FIGURE 4-15   Effect of permeability on water saturation profile. *(After Cole, F., 1969).*

Example 4-6

A four-layer oil reservoir is characterized by a set of reservoir capillary pressure-saturation curves as shown in Figure 4-16. The following additional data are also available.

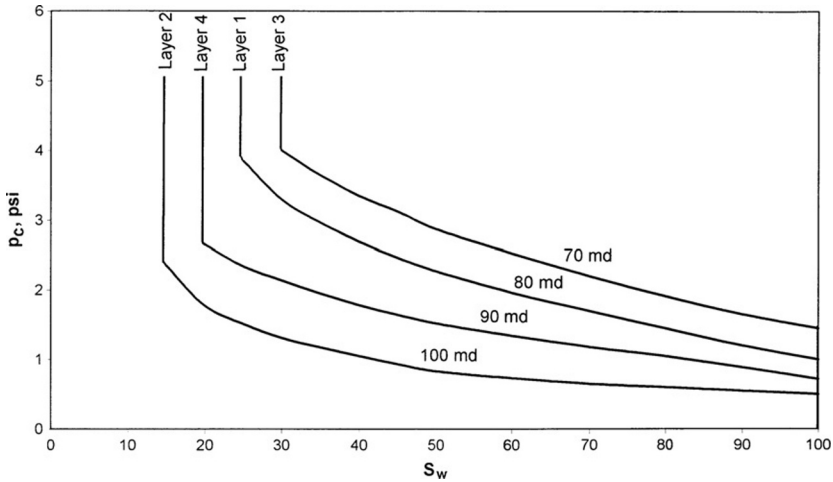| Layer	Depth, ft	Permeability, md
1	4000–4010	80
2	4010–4020	100
3	4020–4035	70
4	4035–4060	90

FIGURE 4-16 Variation of p_c with k.

WOC $= 4,060$ ft
Water density $= 65.2$ lb/ft^3
Oil density $= 55.2$ lb/ft^3

Calculate and plot water saturation versus depth for this reservoir.

Solution

Step 1. Establish the FWL by determining the displacement pressure p_d for the bottom layer, i.e., Layer 4, and apply Equation 4-37:

○ $p_d = 0.75$ psi

$$FWL = 4,060 + \frac{(144)(0.75)}{(65.2 - 55.2)} = 4,070.8 \, ft$$

Step 2. The top of the bottom layer is located at a depth of 4,035 ft, which is 35.8 ft above the FWL. Using that height h of 35.8 ft, calculate the capillary pressure at the top of the bottom layer.

$$p_c = \left(\frac{h}{144}\right)\Delta\rho = \left(\frac{35.8}{144}\right)(65.2 - 55.2) = 2.486 \, psi$$

○ From the capillary pressure-saturation curve designated for Layer 4, read the water saturation that corresponds to a p_c of 2.486 to give $S_w = 0.23$.
○ Assume different values of water saturations and convert the corresponding capillary pressures into height above the FWL by applying Equation 4-34.

$$h = \frac{144 \, p_c}{\rho_w - \rho_o}$$

S_w	P_c, psi	h, ft	Depth = FWL – h
0.23	2.486	35.8	4035
0.25	2.350	33.84	4037
0.30	2.150	30.96	4040
0.40	1.800	25.92	4045
0.50	1.530	22.03	4049
0.60	1.340	19.30	4052
0.70	1.200	17.28	4054
0.80	1.050	15.12	4056
0.90	0.900	12.96	4058

Step 3. The top of Layer 3 is located at a distance of 50.8 ft from the FWL (i.e., $h = 4{,}070.8 – 4{,}020 = 50.8$ ft). Calculate the capillary pressure at the top of the third layer:

-
$$p_c = \left(\frac{50.8}{144}\right)(65.2 - 55.2) = 3.53\,\text{psi}$$

- The corresponding water saturation as read from the curve designated for Layer 3 is 0.370.
- Construct the following table for Layer 3.

S_w	p_c, psi	h, ft	Depth = FWL – h
0.37	3.53	50.8	4020
0.40	3.35	48.2	4023
0.50	2.75	39.6	4031
0.60	2.50	36.0	4035

Step 4. ○ Distance from the FWL to the top of Layer 2 is:

$$h = 4{,}070.8 – 4{,}010 = 60.8\,\text{ft}$$

- $p_c = \left(\dfrac{60.8}{144}\right)(65.2 - 55.2) = 4.22\,\text{psi}$
- S_w at p_c of 4.22 psi is 0.15.
- Distance from the FWL to the bottom of the layer is 50.8 ft that corresponds to a p_c of 3.53 psi and S_w of 0.15. This indicates that the second layer has a uniform water saturation of 15%.

Step 5. For Layer 1, distance from the FWL to the top of the layer:

- $h = 4{,}070.8 – 4{,}000 = 70.8$ ft
- $p_c = \left(\dfrac{70.8}{144}\right)(10) = 4.92\,\text{psi}$

- S_w at the top of Layer 1 = 0.25
- The capillary pressure at the bottom of the layer is 3.53 psi with a corresponding water saturation of 0.27.

Step 6. Figure 4-17 documents the calculated results graphically. The figure indicates that Layer 2 will produce 100% oil while all remaining layers produce oil and water simultaneously.
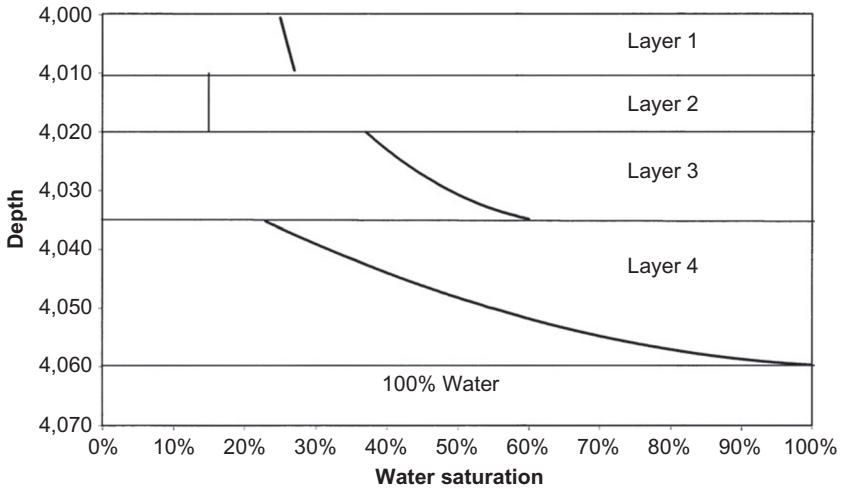
FIGURE 4-17 Water saturation profile.

Leverett J-Function

Capillary pressure data are obtained on small core samples that represent an extremely small part of the reservoir and, therefore, it is necessary to combine all capillary data to classify a particular reservoir. The fact that the capillary pressure-saturation curves of nearly all naturally porous materials have many features in common has led to attempts to devise some general equation describing all such curves. Leverett (1941) approached the problem from the standpoint of dimensional analysis.

Realizing that capillary pressure should depend on the porosity, interfacial tension, and mean pore radius, Leverett defined the dimensionless function of saturation, which he called the J-function, as

$$J(S_w) = 0.21645 \frac{p_c}{\sigma} \sqrt{\frac{k}{\phi}} \qquad (4\text{-}36)$$

where

$J(S_w)$ = Leverett J-function
p_c = capillary pressure, psi
σ = interfacial tension, dynes/cm
k = permeability, md
ϕ = fractional porosity

In doing so, Leverett interpreted the ratio of permeability, k, to porosity, ϕ, as being proportional to the square of a mean pore radius.

The J-function was originally proposed as a means of converting all capillary-pressure data to a universal curve. There are significant differences in correlation of the J-function with water saturation from formation to
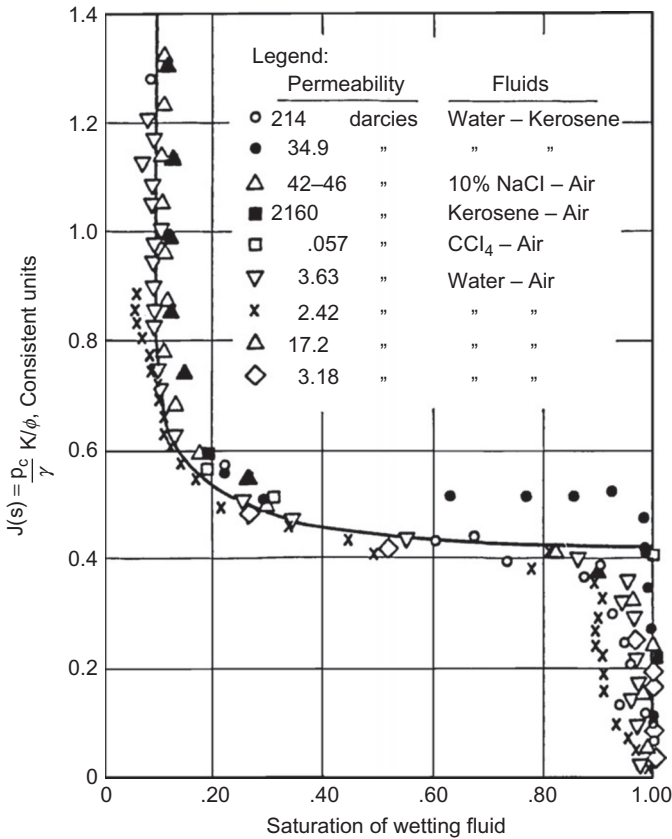
FIGURE 4-18 The Leverett J-function for unconsolidated sands. *(After Leverett, 1941.)*

formation, so that no universal curve can be obtained. For the same formation, however, this dimensionless capillary-pressure function serves quite well in many cases to remove discrepancies in the p_c versus S_w curves and reduce them to a common curve. This is shown for various unconsolidated sands in Figure 4-18.

Example 4-7

A laboratory capillary pressure test was conducted on a core sample taken from the Nameless Field. The core has a porosity and permeability of 16% and 80 md, respectively. The capillary pressure-saturation data are given as follows:

S_w	p_c, psi
1.0	0.50
0.8	0.60
0.6	0.75
0.4	1.05
0.2	1.75

The interfacial tension is measured at 50 dynes/cm. Further reservoir engineering analysis indicated that the reservoir is better described at a porosity value of 19% and an absolute permeability of 120 md. Generate the capillary pressure data for the reservoir.

Solution

Step 1. Calculate the J-function using the measured capillary pressure data.

$$J(S_w) = 0.21645 (p_c/50) \sqrt{80/0.16} = 0.096799 \, p_c$$

S_w	p_c, psi	$J(S_w) = 0.096799 \, (p_c)$
1.0	0.50	0.048
0.8	0.60	0.058
0.6	0.75	0.073
0.4	1.05	0.102
0.2	1.75	0.169

Step 2. Using the new porosity and permeability values, solve Equation 4-36 for the capillary pressure p_c.

$$p_c = J(S_w)\sigma / \left[0.21645 \sqrt{\frac{k}{\phi}} \right]$$

$$p_c = J(S_w)50 / \left[0.21645 \sqrt{\frac{120}{0.19}} \right]$$

$$p_c = 9.192 \, J(S_w)$$

Step 3. Reconstruct the capillary pressure-saturation table.

S_w	$J(S_w)$	$p_c = 9.192 \, J(S_w)$
1.0	0.048	0.441
0.8	0.058	0.533
0.6	0.073	0.671
0.4	0.102	0.938
0.2	0.169	1.553

Converting Laboratory Capillary Pressure Data

For experimental convenience, it is common in the laboratory determination of capillary pressure to use air-mercury or air-brine systems, rather than the actual water-oil system characteristic of the reservoir. Since the laboratory fluid system does not have the same surface tension as the reservoir system, it becomes necessary to convert laboratory capillary pressure to reservoir capillary pressure. By assuming that the Leverett J-function is a property of rock and does not change from the laboratory to the reservoir, we can calculate reservoir capillary pressure as show below.

$$(p_c)_{res} = (p_c)_{lab} \frac{\sigma_{res}}{\sigma_{lab}}$$

Even after the laboratory capillary pressure has been corrected for surface tension, it may be necessary to make further corrections for permeability and porosity. The reason for this is that the core sample that was used in performing the laboratory capillary pressure test may not be representative of the average reservoir permeability and porosity. If we assume that the J-function will be invariant for a given rock type over a range of porosity and permeability values, then the reservoir capillary pressure can be expressed as

$$(p_c)_{res} = (p_c)_{lab} \frac{\sigma_{res}}{\sigma_{lab}} \sqrt{(\phi_{res} k_{core})/(\phi_{core} k_{res})} \qquad (4\text{-}37)$$

where

$(p_c)_{res}$ = reservoir capillary pressure
σ_{res} = reservoir surface or interfacial tension
k_{res} = reservoir permeability
ϕ_{res} = reservoir porosity
$(p_c)_{lab}$ = laboratory measured capillary pressure
ϕ_{core} = core porosity
k_{core} = core permeability

PERMEABILITY

Permeability is a property of the porous medium that measures the capacity and ability of the formation to transmit fluids. The rock permeability, k, is a very important rock property because it controls the directional movement and the flow rate of the reservoir fluids in the formation. This rock characterization was first defined mathematically by Henry Darcy in 1856. In fact, the equation that defines permeability in terms of measurable quantities is called **Darcy's Law.**

Darcy developed a fluid flow equation that has since become one of the standard mathematical tools of the petroleum engineer. If a horizontal linear flow of an incompressible fluid is established through a core sample of length L and a cross-section of area A, then the governing fluid flow equation is defined as

$$v = -\frac{k}{\mu} \frac{dp}{dL} \qquad (4\text{-}38)$$

where

v = apparent fluid flowing velocity, cm/sec
k = proportionality constant, or permeability, Darcy's
μ = viscosity of the flowing fluid, cp
dp/dL = pressure drop per unit length, atm/cm

The velocity, v, in Equation 4-38 is not the actual velocity of the flowing fluid but is the apparent velocity determined by dividing the flow rate by the cross-

sectional area across which fluid is flowing. Substituting the relationship, q/A, in place of v in Equation 4-38 and solving for q results in

$$q = -\frac{kA}{\mu}\frac{dp}{dL}$$

(4-39)

where

q = flow rate through the porous medium, cm³/sec
A = cross-sectional area across which flow occurs, cm²

With a flow rate of one cubic centimeter per second across a cross-sectional area of one square centimeter with a fluid of one centipoise viscosity and a pressure gradient at one atmosphere per centimeter of length, it is obvious that k is unity. For the units described above, k has been arbitrarily assigned a unit called *Darcy* in honor of the man responsible for the development of the theory of flow through porous media. Thus, when all other parts of Equation 4-39 have values of unity, k has a value of one Darcy.

One Darcy is a relatively high permeability as the permeabilities of most reservoir rocks are less than one Darcy. In order to avoid the use of fractions in describing permeabilities, the term *millidarcy* is used. As the term indicates, one millidarcy, i.e., 1 md, is equal to one-thousandth of one Darcy or,

1 Darcy = 1000 md

The negative sign in Equation 4-39 is necessary as the pressure increases in one direction while the length increases in the opposite direction.

Equation 4-39 can be integrated when the geometry of the system through which fluid flows is known. For the simple linear system shown in Figure 4-19, the integration is performed as follows:

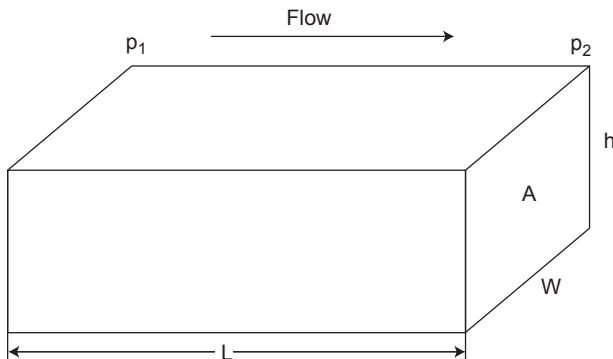$$q \int_{0}^{L} dL = -\frac{kA}{\mu} \int_{p_1}^{p_2} dp$$

FIGURE 4-19   Linear flow model.

Integrating the above expression yields:

$$qL = -\frac{kA}{\mu}(p_2 - p_1)$$

It should be pointed out that the volumetric flow rate, q, is constant for liquids because the density does not change significantly with pressure.

Since p_1 is greater than p_2, the pressure terms can be rearranged, which will eliminate the negative term in the equation. The resulting equation is:

$$q = \frac{kA(p_1 - p_2)}{\mu L} \tag{4-40}$$

Equation 4-40 is the conventional linear flow equation used in fluid flow calculations.

Standard laboratory analysis procedures will generally provide reliable data on permeability of core samples. If the rock is not homogeneous, the whole core analysis technique will probably yield more accurate results than the analysis of core plugs (small pieces cut from the core). Procedures that have been used for improving the accuracy of the permeability determination include cutting the core with an oil-base mud, employing a pressure-core barrel, and conducting the permeability tests with reservoir oil.

Permeability is reduced by overburden pressure, and this factor should be considered in estimating permeability of the reservoir rock in deep wells because permeability is an isotropic property of porous rock in some defined regions of the system, that is, it is directional. Routine core analysis is generally concerned with plug samples drilled parallel to bedding planes and, hence, parallel to direction of flow in the reservoir. These yield horizontal permeabilities (k_h).

The measured permeability on plugs that are drilled perpendicular to bedding planes are referred to as vertical permeability (k_v). Figure 4-20 shows a schematic illustration of the concept of the core plug and the associated permeability.

As shown in Figure 4-20, there are several factors that must be considered as possible sources of error in determining reservoir permeability. These factors are:

1. Core sample may not be representative of the reservoir rock because of reservoir heterogeneity.
2. Core recovery may be incomplete.
3. Permeability of the core may be altered when it is cut, or when it is cleaned and dried in preparation for analysis. This problem is likely to occur when the rock contains reactive clays.
4. Sampling process may be biased. There is a temptation to select the best parts of the core for analysis.

Permeability is measured by passing a fluid of known viscosity p through a core plug of measured dimensions (A and L) and then measuring flow rate q and pressure drop Δp. Solving Equation 4-40 for the permeability, gives:

$$k = \frac{q \mu L}{A \Delta p}$$

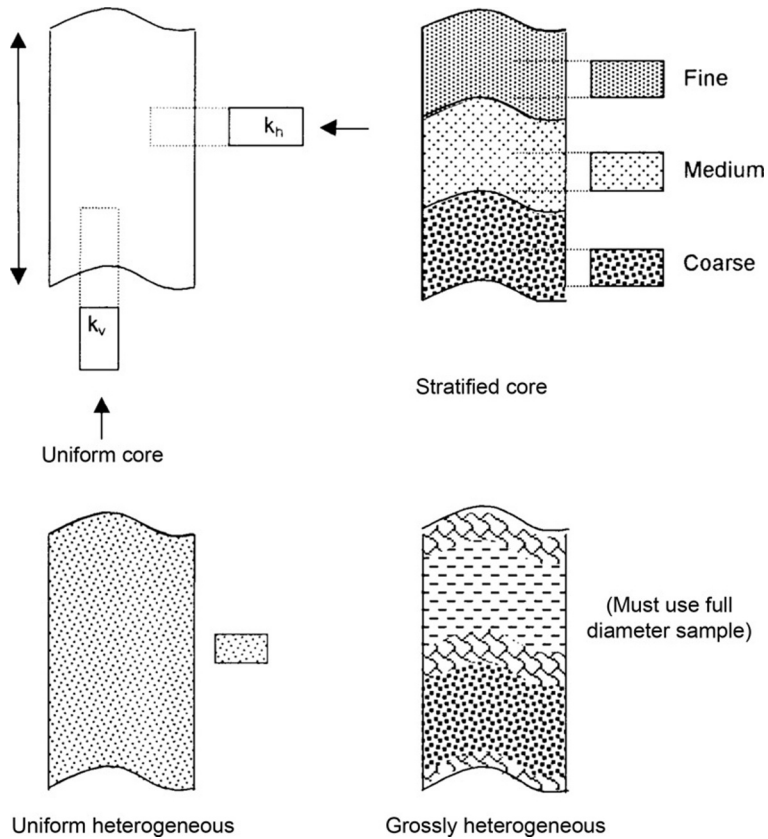**FIGURE 4-20**   Representative samples of porous media.

where

L = length of core, cm
A = cross-sectional area, cm^2

The following conditions must exist during the measurement of permeability:

○   Laminar (viscous) flow
○   No reaction between fluid and rock
○   Only single phase present at 100% pore space saturation

This measured permeability at 100% saturation of a single phase is called the *absolute permeability* of the rock.

Example 4-8

A brine is used to measure the absolute permeability of a core plug. The rock sample is 4 cm long and 3 cm^2 in cross section. The brine has a viscosity of 1.0 cp and is flowing a constant rate of 0.5 cm^3/sec under a 2.0 atm pressure differential. Calculate the absolute permeability.

Solution

Applying Darcy's equation, i.e., Equation 4-40, gives:

$$0.5 = \frac{(k)\,(3)\,(2)}{(1)\,(4)}$$

$$k = 0.333\,\text{Darcys}$$

Example 4-9

Rework the above example assuming that an oil of 2.0 cp is used to measure the permeability. Under the same differential pressure, the flow rate is 0.25 cm³/sec.

Solution

Applying Darcy's equation yields:

$$0.5 = \frac{(k)\,(3)\,(2)}{(1)\,(4)}$$

$$k = 0.333\,\text{Darcys}$$

Dry gas is usually used (air, N_2, He) in permeability determination because of its convenience, availability, and to minimize fluid-rock reaction.

The measurement of the permeability should be restricted to the low (laminar/viscous) flow rate region, where the pressure remains proportional to flow rate within the experimental error. For high flow rates, Darcy's equation as expressed by Equation 4-40 is inappropriate to describe the relationship of flow rate and pressure drop.

In using dry gas in measuring the permeability, the gas volumetric flow rate q varies with pressure because the gas is a highly compressible fluid. Therefore, the value of q at the average pressure in the core must be used in Equation 4-40. Assuming the used gases follow the ideal gas behavior (at low pressures), the following relationships apply:

$$p_1 V_1 = p_2 V_2 = p_m V_m$$

In terms of the flow rate q, the above equation can be equivalently expressed as:

$$p_1 q_1 = p_2 q_2 = p_m q_m \qquad (4\text{-}41)$$

with the mean pressure p_m expressed as:

$$p_m = \frac{p_1 + p_2}{2}$$

where

p_1, p_2, p_m = inlet, outlet, and mean pressures, respectively, atm
V_1, V_2, V_m = inlet, outlet, and mean gas volume, respectively, cm³