continue until convergence is reached.

p^*_{ws}	$\bar{p}$	$\bar{Z}$	p_{ws}
5000	4500	.932	4883
4883	4442	.928	4887
4887	4444	.928	4887

The calculation could also be made by estimating an initial value for Z and comparing calculated and estimated values until convergence on Z is obtained.

- **Cullender and Smith Method:**

The method presented by Cullender and Smith takes into account the variation of temperature with depth and the variation of Z with pressure and temperature. From Equation 5-18,

$$\int_{p_{ts}}^{p_{ws}} \frac{TZ}{p}\,dp = \frac{M}{R}\int_{o}^{H} dh = \frac{MH}{R} = 0.01875\,\gamma_g\,H.$$

The integral is written in short notation as

$$\int_{p_{ts}}^{p_{ws}} \frac{TZ}{p}\,dp = \int_{p_{ts}}^{p_{ws}} I\,dp = 0.01875\,\gamma_g\,H$$

Using a series expansion, the value of the integral is approximated by

$$2\int I\,dp = (p_{ms} - p_{ts})(I_{ms} + I_{ts}) + (p_{ws} - p_{ms})(I_{ws} + I_{ms}), \quad \ldots\ldots\ldots (5.21)$$

where

p_{ms} = pressure at mid-point of well, $H/2$,
I_{ms} = I evaluated at p_{ms}, $\bar{T}$,
I_{ts} = I evaluated at p_{ts}, T_s,
I_{ws} = I evaluated at p_{ws}, T_f.

The calculation procedure consists of dividing the well into two equal segments of length, H/2, finding the pressure p_{ms} at H/2 and using this value to calculate p_{ms}. I_{ts} can be evaluated from known surface conditions; that is,

$$p_{ms} = p_{ts} + \frac{.01875\,\gamma_g\,H}{I_{ms} + I_{ts}},$$

$$p_{ws} = p_{ms} + \frac{.01875\,\gamma_g\,H}{I_{ms} + I_{ws}}.$$

Example 5-3:

Work Example 5-2 using the Cullender and Smith method.

Solution:

Temperature at any depth $h = 70 + \frac{220 - 70}{10000} h =$
$70 + .015h$

Calculate I_{ts}:

At $T = 70$, $p = 4000$, $Z = .84$

$$I_{ts} = \frac{TZ}{p} = \frac{530(.84)}{4000} = 0.1113$$

Estimate p_{ms}:

$$p^*_{ms} = p_{ts}\,(1 + 2.5 \times 10^{-5}\,H/2)$$
$$= 4000\,(1 + 2.5 \times 10^{-5}\,(5000)) = 4500 \text{ psia}$$

$\bar{T} = 70 + .015(5000) = 145$

$Z = 0.93$

Calculate I_{ms}:

$$I_{ms} = \frac{\bar{T}Z}{p_{ms}} = \frac{605\,(.93)}{4500} = 0.1250$$

Calculate p_{ms}:

$$p_{ms} = p_{ts} + \frac{0.01875\,\gamma_g\,H}{I_{ms} + I_{ts}}$$

$$= 4000 + \frac{0.01875(.6)(10000)}{0.1250 + 0.1113}$$

$p_{ms} = 4000 + 476 = 4476$

This is not close enough to the estimated value of 4500 psia, therefore set $p^*_{ms} \doteq 4476$ and repeat.

At $\bar{T} = 145$, $p^*_{ms} = 4476$, $Z = .93$

$$I_{ms} = \frac{605\,(.93)}{4476} = 0.1257$$

$p_{ms} = 4000 + 475 = 4475$, which is close enough.

Estimate p_{ws}:

$$p^*_{ws} = p_{ms}\,(1 + 2.5 \times 10^{-5}\,H/2)$$
$$= 4475\,[1 + 5000(2.5 \times 10^{-5})]$$

$p^*_{ws} = 5034$ psia, $T = 220°F$, $Z = 1.006$

Calculate I_{ws}

$$I_{ws} = \frac{680(1.006)}{5034} = 0.1359$$

$$p_{ws} = p_{ms} + \frac{0.01875\,\gamma_g H}{I_{ms} + I_{ws}}$$

$$= 4475 + \frac{112.5}{0.1257 + 0.1359} = 4905 \text{ psia}$$

For the second trial, $Z = 0.998$

$$I_{ws} = \frac{680(.998)}{4905} = 0.1384$$

$$p_{ws} = 4475 + 426 = 4901 \text{ psia}$$

This compares to 4887 calculated using the average pressure and temperature method.

5.2.2 Flowing Bottom-Hole Pressure

For a flowing well the velocity is not zero, and ignoring acceleration, Equation 5-15 becomes, for a well inclined at an angle Ø from the vertical,

$$\frac{dp}{dL} = \frac{g}{g_c}\rho \cos \emptyset + \frac{f \rho v^2}{2 g_c d} \quad \text{………….…….. (5.22)}$$

Several methods have been presented for integrating Equation 5-22 depending on the assumptions made for handling temperature and Z-factor. Only the average pressure and temperature and Cullender and Smith methods will be discussed.

- **Average Pressure and Temperature Method:**

Substituting the expression for gas density in terms of p, T, and Z into Equation 5-22 results in

$$\frac{dp}{dL} = \frac{pM}{ZRT}\left(\cos \phi + \frac{f v^2}{2 g_c d}\right) \quad \text{……………. (5.23)}$$

Integration of Equation 5-23 assuming an average temperature in the flow string and evaluating Z at average conditions of pressure and temperature gives

$$p_{wf}^2 = p_{tf}^2 \, \text{EXP}(S) + \frac{25 \gamma_g q^2 \bar{T} \bar{Z} f (MD)(\text{EXP}(S) - 1)}{S d^5}, \quad \text{………… (5.24)}$$

where

p = psia,
$S = 0.0375 \gamma_g (TVD)/\bar{T}\bar{Z}$,
MD = measured depth, ft,
TVD = true vertical depth, ft,
$\bar{T}$ = °R,
q = MMscfd,
d = inches, and
$f = f(N_{Re}, \epsilon/d)$ (Jain or Colebrook equation)

The solution procedure is the same as for a shut-in well except for evaluation of the friction factor, which requires calculating a Reynolds number and estimating pipe roughness. Iteration is required since Z must be evaluated at $\bar{p} = (p_{tf} + p_{wf})/2$

Dividing the well into several length increments and using the procedure described earlier will give more ac- curate results. Actually, any of the methods will give identical results if the well is divided into short enough increments.

Convergence is sometimes obtained faster if iteration is performed on the Z-factor rather than the unknown pressure. The procedure for this method is:

1. Estimate Z* (A good first estimate is 0.9).

2. Calculate the unknown pressure using Equation 5-24 with Z = Z*

3. Calculate the average pressure, $\bar{p} = (P_{tf} + P_{wf})/2$.

4. Evaluate Z at $\bar{p}$ and $\bar{T}$.

5. Compare Z and Z*. If not close enough, set Z* = Z and go to Step 2. Repeat until abs(Z - Z*)/Z< 0.001 or any other tolerance preferred. When the tolerance is met, the pressure calculated in Step 2 is the correct value.

Example 5-3a:

Use the average pressure and temperature method to calculate the flowing bottom-hole pressure for the following directional well:

$$\gamma_g = 0.75, \quad MD = 10{,}000 \text{ ft}, \quad TVD = 7{,}000 \text{ ft}$$
$$T_s = 110°\text{F}, \quad T_f = 245°\text{F}, \quad p_{tf} = 2000 \text{ psia},$$
$$q_{sc} = 4.915 \text{ MMscfd}, \quad d = 2.441 \text{ in.}, \quad \epsilon = 0.0006 \text{ in.},$$
$$\bar{\mu} = 0.012 \text{ cp}$$

Solution:

In terms of mass flow rate, the Reynolds number is

$$N_{Re} = \frac{C\,\gamma_g\,q_{sc}}{\mu\,d} \qquad \text{.................. (5.25)}$$

Where

Variable	Units: Field	Units: SI
q_{sc} = gas flow rate	MMscfd	MM m³/day
γ_g = gas gravity	—	—
μ = gas viscosity	cp	kg/m-sec
d = pipe inside diameter	in.	m
C = constant	20011	17.96

$$N_{Re} = \frac{20011\ \gamma_g q_{sc}}{\bar{\mu}\ d} = \frac{20011(0.75)(4.915)}{0.012(2.441)}$$

$$= 2.518 \times 10^6$$

From Equation 5-14 $f = 0.015$

(1) Estimate $Z^* = 0.9$

$$S = \frac{0.0375(0.75)(7000)}{638\ Z^*} = \frac{0.3086}{Z^*}$$

(2) $$p_{wf}^2 = (2000)^2\ \text{EXP}\ (0.3086/Z^*) + \frac{25(.75)(4.915)^2(638)\ Z^*\ (.015)}{\frac{0.3086}{Z^*}(2.441)^5}$$

$$\frac{(10{,}000)[\text{EXP}\ (0.3086/Z^*) - 1]}{\frac{0.3086}{Z^*}(2.441)^5}$$

$$p_{wf}^2 = 4 \times 10^6\ \text{EXP}\ (0.3086/Z^*) + 1.621 \times 10^6\ (Z^*)^2\ (\text{EXP}(.3086/Z^*) - 1)$$

For $Z^* = 0.9$, $p_{wf}^2 = 5.636 \times 10^6 + 536{,}966$

$p_{wf}^2 = 6.173 \times 10^6$, $p_{wf} = 2485$ psia

(3) $$\bar{p} = (p_{tf} + p_{wf})/2 = \frac{2000 + 2485}{2} = 2242 \text{ psia}$$

(4) At $p = 2242$ psia and $T = 178°F$, $Z = 0.806$

(5) $$\frac{\text{abs}(Z - Z^*)}{Z} = \frac{0.9 - 0.806}{0.806} = 0.117,$$

which is too large.

(2)′ For $Z^* = 0.806$, $p_{wf}^2 = 5.866 \times 10^6 + 491{,}187$

$p_{wf}^2 = 6.357 \times 10^6$, $p_{wf} = 2521$ psia

(3)′ $$\bar{p} = \frac{2000 + 2521}{2} = 2261 \text{ psia}$$

(4)′ At $p = 2261$ psia and $T = 178°F$, $Z = 0.805$

(5)′ $$\frac{\text{abs}(Z - Z^*)}{Z} = \frac{\text{abs}\ (.805 - .806)}{.805} = 0.001,$$ which

is close enough.

Therefore, $p_{wf} = 2521$ psia.

• Cullender and Smith Method:

Derivation of the Cullender and Smith method for flowing wells begins with Equation 5-23. The following substitutions are made for velocity:

$$v = \frac{q}{A},$$

$$q = q_{sc}\frac{p_{sc}\,T\,Z}{T_{sc}\,pZ_{sc}},$$

which gives

$$\frac{dp}{dL} = \frac{pM\cos\emptyset}{ZRT} + \frac{MTZ\,p_{sc}^{2}\,f\,q_{sc}^{2}}{Rp\,T_{sc}^{2}\,2\,g_c d\,A^2}$$

or

$$\frac{p}{ZT}\frac{dp}{dh} = \frac{M}{R}\left[\left(\frac{p}{ZT}\right)^2\cos\emptyset + C\right],$$

where

$$C = \frac{8\,p_{sc}^{2}\,q_{sc}^{2}\,f}{T_{sc}^{2}\,g_c\,\pi^2\,d^5}$$

Which is constant for a given flow rate in a particular pipe size. Separating the variables gives:

$$\int_{p_{tf}}^{p_{wf}} \frac{\frac{p}{ZT}\,dp}{\left(\frac{p}{ZT}\right)^2\cos\emptyset + C} = \frac{M}{R}\int_{o}^{MD} dL, \quad \ldots\ldots\ldots\ldots (5\text{-}26)$$

Which is applicable for any consistent set of units. Substituting field units and integrating the right-hand side of Equation 5-26 gives:

$$\int_{p_{tf}}^{p_{wf}} \frac{\frac{p}{ZT}\,dp}{0.001\left(\frac{p}{ZT}\right)^2\frac{TVD}{MD} + F^2} = 18.75\,\gamma_g\,MD \quad \ldots\ldots\ldots\ldots\ldots (5\text{-}27)$$

where

$$F^2 = \frac{0.667\, f\, q_{sc}^{2}}{d^5}, \quad \text{…………..} (5.28)$$

and $\dfrac{TVD}{MD} = \cos\phi.$

Writing Equation 5-27 in short notation and dividing the well into two increments of length H/2 gives:

Upper half of well:

$$18.75\, \gamma_g (MD) = (p_{mf} - p_{tf})(I_{mf} + I_{tf}),$$

Lower half of well:

$$18.75\, \gamma_g (MD) = (p_{wf} - p_{mf})(I_{wf} + I_{mf}),$$

where

$$I = \frac{\dfrac{p}{TZ}}{0.001\left(\dfrac{p}{TZ}\right)^2 \dfrac{TVD}{MD} + F^2} \quad \text{………..} (5\text{-}29)$$

The solution procedure is similar to that for the static case, but is more involved because of the more complicated definition of I. For practical purposes, F can be considered a constant since the only variable in the Reynolds number used in evaluating f is gas viscosity. Viscosity is a function of pressure, but for simplification of the calculations it can be evaluated at $\bar{T}$ and the known pressure.

Example 5-4:

The following data pertain to a flowing gas well. Use the Cullender and Smith method to calculate the flowing bottom-hole pressure.

$\gamma_g = 0.75$, $H = 10{,}000$ ft, $T_s = 110°F$,
$T_f = 245°F$, $p_{tf} = 2000$ psia, $q_{sc} = 4.915$ MMscfd,
$d = 2.441$ in., $\epsilon = 0.0006$ in., $\bar{\mu} = 0.012$ cp
$\phi = 0°$

Solution:

Calculate f and F^2:

$$N_{Re} = \frac{20011(0.75)(4.915)}{0.012(2.441)} = 2.518 \times 10^6$$

From Equation 5-14 $f = 0.015$

$$F^2 = \frac{0.667\ (0.015)(4.915)^2}{(2.441)^5} = 0.00279$$

Calculate I_{tf}:

At $p = 2000$ psia, $T = 110°F$, $Z = 0.71$

$$\frac{p}{TZ} = \frac{2000}{(570)(.71)} = 4.942$$

$$I_{tf} = \frac{4.942}{(0.001)(4.942)^2 + 0.00279} = 181.60$$

Estimate p^*_{mf}: (First Trial)

$$p^*_{mf} = 2000\ (1 + 2.5 \times 10^{-5}(5000)) = 2250 \text{ psia}$$

Calculate I_{mf}:

At $p = 2250$, $T = 110 + 67.5 = 178$, $Z = 0.797$

$$\frac{p}{TZ} = \frac{2250}{(638)(.797)} = 4.425$$

$$I_{mf} = \frac{4.425}{(0.001)(4.425)^2 + 0.00279} = 197.81$$

Calculate p_{mf}:

$$p_{mf} = p_{tf} + \frac{18.75\ \gamma_g H}{I_{mf} + I_{tf}} = 2000 + \frac{18.75(.75)(10000)}{197.81 + 181.60}$$

$p_{mf} = 2000 + 371 = 2371$ (not close enough to p^*_{mf})

Calculate I_{mf}: (Second Trial)

At $p = 2371$, $T = 178$, $Z = 0.796$

$$\frac{p}{TZ} = \frac{2371}{638(.796)} = 4.669$$

$$I_{mf} = \frac{4.669}{0.001(4.669)^2 + 0.00279} = 189.88$$

Calculate p_{mf}:

$$p_{mf} = 2000 + \frac{140625}{189.88 + 181.60} = 2379$$

Calculate I_{mf}: (Third Trial)

At $p = 2379$, $T = 178$, $Z = 0.796$

$$\frac{p}{TZ} = \frac{2379}{638(.796)} = 4.684$$

$$I_{mf} = \frac{4.684}{0.001(4.684)^2 + 0.00279} = 189.41$$

Calculate p_{mf}:

$$p_{mf} = 2000 + \frac{140625}{189.41 + 181.60} = 2379 \text{ psia}$$

Therefore, the pressure at the mid-point of the well is 2379 psia. The value of p_{wf} is now calculated.
Estimate p^*_{wf}:

$$p^*_{wf} = 2379\ (1 + 2.5 \times 10^{-5}(5000)) = 2676$$

Calculate I_{wf}:

At $p = 2676$, $T = 245$, $Z = 0.867$

$$\frac{p}{TZ} = \frac{2676}{705(0.867)} = 4.378$$

$$I_{wf} = \frac{4.378}{(0.001)(4.378)^2 + 0.00279} = 199.39$$

Calculate p_{wf}: (First Trial)

$$p_{wf} = p_{mf} + \frac{140625}{199.39 + 189.41} = 2379 + 362 = 2741$$

Calculate I_{wf}: (Second Trial)

At $p = 2741$, $T = 245$, $Z = 0.868$

$$\frac{p}{TZ} = \frac{2741}{705(0.868)} = 4.479$$

$$I_{wf} = \frac{4.479}{(0.001)(4.479)^2 + 0.00279} = 196.00$$

Calculate p_{wf}:

$$p_{wf} = 2379 + \frac{140625}{196.00 + 189.41} = 2744 \text{ psia}$$

This is close enough to the previously calculated value of 2741 psia. Therefore, the flowing bottom-hole pressure is 2744 psia.